Task: The Speeding Problem **Algebra 1/Core Math I**

The city of Cautionville has decided to utilize a new formula for calculating the fine for speeding within their city limits. Speeding violations will be categorized in two ways: regular speeding violations and reckless driving speeding violations. To calculate the charge for each regular speeding violation, the city is enforcing a fee of $60 for each speeding ticket issued. In addition, there will be a charge of $8 for every mile per hour driven that exceeds the city-wide speed limit. The maximum charge for a regular speeding ticket is $300. Anything beyond the $300 amount is considered to be in the reckless driving category and enforces larger penalties and a mandatory court appearance for further potential consequences. The city-wide speed limit is 30 miles per hour.

A. Based on the information above, determine the cost for each of the speeds below. Show how you determined your answers.

   27 miles per hour

   38 miles per hour

   45 miles per hour

B. Write an equation that could be used to determine the total cost of a regular speeding violation ticket. Be sure to define each variable.

   Use your equation to find the cost for a person driving 25 miles per hour over the speed limit. Show your work.

   Use your equation to find the speed that would result in a $212 fine. Show your work.

C. Does your equation represent a function? Why or why not? If it does, what would be the domain and range of the function?

D. At what speeds does the city of Cautionville consider a driver to be reckless? Explain how you determined your answer.

Teacher Notes	
This problem is written to have students work with linear equations in a real-world context. Part A is designed to provide an entry point for all students and to indirectly introduce the idea of the domain in context. Some students will need the numeric structures of Part A to develop the equation later. Part B moves students into writing a generalization (equation) for the scenario. It is important to note that there are at least two different equations that represent the scenario. The variable could be defined as strictly the speed a car is going or as the speed driven over the city-wide speed limit. Part C builds into the idea of identifying functions and defining the domain and range based on the context. To address more of IF.A.1, consider introducing or reviewing function notation. Finally, Part D is designed to begin the transition from equations to inequalities.	
Common Core State Standards for Mathematical Content	**Common Core State Standards for Mathematical Practice**
A – CED.A.1 Create equations ~~and inequalities~~ in one variable and use them to solve problems. *Include equations arising from linear and quadratic functions, and simple rational and exponential functions.* A – CED.A.3 Represent constraints by equations ~~or inequalities, and by systems of equations and/or inequalities~~, and interpret solutions as viable or nonviable options in a modeling context. *For example, represent inequalities describing nutritional and cost constraints on combinations of different foods.* A – REI.B.3 Solve linear equations and ~~inequalities~~ in one variable~~, including equations with coefficients represented by letters~~. F – IF.B.5 Relate the domain of a function ~~to its graph and, where applicable,~~ to the quantitative relationship it describes. *For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.* F – IF.A.1 Understand that a function from one set (called the domain) to another set (called the range) assigns to each element or the domain exactly one element to the range. If f is a function and x is an element of its domain, then f(x) denotes the output of *f* corresponding to the input x. ~~The graph of *f* is the graph of the equation y = f(x)~~ F – BF.A.1.a Write a function that describes a relationship between two quantities - Determine an explicit expression, a recursive process, or steps for calculation from a context.	1. Make sense of problems and persevere in solving them. 2. Reason abstractly and quantitatively. 3. Construct viable arguments and critique the reasoning of others. 4. Model with mathematics. 5. Use appropriate tools strategically. 6. Attend to precision. 7. Look for and make use of structure. 8. Look for and express regularity in repeated reasoning.

Essential Understandings	
• Equations can be used to model real-world scenarios. The variables used in the equation must be defined. • A function assigns to each element of the domain exactly one element of the range. (HSF – IF.A.1) • Solutions to an equation must be considered viable based on the domain of the function and context of the scenario.	
Explore Phase	
Possible Solution Paths	**Assessing and Advancing Questions**
Part A. Students may calculate the speeds whether by adding the fee at the beginning or at the end. Below are a few examples. 27 mph 60 + 8(-3) or 8(-3) + 60 60 + 8(27-30) or 8(27-30) + 60 Some may not calculate and justify that there would be no violation since the driver was not speeding. 38 mph 60 + 8(8) or 8(8) + 60 60 + 8(38-30) or 8(38-30) + 60 45 mph 60 + 8(15) or 8(15) + 60 60 + 8(45-30) or 8(45-30) + 60	Assessing Questions: • Describe what each number in your calculations represents. Where is the fee? Where is the cost charged for every mile per hour over the speed limit? • How did you determine what to multiply the $8 by? • How do you know that the 27 miles per hour would not generate a ticket? or Why did you not calculate a number for the 27 miles per hour? Advancing Questions: • Would it matter if the $60 fee was added before or after multiplying? Explain. • How could you write a number sentence that would clearly represent the scenario?
Part B. Students may write different equations based on how the variable is defined. Below are two examples.	Assessing Questions: • Describe what each part of your equation represents. What does the 60 represent? the 8? What is the x? the y? • Talk me through how you solved the problem.

y = 8x + 60 or y = 60 + 8x where x = the speed driven over the speed limit y = 8(x-30) + 60 or y = 60 + 8(x-30) where x = speed driven Students will calculate the fine by substituting a value for the domain value of their equation. Examples: y = 8(25) + 60 = $260 or y = 8(55-30) + 60 = $260 Students will find the speed that will result in a $212 dollar fine by substituting 212 for the range value and solving their equation. Examples: 8x + 60 = 212 8x + 60 – 60 = 212 – 60 8x = 152 8x ÷ 8 = 152 ÷ 8 x = 19 19 + 30 = 49 mph or 8(x – 30) + 60 = 212 8x – 240 + 60 = 212 8x – 180 = 212 8x – 180 + 180 = 212 + 180 8x = 392 8x ÷ 8 = 392 ÷ 8 x = 49 mph	• How does your solution relate to the scenario? Advancing Questions: • How could you use your number sentences in Part A to help write an equation? • How could you write an equation that only uses the speed driven? or How could you write an equation that uses the speed driven over the speed limit?
Part C. Students will need to justify whether their equation represents a function or not. Some possible solutions are: Some may justify that every domain (speed) has exactly one range (cost of Violation) for the constraints in the scenario. Some may graph their equation and show that it meets the Vertical Line Test. Students will need to define the domain and range. Please note that this does not specify the type of notation that may be used by your curriculum resources. Some possible solutions could be: Domain: $30 < x \leq 60$ where x represents the speed driven	Assessing Questions: • How do you know that your equation represents a function? Justify your reasoning. • How did you determine your domain? range? Advancing Questions: • At what speed could you first receive a ticket? How does this relate to domain and range? • How could you use the calculations you did above to help you with this problem?

$0 < x \leq 30$ where x represents the speed driven over the speed limit Range: $0 < y \leq 300$ where y represents the cost of a ticket Domain: $31 \leq x \leq 60$ where x represents the speed driven and integer values $1 \leq x \leq 30$ where x represents the speed driven over the speed limit and integer values	
Part D. Students may write this in words or mathematical symbols. Below are a few samples: For students who wrote their equation based on speed driven over the speed Limit - Any speeds that exceed 30 mph over the speed limit or $x > 30$ For students who wrote their equation based on the speed driven – Any speeds that exceed 60 mph or $x > 60$	Assessing Questions: • Show me how your equation and the scenario support your claim for both the domain and range? Advancing Questions: • Could you have speeds that are not integers? How would that affect your domain and range?
Possible Student Misconceptions	
1. In Part A, students may use the speed driven only and not the speed over the speed limit to calculate the cost of the ticket. Example: $8(27) + 60 = 276$, instead of $8(-3) + 60 = 36$ $8(38) + 60 = 364$, instead of $8(8) + 60 = 124$	Assessing Questions: • Talk me through how you solved the problem. • What does the 27 miles per hour represent? What does the 38 miles per hour represent? How do they relate to the scenario? Advancing Questions: • Would you get a ticket at 27 miles per hour? 38 miles per hour? Explain.
2. Students may use the equation $y = 8x + 60$ and assume that the solution to the equation in Part B is the final solution. In this case, x would represent the speed driven over the speed limit, not the actual speed driven. They may not reason to add the 30 mph to the solution.	Assessing Questions: • Talk me through how you solved the problem. • What does x represent in your equation? • What did your solution represent?

Example: For a ticket cost of $ 212 8x + 60 = 212 8x + 60 – 60 = 212 – 60 8x = 152 8x ÷ 8 = 152 ÷ 8 x = 19 **<u>19 + 30 = 49 mph</u>**	Advancing Questions: • I noticed that the solution is 19 mph. Does that make sense in the context of the problem? Explain. • How does your solution relate to the speeds in Part A?
3. Students may justify their equation to be a function by the Vertical Line Test and not see the domain and range in context.	Assessing Questions: • How does your graph relate to the scenario? • What is the domain and range of your graph? Advancing Questions: • According to your graph, about how fast would you be going to get a ticket for $380? How many speeds could you drive in order to get a ticket for this amount? Explain.
Entry/Extensions	**Assessing and Advancing Questions**
If students can't get started….	Assessing Questions: • What is the question asking? • Describe what each number in your calculations represents. Where is the fee? Where is the cost charged for every mile per hour over the speed limit? • Talk me through how you would find the cost of a ticket. • How did you determine what to multiply the $8 by? • How do you know that the 27 miles per hour would not generate a ticket? or Why did you not calculate a number for the 27 miles per hour? Advancing Questions: • How could you use your number sentences in Part A to help write an equation? • What do you notice is changing and what is staying

	the same in the scenario?
If students finish early….	Assessing Questions: • Show me how your equation relates to the scenario. • Why did you decide to use this equation? • How do you know that your equation represents a function? • How does your domain and range relate to the scenario? Advancing Questions: • Group or pair students who have different equations – How are your methods similar and different? How are all of your equations related? • The mayor of Cautionville wants to change the fee for a regular speeding violation, but keep the same maximum cost and criteria (miles per hour over the speed limit) for distinguishing between a regular and reckless violation. Write an equation that would allow this to happen. Mathematically show how your equation fits the mayor's criteria.

Discuss/Analyze

Whole Group Questions

Select and Sequence refers to when a teacher anticipates possible student strategies ahead of time and then selects and determines the order in which the students' math ideas/strategies will be shared during the whole group discussion. The purpose of this is to determine which ideas will most likely leverage and advance student thinking about the core math idea(s) of the lesson.

During a whole group discussion, students are sharing their strategies that have been pre-selected and sequenced by the teacher. Strategies to consider sharing in order to advance student thinking are:

- Methods of Calculation in Part A: You may want to consider having students share both calculations (using speed driven and

speed driven over the speed limit) and reasoning about the 27 miles per hour.

- Different Types of Equations and solutions: Share an equation where the variable is defined as the speed driven and one where the variable is defined as the speed driven over the speed limit.
- Justifications of the Function: Share various rationales proving their equation is a function (i.e. graph, table, mapping, etc.)
- Domains and Ranges.

There are lots of rich discussions that need to occur during the Whole Group Share. A discussion around how the 27 miles per hour question relates to the context of the scenario could be a bridge to having students look at the domain. The importance of labeling variables and how the two different equations produce different solutions, but can be used to solve the problem should also be discussed. Part C lends itself to a discussion around the domain and range and what type of numbers would fit the scenario. Finally, Part D could be used to introduce the relationships between equations and inequalities.

Questions to pose during the discussion:

- How did you calculate the cost of each ticket?
- Would you need to calculate the 27 miles per hour? Why or why not?
- How did you use the calculations in Part A to write an equation in Part B?
- How did you define your variables in your equation? Why do the labels matter?
- How does your equation relate to the scenario? Where is each part of your equation in the scenario?
- Do the equations shared have the same solution? Explain.
- Does your equation represent a function? Justify your reasoning.
- How did you define the domain and range of your function? How does your domain and range relate to the scenario?
- Could you receive a ticket for going 30.2 miles per hour? Explain. How does this relate to your domain and range?
- How did you determine the speeds at which a driver is considered reckless?

The Speeding Problem Name _____

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