Task: Comparing Shapes	**Geometry**

On a piece of graph paper with a coordinate plane, draw three non-collinear points and label them A, B, C. (Do not use the origin as one of your points.) Connect these points to make a triangle. For each point, take half of the x and y-coordinates and label these new points A', B', C'. Connect these points to make another triangle.

1. Compare the distance from the origin to point A' and from the origin to point A. Do the same for points B' and B, and for points C' and C. Describe any relationships you notice.
2. Find the perimeter of triangle ABC and find the perimeter of triangle A'B'C'. Describe any relationships that you notice.
3. Suppose you repeated the directions, but you took a third of the x and y coordinates. Make a conjecture about what would happen to the relationships you noticed in parts 1 and 2.
4. Suppose you repeated the directions, but used a different shape (e.g. quadrilateral, pentagon, hexagon). Make a conjecture about what would happen to the relationships you noticed in parts 1 and 2.
5. Verify your conjectures for numbers 3 and 4.

Extension: If students know how to find the area of non-right triangles, include this after part 2. Students will compare the area of triangle A'B'C' with the area of triangle ABC. Student should then finish the other parts of the task, making conjectures and proving them, including the areas.

Teacher Notes:

Students will need to know how to find distance between two points in this task.
Because students can place points wherever they want, they should see these relationships will hold in general for any ratio and any shape placed anywhere in the plane.
This task will provide a good foundation for studying similarity and for looking at similarity through dilations from a point.

Common Core State Standards for Mathematical Content	**Common Core State Standards for Mathematical Practice**
G-GPE.B.6 Find the point on a directed line segment between two given points that partitions the segment in a given ratio. G-GPE.B.7 Use coordinates to compute perimeters of polygons and areas of triangles and rectangles, e.g., using the distance formula.★	1. Make sense of problems and persevere in solving them. 2. Reason abstractly and quantitatively. 3. Construct viable arguments and critique the reasoning of others. 4. Model with mathematics. 5. Use appropriate tools strategically. 6. Attend to precision. 7. Look for and make use of structure. 8. Look for and express regularity in repeated reasoning.

Essential Understandings

- The perimeter of figures whose side lengths are in an n:m ratio will also be in an n:m ratio because addition preserves this ratio.
- Behind every proof is a proof idea.
- Empirical verification is an important part of the process of proving, but it can never, by itself, constitute a proof.
- Geometry uses a wide variety of kinds of proofs.

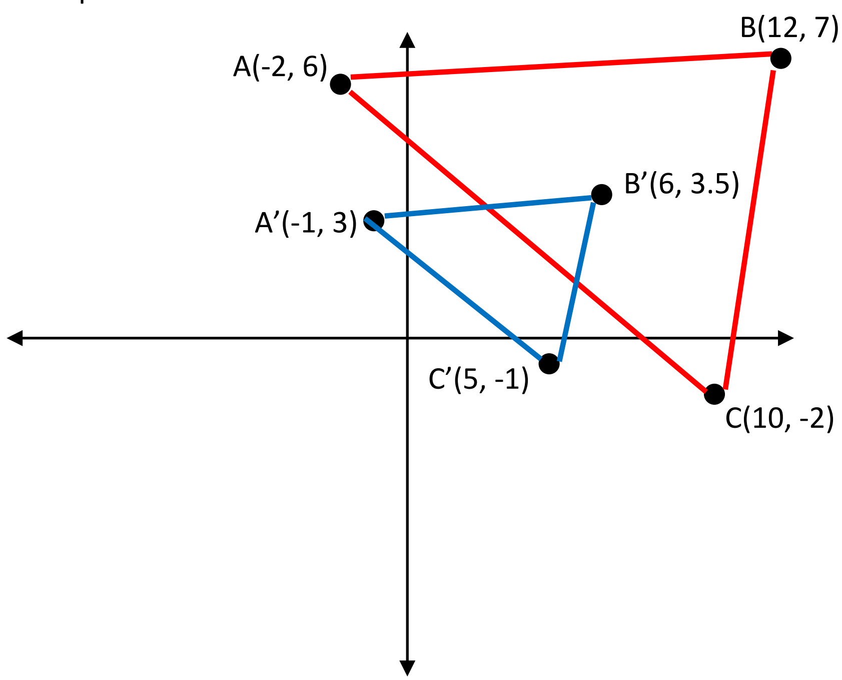
Explore Phase

Possible Solution Paths

Examining Relationships

Students can place points A, B, C anywhere in the plane.

For example:



The distance from the origin to A is

$$d = \sqrt{(-2)^2 + 6^2} = \sqrt{4+36} = \sqrt{40} = 2\sqrt{10}$$

The distance from the origin to A’ is

$$d = \sqrt{(-1)^2 + 3^2} = \sqrt{1+9} = \sqrt{10}$$

The distance from the origin to A’ is half the distance from the origin to A.
The steps would be similar for points B and B’, and for points C and C’.

Assessing and Advancing Questions

Assessing Questions

Tell me why you placed your points there.
How did you know where to place points A’, B’, C’?

Advancing Questions

How can you find the distance between two points?
How do you calculate perimeter?

Depending on where students placed their points, they might be able to count without using the distance formula, but not being able to use the origin should require them to use it somewhere. Students should notice that the perimeter of the triangle with the prime points is half of the perimeter of the original triangle.	
<u>Making Conjectures</u> Students should make a conjecture that if you divide the x and y-coordinates by any number, that number will define the ratio from the origin to the new point and the origin to the original point. Similarly, students should conjecture the relationships will be maintained with any ratio and the perimeter of any polygon.	<u>Assessing Questions</u> Tell me about your conjectures. <u>Advancing Questions</u> Do you think your conjecture will hold by dividing the coordinates by any number? Why or why not? How could you show this for any value? For any polygon?
<u>Proving Conjectures</u> <u>Ratios from the Origin</u> Suppose students divide the x and y-coordinates by n: Let the coordinates for A be (r, s) and the coordinates for a be $\left(\frac{r}{n}, \frac{s}{n}\right)$. The distance from the origin to A is $d = \sqrt{r^2 + s^2}$ The distance from the origin to A' is $d = \sqrt{\left(\frac{r}{n}\right)^2 + \left(\frac{s}{n}\right)^2} = \sqrt{\frac{r^2 + s^2}{n^2}} = \frac{1}{n}\sqrt{r^2 + s^2}$ The distance from the origin to A' is $\frac{1}{n}$ the distance from the origin to A. (Because this verification is done in general, it will hold for any points B and C. If students use specific points, ask how they can generalize their thinking to account for all points). <u>Ratio of Perimeters</u> Let the coordinates for A be (r, s) and the coordinates for B be (p, q). The distance from A to B is $d = \sqrt{(p - r)^2 + (q - s)^2}$	<u>Assessing Questions</u> I see you picked some new points and tried a different situation. Tell me how you are thinking about verifying your conjectures. <u>Advancing Questions</u> Can you verify your conjecture will be true for the ratio of everyone in the class? Can you verify your conjecture for any polygon? How would you do that? How can you express your reasoning? Would it be easier to explain in words, with algebra, or some other way?

The distance from A' to B' is $d=\sqrt{\left(\frac{p-r}{n}\right)^2+\left(\frac{q-s}{n}\right)^2}=\sqrt{\frac{(p-r)^2+(q-s)^2}{n^2}}=\frac{1}{n}\sqrt{(p-r)^2+(q-s)^2}$ Students can repeat for the distance from A to C, B to C and A' to C', B' to C'. When summing for the perimeter, the $\frac{1}{n}$ will factor out for triangle A'B'C'. These ideas hold for any polygon.	
Possible Student Misconceptions	
Students may connect A' to A, B' to B, C' to C and not have an accurate picture for the task.	<u>Assessing Questions</u> Let's read the problem together. What is it that you need to do first? <u>Advancing Questions</u> If you are to divide the coordinates by two, what kinds of numbers might you choose that would make this problem a little easier. Tell me how you think you should use these points to make two triangles.
Students may have a hard time thinking about how to verify their conjectures generally. Students may prove a specific case.	<u>Assessing</u> Tell me about how you made this conjecture. What do you think is happening? What do you think is happening with your classmates' relationships? <u>Advancing</u> I see you are verifying your conjecture for a quadrilateral you made. How can you verify your conjecture for any quadrilateral?
Entry/Extensions	**Assessing and Advancing Questions**
If students can't get started….	<u>Assessing Questions</u> Let's read the problem together. What is it that you need to do first? <u>Advancing Questions</u> Do you remember how to use the distance formula? Show me how to find the distance between these two points.
If students finish early….	<u>Assessing Questions</u> What did you notice about the relationships in general? What kind of proof did you use?

	<u>Advancing Questions</u> What do you think will happen with the areas of the triangles? Why? Can you prove this?

Discuss/Analyze

Whole Group Questions

- What relationships did you notice for the ratio from the origin to the prime points and the original points? Why do you think this is the case?
- What about the ratio of the perimeters?
- What can we say if these relationships were true for everyone's picture in the class, but everyone graphed three different points to start with?
- How does this make you think about an idea for a proof?
- Let's talk about your conjectures. What did you think would happen by dividing the coordinates by a number different than 2? Did anyone try 3 or 4? What effect would this have on the different ratios?
- I noticed that some of you verified your conjecture with a different case. Remember we just said that everyone in the class could have a different picture but noticed all of the same relationships. We want to be able to verify a conjecture that will work for any case. How can we do that?
- Why do you think it is important to prove something that will work for any case rather than a specific one?

Comparing Shapes Task **Name: ______________________________**

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