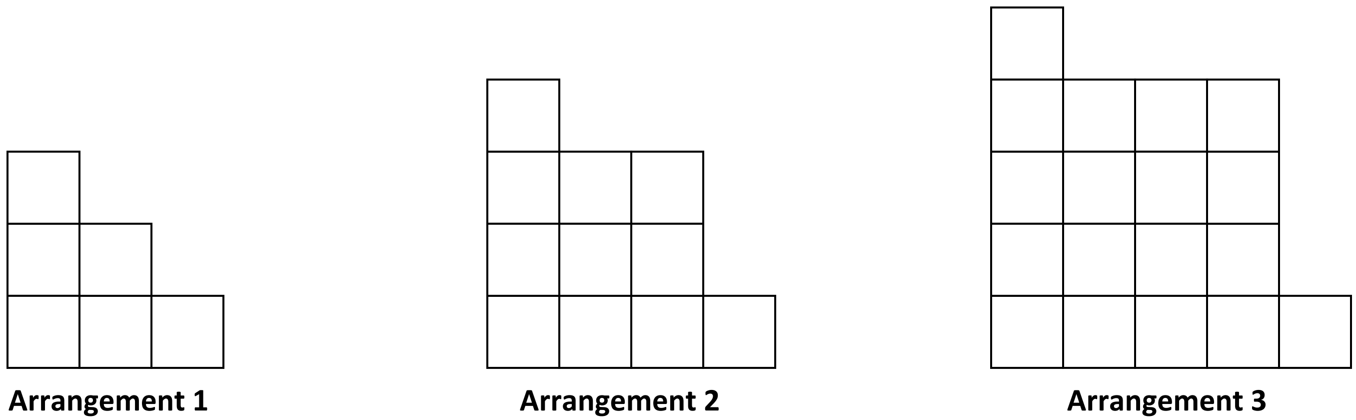
Task: Will and Latisha's Tile Problem | **Algebra I**

In math class, Will and Latisha were challenged to create their own pattern with tiles. Latisha built the first three arrangements of their pattern as shown below. Both Will and Latisha decided to investigate continuing the pattern.



A. Will and Latisha wondered how many tiles would be needed to build each of the arrangements below. If the pattern continued, determine how many tiles would be needed for each arrangement and show how you determined your answer.

5^{th} Arrangement

10^{th} Arrangement

101^{st} Arrangement

B. They decided that it would be much more efficient if they could use what they know to generalize a way to find the number of tiles needed for any arrangement. Write a function rule that could be used to find the number of tiles needed to build any arrangement in this pattern. Show how your function rule relates to the arrangements.

C. Will claims that there is an arrangement in the pattern that would take 56,646 tiles to build. Is he correct? Use mathematics to justify your answer.

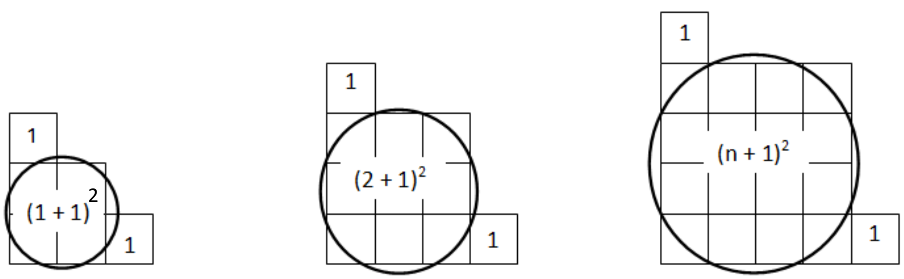
Teacher Notes

- It is important for you as the teacher to do the math prior to using this task with your students. You will need to make sense of the various solution paths in order to anticipate student misconceptions and ideas.
- Multiple tools will need to be made available for students to make sense of the problem. Some to consider are: graph or grid paper, pencil-and-paper, color tiles, and graphing calculator/software. Although color tiles are not necessary, the kinesthetic practice of building the arrangements may help some students. This can be simulated by having students draw the arrangements. Although the use of the graphing calculator makes the task richer, students can work this problem at least three different ways without a graphing calculator.
- It will be very important to have students make sense of the visual arrangement models and how they relate to their expressions before moving to calculations. This will help students visualize their expressions and add to the concept of equivalence. Make sure that multiple visual representations occur before moving to calculations.
- This task lends itself to important mathematical discussions that may not be a part of the mathematical goal for this lesson. As an extension, some discussions that could occur include properties, equivalence, and how Completing the Square and the Quadratic Formula are related (this would fully address standard REI.B.4 a).

Common Core State Standards for Mathematical Content	**Common Core State Standards for Mathematical Practice**
A–CED.A.1 Create equations ~~and inequalities~~ in one variable and use them to solve problems. *Include equations arising from linear and quadratic functions, and simple rational and exponential functions.* A–CED.A.3 Represent constraints by equations ~~or inequalities, and by systems of equations and/or inequalities~~, and interpret solutions as viable or nonviable options in a modeling context. *For example, represent inequalities describing nutritional and cost constraints on combinations of different foods.* A–REI.B.4 Solve quadratic equations in one variable.	1. Make sense of problems and persevere in solving them. 2. Reason abstractly and quantitatively. 3. Construct viable arguments and critique the reasoning of others. 4. Model with mathematics. 5. Use appropriate tools strategically. 6. Attend to precision. 7. Look for and make use of structure. 8. Look for and express regularity in repeated reasoning.

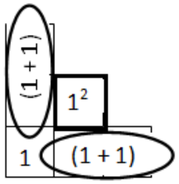
Essential Understandings

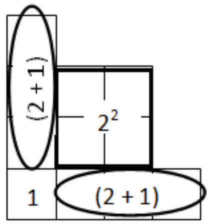
- Quadratic equations can be used to model scenarios.
- Solutions to a quadratic equation must be considered viable based on the domain and context of the function.
- There are multiple methods to solve quadratic equations. The efficiency and accuracy of each method will vary based on the tools available and the solutions represented by the equation.
- Equivalent expressions can be simplified to the same form.

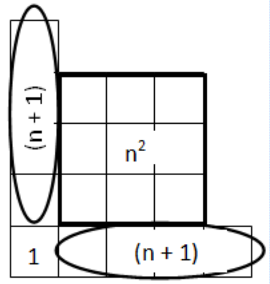
Explore Phase	
Possible Solution Paths	**Assessing and Advancing Questions**
Part A. 5^{th} Arrangement: 38 10^{th} Arrangement: 123 101^{st} Arrangement: 10,406 Some students may just build with tiles or draw the arrangement on graph paper and count the tiles. (5^{th} and 10^{th}) Students may use various calculations based on how they visualize the arrangements. These concrete calculations can lead to Part B (the general case).	Assessing Questions: • Build the next arrangement. Build the next arrangement (5^{th}). How did you know to build the arrangement that way? • Describe how you found the number of tiles. What do you notice about the relationship between the arrangement number and the number of tiles in the arrangement? Advancing Questions: • How could you use what you know about the 5^{th} and 10^{th} arrangements to find the number of tiles for the 20^{th} arrangement? 50^{th} arrangement? 101^{st} arrangement? • What do you notice is changing and what is staying the same about each arrangement?
Part B. Below are some possible ways that students may be thinking about their function rule. 1. Partitioned: $(n + 1)^2 + 2$ n = number of arrangement  $(1+ 1)^2 + 2$ $(2+ 1)^2 + 2$ $(n + 1)^2 + 2$	Assessing Questions: • Show me that your expression works for the known arrangements. • Describe how you are visualizing the partitions of the arrangements. Why did you decide to partition your drawing this way? • Where is the $(n + 1)^2$ in your arrangement? Where is the 2? Advancing Questions: • How could you partition the arrangements differently? What expression would represent this? • How does your new expression relate to your first expression?

2. Partitioned: $n^2 + 2(n + 1) + 1$

n= number of arrangement







$(1)^2 + (1+1) + (1+1) + 1$

Or

$(1)^2 + 2(1+1) + 1$

$(2)^2 + (2+1) + (2+1) + 1$

$(2)^2 + 2(2+1) + 1$

$(n)^2 + (n+1) + (n+1) + 1$

$(n)^2 + 2(n+1) + 1$

Assessing Questions:

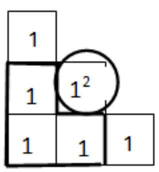
- Show me that your expression works for the known arrangements.
- Describe how you are visualizing the partitions of the arrangements. Why did you decide to partition your drawing this way?
- Where is the n^2 in your arrangement? Where is the $2(n + 1)$? Where is the 1?

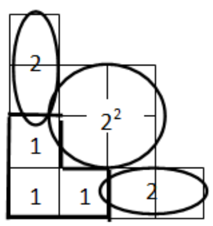
Advancing Questions:

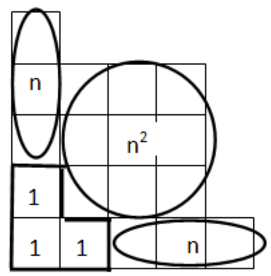
- How could you partition the arrangements differently? What expression would represent this?
- How does your new expression relate to your first expression?

3. Partitioned: $n^2 + 2n + 3$

n = number of arrangement







$(1)^2 + (1+1) + (1+1) + 3$

Or

$(1)^2 + 2(1) + 3$

$(2)^2 + (2+2) + 3$

$(2)^2 + 2(2) + 3$

$(n)^2 + (n + n) + 3$

$(n)^2 + 2(n) + 3$

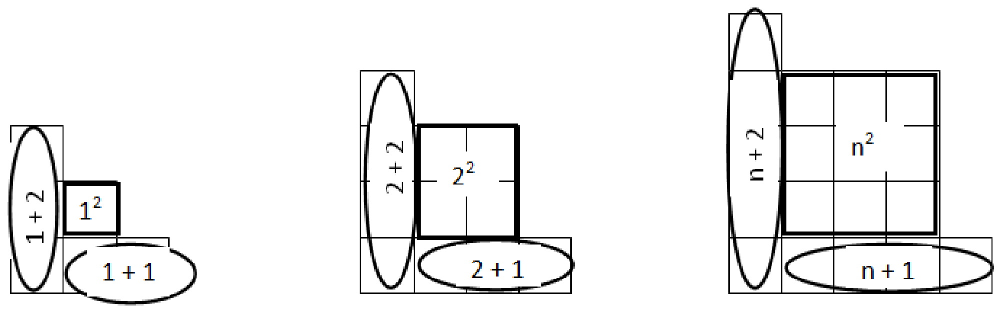
Assessing Questions:

- Show me that your expression works for the known arrangements.
- Describe how you are visualizing the partitions of the arrangements. Why did you decide to partition your drawing this way?
- Where is the n^2 in your arrangement? Where is the 2n? Where is the 3?

Advancing Questions:

- How could you partition the arrangements differently? What expression would represent this?
- How does your new expression relate to your first expression?

4. Partitioned: $n^2 + (n + 1) + (n + 2)$



$(1)^2 + (1 + 1) + (1 + 1)$ $(2)^2 + (2 + 1) + (2 + 2)$ $n^2 + (n + 1) + (n + 2)$

Assessing Questions:

- Show me that your expression works for the known arrangements.
- Describe how you are visualizing the partitions of the arrangements. Why did you decide to partition your drawing this way?
- Where is the n^2 in your arrangement? Where is the $n + 1$? Where is the $n + 2$?

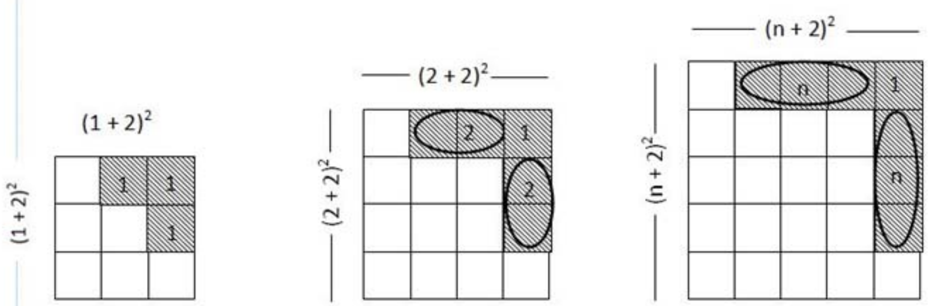
Advancing Questions:

- How could you partition the arrangements differently? What expressions would represent this?
- How does your new expression relate to your first expression?

5. Add tiles to complete the square and then subtract the added tiles.

$(n + 2)^2 - (2n + 1)$

n = number of arrangement



$(1+2)^2 - (1+1+1)$ $(2 + 2)^2 - (2+2+1)$ $(n + 2)^2 - (n + n + 1)$

Or

$(1 + 2)^2 - (2 \bullet 1 + 1)$ $(2 + 2)^2 - (2 \bullet 2 + 1)$ $(n+2)^2 - (2n+1)$

Assessing Questions:

- Show me that your expression works for the known arrangements.
- Describe how you are visualizing the partitions of the arrangements. Why did you decide to partition your drawing this way?
- How did adding tiles to your arrangement help you make sense of the pattern?
- Why did you subtract tiles?
- Where is the $(n + 2)^2$ in your arrangement? Where is the $2n + 1$?
- Why did you group the $2n + 1$?

Advancing Questions:

- How could you partition the arrangements differently? What expression would represent this?
- How does your new expression relate to your first expression?

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Part C.
Below are several methods that students may use to solve the problem.

1. Completing the Square – Note the connections to some of the representations.

$n^2 + 2n + 3 = 56,646$
$n^2 + 2n = 56,643$
$n^2 + 2n + 1 = 56,643 + 1$
$n^2 + 2n + 1 = 56,644$
$(n + 1)^2 = 56,644$
$\sqrt{(n + 1)^2} = \sqrt{56,644}$
$n + 1 = \pm 238$
$n = 237$
$n = -239$
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Assessing Questions:
• Why did you decide to use Completing the Square?
• Talk me through how you solved the problem.
• Why are there two potential solutions to this problem?
• Given the scenario, which value for n would make sense? Why?

Advancing Questions:
• How could you work the problem a different way?
• How would you know which method to use?
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2. Quadratic Formula

$n^2 + 2n + 3 = 56,646$
$n^2 + 2n - 56,643 = 0$
$n = \frac{-2 \pm \sqrt{2^2 - 4(1)(-56,643)}}{2}$
$n = \frac{--2 \pm 476}{2}$
$n = -239$
$n = 237$
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Assessing Questions:
• Why did you decide to use the Quadratic Formula?
• What would be the values for *a*, *b*, and *c*?
• Talk me through how you solved the problem.
• Why are there two potential solutions to this problem?
• Given the scenario, which value for *n* would make sense? Why?

Advancing Questions:
• How could you work the problem a different way?
• How would you know which method to use?
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3. Some may use the generated function rule of $(n + 1)^2 + 2$ and solve by Taking the Square Root.

$$(n+1)^2 + 2 = 56,646$$

$$(n+1)^2 = 56,644$$

$$\sqrt{(n+1)} = \sqrt{56,644}$$

$$n + 1 = \pm 238$$

$$n = -239$$

$$n = 237$$

Assessing Questions:

- Why did you decide to use this form of the equation?
- Talk me through how you solved the problem.
- Why are there two potential solutions to this problem?
- Given the scenario, which value for *n* would make sense? Why?

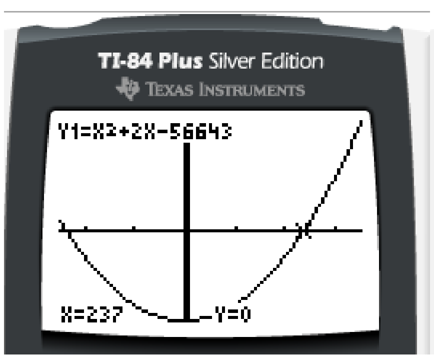
Advancing Questions:

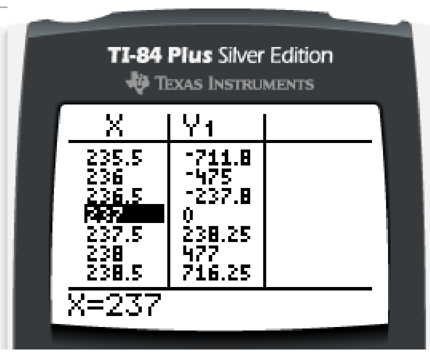
- How could you work the problem a different way?
- How would you know which method to use?

4. Graph or Table – finding the x-intercepts (roots)

$n^2 + 2n + 3 = 56,646$

$n^2 + 2n - 56,643 = 0$





Assessing Questions:

- Why did you decide to use graphing to solve this problem? Or, why did you decide to use a table to solve this problem?
- Why did you decide to use this equation?
- How did you use your calculator to solve this problem? Show me the process.
- What does the x-value represent in the graph/table? What does the x-value represent in the scenario?
- Why are there two potential solutions to this problem?
- Given the scenario, which value of x would make sense? Why?

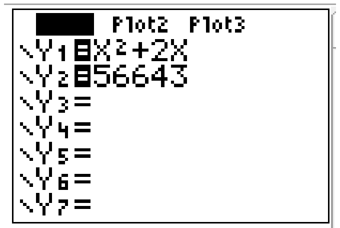
Advancing Questions:

- What if you decided to use $x^2 + 2x = 56,643$?
  a. How would this change the graph/table?
  b. How are the graphs related?
- How could you work the problem another way?
- How would you know which method to use?

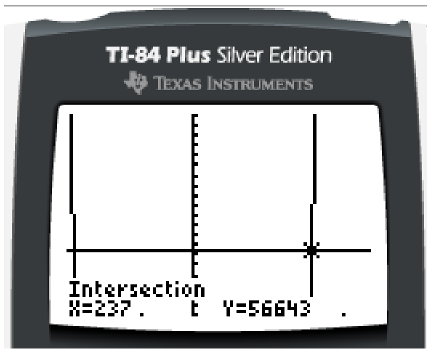
5. Graphing or Table – finding the intersection

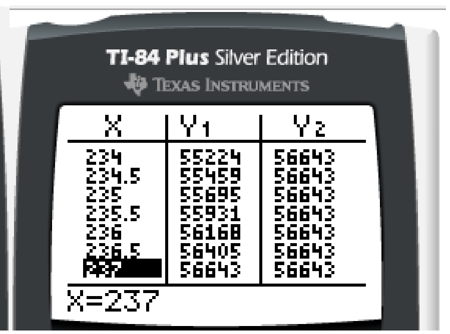
$n^2 + 2n + 3 = 56{,}646$

$n^2 + 2n = 56{,}643$



Equation





Assessing Questions:

- Why did you decide to use graphing to solve this problem? Or, why did you decide to use a table to solve this problem?
- Why did you decide to use this equation?
- How did you use your calculator to solve this problem? Show me the process.
- What does the x-value represent in the graph/table? What does the x-value represent in the scenario?
- Why are there two potential solutions to this problem?
- Given the scenario, which value of x would make sense? Why?

Advancing Questions:

- What if you decided to use $x^2 + 2x – 56{,}643 = 0$?
  - c. How would this change the graph/table?
  - d. How are the graphs related?
- How could you work the problem another way?
- How would you know which method to use?

Possible Student Misconceptions

1. The function rule that students generate may work for a specific arrangement but not in the general case.

Assessing Questions:

- Have students test their expression with the known (built) arrangements. Does your function rule work? Why or why not?

Advancing Question:

- Have students show how each part of the expression relates to the arrangement drawing.

2. Students may let the variable represent the number of tiles in the arrangement, not the arrangement number.	Assessing Questions: • What does the variable represent in your expression? Advancing Question: • How could you define your variable in a way that would allow you to find the number of tiles in the 20^{th} arrangement? 40^{th} arrangement? 101^{st} arrangement?
3. Students may approach the sequence as recursive. Although this is a way of representing a sequence, the intent of this task is to generate an explicit form.	Assessing Questions: • What does the variable represent in your expression? • Why did you choose to represent the sequence with this approach? • What limitations does this recursive approach have in this particular problem? Advancing Question: • How could you generate a function rule that does not rely on needing to know the number of tiles in the prior arrangement?
Entry/Extensions	**Assessing and Advancing Questions**
If students can't get started….	Assessing Questions: • What is the question asking? • Have students draw or build the next few arrangements and describe how they determined the arrangement. • Describe how you are visualizing the partitions of the arrangements. Why did you decide to partition your drawing this way? • Have students mark/show what each part of the arrangement represents from their expression. Advancing Questions: • Have students test their expression with the known (built) arrangements. Does your expression work? Why or why not?

	• How could you use what you know about the 5th and 10th arrangements to find the number of tiles for the 20th arrangement? 50th arrangement? 101st arrangement? • What do you notice is changing and what is staying the same about each arrangement?
If students finish early….	Assessing Questions: • Show me how your function rule relates to the arrangement. • Why did you decide to partition your drawing this way? • Why did you decide to use this method to solve the problem? • Why are there two potential solutions to this problem? • Given the scenario, which value of x would make sense? Why? Advancing Questions: • Group or pair students who are finished – How are your methods similar and different? How are all of your equations related? • Are any of the forms of the equations easier to solve than others? Why?

Discuss/Analyze

Whole Group Questions

Select and Sequence refers to when a teacher anticipates possible student strategies ahead of time and then selects and determines the order in which the students' math ideas/strategies will be shared during the whole group discussion. The purpose of this is to determine which ideas will most likely leverage and advance student thinking about the core math idea(s) of the lesson.

During a whole group discussion, students are sharing their strategies that have been pre-selected and sequenced by the teacher. Strategies to consider sharing in order to advance student thinking are:

- Drawing/Model: Various arrangements (especially solutions 1, 3, and 5). Function rules should be charted and could provide additional practice by having students verify that all are truly equivalent.
- Methods of Solving: Completing the Square, Quadratic Formula, Taking Square Roots, Graphs, and Tables.

You may want to consider charting on the board or chart paper the different expanded equations throughout the Whole Group Share time. A discussion around the different forms of the equation can lead to valuable property discussions, additional practice with simplifying expressions, and the usefulness of the different forms. In some cases, the simplest form may not be the simplest to solve. This could lead into a discussion of utilizing Completing the Square. Students should see that each expression simplifies to the same expression ($n^2 + 2n + 3$).

Questions to pose during the discussion:

- Why did you decide to partition your arrangement this way?
- How does your function rule relate to your drawing? Where is each part of your expression in the arrangement?
- How does each of the expressions relate to each other?
- Why did you decide to use your method to solve the problem? What advantages/disadvantages did your method have?
- Were any of the forms easier to solve? Why?
- Your equation had two answers. How did you determine which answer to use?