Tennessee Department of Education

Task: The Leader of the Pack **7th Grade**

Micah and his best friend Tobias are motorcycle enthusiasts. They found a table of the 4 fastest bikes and their top speeds online, and they want to know how long it would take to travel the 5.2 miles from Micah's house to Tobias's on each bike if they traveled at top speed.

a) Finish the table to the right. What do you notice about the relationship between speed and time?

b) After looking at the table, Micah and Tobias theorize that with faster and faster speeds, they could eventually cover the distance between their houses in 0 seconds. Create a table that illustrates their theory and then tell whether you believe the distance could be covered in 0 seconds. Use mathematical reasoning and concepts to support your claims.

Motorcycle	Top Speed	Time (in seconds)
Dodge Tomahawk	350 mph	
Suzuki Hayabusa	248 mph	
MTT Turbine Superbike	227 mph	
Honda Blackbird	190 mph	

Teacher Notes:

In part a) of the task, students will simply do calculations and complete the table. The pattern that emerges suggests that faster speeds mean shorter travel times; however, this pattern does not continue until the time is 0. Students should be guided through the assessing and advancing questions to realize that for a constant distance in the equation $D = rt$ a time of zero can only reasonably relate to a distance of 0, which leads to a whole group question about the mysteries of division by zero.

Tennessee State Standards for Mathematical Content	Tennessee State Standards for Mathematical Practice
7.NS.A.2 Apply and extend previous understanding of multiplication and division and of fractions to multiply and divide rational numbers. **7.NS.A.2b** Understand that integers can be divided, provided that the divisor is not zero, and every quotient of integers (with non-zero divisor) is a rational number. If p and q are integers, then $-(p/q) = (-p)/q = p/(-q)$. Interpret quotients of rational numbers by describing real- world contexts.	1. Make sense of problems and persevere in solving them. 2. Reason abstractly and quantitatively. 3. Construct viable arguments and critique the reasoning of others. 4. Model with mathematics. 5. Use appropriate tools strategically. 6. Attend to precision. 7. Look for and make use of structure. 8. Look for and express regularity in repeated reasoning.

Essential Understandings

- Equations of the form $a = bx$ can be solved by dividing both sides by b.
- Rational numbers follow the same properties of multiplication and division as integers, whole numbers, and fractions.
- Rounding long decimal numbers is necessary in many real world problems.
- Dividing a non-zero constant by larger and larger quantities causes the quotient to get smaller and smaller, but it will never be zero.
- Division by zero is undefined, because $\frac{a}{0} = b$ implies $a = b \cdot 0$, which is not true when $a \neq 0$ and has infinitely many solutions when $a = 0$.

Explore Phase

Possible Solution Paths	Assessing and Advancing Questions

a)

Solution Path 1:

For part a), students may reason through how to calculate the time that it would take to travel 5.2 miles at each speed based on previous experiences with distance problems, or by thinking about what the units "miles per hour" mean. They will find each time (shown in the completed table below) by dividing the distance by the top speed the motorcycle can travel. However, they must note that since the units on the speed were miles per hour, the units on the time are hours. To convert hours to seconds, they must multiply by 60 to get to minutes and then by 60 again to find seconds, which is the equivalent of multiplying by $60 \times 60 = 3600$.

Motorcycle	Top Speed	Time (in seconds)
Dodge Tomahawk	350 mph	$0.014857hr \approx 53\text{sec}$
Suzuki Hayabusa	248 mph	$0.020968hr \approx 75\text{sec}$
MTT Turbine Superbike	227 mph	$0.022907hr \approx 82\text{sec}$
Honda Blackbird	190 mph	$0.027368hr \approx 98\text{sec}$

Note that students should decide what decimal place is appropriate to round to and convert to seconds, no matter what method they use. There is no exact correct number of decimal places to use, but they should use at least 4, since they will be multiplying by a number that's at least 1,000 ($60 \times 60 = 3600$).

The pattern shown in the table above is that as the speed increases, the time needed to travel the same distance decreases.

Assessing Questions:

- How did you decide how to calculate the time it would take to travel the 5.2 miles at each speed?
- What clues did you see that helped you decide to do it this way?
- How did you convert the time to seconds after you had found it in hours?
- Is there a formula that can help you to always know whether to divide or multiply distances, rates, and times?

Advancing Questions:

- What operation is implied by miles per hour?
- How can you convert from hours to seconds?
- Are seconds or hours larger? Should you multiply or divide to change hours to seconds?
- Would it help to use simpler numbers and to develop a method or formula?

a)

Solution Path 2:

An alternate way of solving part a) is to use the formula $D = rt$, which students may recall from previous experience. By using $D = 5.2miles$, altering the rate for each row of the table, and solving for the time, students will find the time (in hours) it would take to travel the distance at each rate. To convert hours to seconds, they must multiply by 60 to get to minutes and then by 60 again to find seconds, which is the equivalent of multiplying by $60 \times 60 = 3600$.

Assessing Questions:

- How did you find the values for the table?
- What units were your answers in initially?
- How did you convert them to seconds?
- What operation did you use to find time from distance and rate?

Advancing Questions:

- Is there a formula that relates distance, rate and time?
- How can the formula $D = rt$ be transformed so that time can be calculated?

b) Students may have to brainstorm together before deciding how to approach this portion of the problem, as they are not told what rate values to start or end with or by what increments they should increase each time. The table below is an example of an appropriate table, although there are many other appropriate tables:

Rate (mph)	**Time (in seconds)**
400	$0.013 hrs \approx 47 \text{sec}$
500	$0.0104 hrs \approx 37 \text{sec}$
600	$0.0087 hrs \approx 31 \text{sec}$
700	$0.0074 \approx 27 \text{sec}$
800	$0.0065 hrs \approx 23 \text{sec}$
900	$0.0058 hrs \approx 21 \text{sec}$
1,000	$0.0052 hrs \approx 19 \text{sec}$

Any acceptable tables would be similar but may contain different starting or ending values, or different amounts of increase in the rate from row to row. If the numbers they are getting are not changing much, they should increase the rate by more each time. Eventually, they will see that *the amount of time it takes to travel between Micah's and Tobias's houses decreases as the speed increases, and it may eventually be very, very small or perhaps even appear to be equal to 0 on a calculator*. Any claims beyond that should not be made (i.e. "The time will certainly be zero at a high enough speed" would be incorrect.). After their observations have been made, they should be guided through the advancing questions to an understanding that *t* cannot be zero, since $t = 0$ implies that $D = r \cdot 0 = 0$, which is not true. This can be achieved through the Whole Group Questions at the end as well.

Assessing Questions:

- How did you choose the starting value for the rate?
- How did you decide how much to increase the rate to calculate each new time?
- Why did you end the table with the rate you chose?
- Do you agree with Micah and Tobias that eventually the distance could be traveled in 0 seconds?
- Did your calculator ever give you a *t* value of 0?
- What is the value of distance, *D,* in all of the rows of the table?
- How did you solve for time, *t*?
- How can a division problem equal zero (i.e. if you calculate $\frac{a}{b} = a \div b = 0$, what are possible values of *a* and *b*)?
- If you use the equation $D = rt$, what happens if $t = 0$?
- What values of *r* are possible if $t = 0$?
- What manipulations of the equation aren't possible if $t = 0$ (i.e. $D = r \cdot 0$)?
- Can you phrase this idea in terms of the problem? Why can't *t* be zero?

Advancing Questions:

- What do you need to know in order to calculate time?
- What kind of rates should you put in your table?
- What is happening to the time as the rate increases?
- Does time ever equal zero?

Possible Student Misconceptions	**Assessing and Advancing Questions**
When dividing to find the times in part a), students divide the bigger number by the smaller one.	• How can you calculate how far you went if you traveled for a certain time at a certain speed, for instance 65 mph for 2 hours? • Does this give you a clue about how distance, speed, and time related? • Can you write this in a formula and use it to find the time, given the distance and speed?

Since the time is getting smaller and smaller as the speed increases, students believe that the distance could eventually be traveled in 0 seconds.	• What distance can be covered if the time is zero? • What speeds can be used to cover 0 miles in 0 seconds?
Entry/Extensions	**Assessing and Advancing Questions**
If students can't get started….	• If I told you someone was traveling 70 mph for 2 hours, how would you calculate the distance covered? • How are distances, speeds, and the time it takes to travel them related? • How would you use this relationship to complete the table?
If students finish early….	• What concept that we have learned about integers was illustrated in this activity? • Can you write a paragraph explaining how this activity illustrates division concepts? • Under what conditions is $D=rt$ a proportional relationship?
Discuss/Analyze	
Whole Group Questions	
Solving the Equation • In part a), did anyone use an equation to complete the table? • If you did not use an equation, how did you find the times for each speed? • Can someone show me how you got one of the times using the equation? • Can someone else tell me how you got the same time without using the equation? • How are these approaches the same? • What operation was used? Why? • Why do you think that we use formulas and equations to solve problems, when they can be done without them?	
Rounding Choices • As you found the times for the table, were they "nice" decimal numbers? What was not nice about them? • Can a few people tell me how they handled these unruly decimals? • Why did you choose to do it that way? • Did anyone do it differently? • What seems to be the best approach when solving a real-world problem? • Do these numbers represent exact answers? • Why is it okay in this context that they do not?	
Dividing by Large Numbers • As you filled out the table in part b), how did you choose the speeds? • How do the speeds change as you move down the table?	

- How do the times change?
- How is the distance changing?
- What operation is occurring between the distance and the speed in order to give you the time?
- Can you make a general observation about what happens when we divide a constant by larger and larger numbers?
- Will this value ever reach zero? *[This provides a good place to discuss the limitations of calculators and why users must always reason about appropriateness of answers.]*

Dividing by Zero

- What is an equation that would represent what happens when $t = 0$?
- Are there any other distances that could be covered in 0 seconds?
- Are there any other rates that would satisfy the formula?
- What observations can you make about *D* and *r* when $t = 0$?

Name _____

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