Task: Fourth Degree Polynomial	**Algebra II**

Pre-Problem Work: Create up with a second degree polynomial that has an x^2 and a constant term, but not an x term, and that has integer zeros.

Use what you did in this problem to help get you started on the task below:

a) Create a fourth degree polynomial function that meets the following criteria:

- The leading coefficient is positive
- There are four distinct zeros, all of which are integers
- The polynomial has no x^3 or x term

b) Verify that your polynomial meets the above conditions by using equations and a graph.

c) Identify the line of symmetry and explain features of your polynomial that cause this symmetry.

Teacher Notes:

The pre-problem work is optional to help student who have not worked with conjugates. They can apply their understanding of how these factors work in quadratics for the task with a quartic.

Although this task is open ended, all students should come up with graphs that have the same general shape.

This task will lead to discovery of even functions and y-axis symmetry.

There are many extensions to this problem including –
End behavior
Relative/Absolute Extrema

Common Core State Standards for Mathematical Content	**Common Core State Standards for Mathematical Practice**
A-APR.B.3 Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a	**1. Make sense of problems and persevere in solving them.** 2. Reason abstractly and quantitatively.

rough graph of the function defined by the polynomial.	3. Construct viable arguments and critique the reasoning of others. 4. Model with mathematics. **5. Use appropriate tools strategically.** 6. Attend to precision. **7. Look for and make use of structure.** **8. Look for and express regularity in repeated reasoning.**

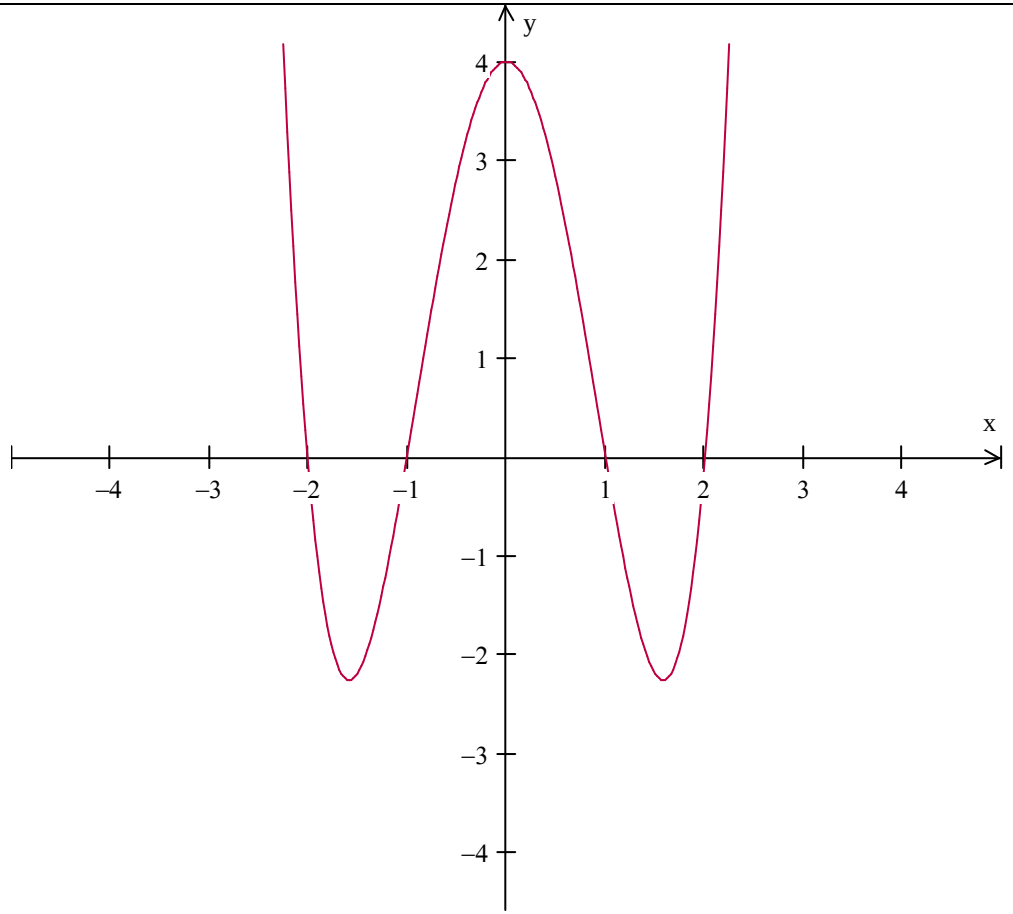
Essential Understandings

- Functions can be represented in multiple ways, including algebraic (symbolic), graphical, verbal, and tabular representations. Links among these different representations are important to studying relationships and change.

Explore Phase

Possible Solution Paths	**Assessing and Advancing Questions**
a) $f(x) = (x-2)(x+2)(x-1)(x+1)$ $= (x^2-4)(x^2-1)$ $= x^4-5x^2+4$	Assessing – Why did you use opposite integers for your factors? Advancing – Is this the only polynomial that would meet the criteria? What would have to happen to the factors to meet the criteria?

b)



Assessing – How did you know that the function crosses the x-axis at the points you have indicated?

Advancing – How do you know that the function doesn't have any more turns in it than you have shown?

c) The line of symmetry is the y-axis or x = 0. The reason why this is true is that because everywhere you put in opposite values for x they get the same y value. EX: $2^4 = 16$ and $(-2)^4 = 16$	Assessing – Why did you say the y axis was the line of symmetry? Advancing – Do all fourth degree polynomials have the y axis as the line of symmetry?
Possible Student Misconceptions	
a) Students may struggle with how to not have the x^3 or x term.	Assessing – What combinations have you tried so far? Advancing – How can we use the information in the pre-problem work to help us?
b) Students may struggle to graph the polynomial without a calculator.	Assessing – Which form of the polynomial are you using to make the graph? Advancing – Is there a better form we could use to make sketching the graph easier?
c) Students may struggle with finding features of the polynomial that cause the symmetry in the graph.	Assessing – Do you understand the concept of line of symmetry? Advancing – Think about raising a number to even and odd powers. What happens when you raise a number to the third power? The fourth power?
Entry/Extensions	**Assessing and Advancing Questions**
If students can't get started….	Assessing – How did you come up with the second degree polynomial in the pre problem work?

	Advancing – How can we use this information to help us in creating the fourth degree polynomial?
If students finish early….	Assessing – What if the leading coefficient of your polynomial was 2. What effect(s) would that have on your graph? Advancing – Can you find the ordered pairs of the lowest points on the graph?

Discuss/Analyze

Whole Group Questions

- What helped you come up with the polynomial without the x^3 or x term?
- Would all the polynomials that would fit these criteria have the same general shape?
- What do you notice about all of the terms in the polynomials groups came up with?
- What do you notice about the symmetry of this graph?
- Can we determine the ordered pairs of the absolute minimums?
- Can we determine the ordered pair of the relative maximum?