Task: Representing the National Debt **7th grade**

Rachel's economics class has been studying the national debt. The day her class discussed it, the national debt was $16,743,576,637,802.93. Rachel wants to find a way to represent this debt in a meaningful way to the class, so she decides to model the debt using jelly beans. If Rachel lets each jelly bean represent $1, what would be the volume of jelly beans? Rachel decides to use railroad boxcars to "hold" her jelly bean representation of the national debt.

a) Describe two different ways Rachel could estimate the volume of a jelly bean.
b) Before you begin your calculations, how many railroad boxcars do you think would be needed to hold Rachel's jelly beans? Why?
c) Suppose that Rachel decides to use a popular size of railroad boxcar to hold her jelly beans. This particular boxcar has a capacity of 177.75 cubic meters. The jelly bean that Rachel has decided to use has an average volume of 3.53 cubic centimeters. How many boxcars would Rachel need to hold her jelly bean representation of the national debt? Assume that the jelly beans are packed so that no air space is left. You may want to round the national debt to the nearest whole dollar. Explain any conversions you need.

Teacher Notes:

Students may not be familiar with the concept of the "national debt", so teachers may want to spend a few minutes setting up the task by describing this concept.

Students will need access to a calculator for this task in order to do the calculations in a timely manner. Students will also need to be familiar with conversions between metric units of measurement.

Measurements quoted in the task are estimates, so the student's final answer will be an estimate as well. There are a couple of places where a student could use a rounded number in the calculation, so final answers might vary from student to student, depending on how the units are rounded.

In the task, an assumption is made that the jelly beans are packed in such a way that no volume is "wasted" in the boxcar with airspace between the jelly beans. In reality, an estimated 20%-30% of the volume of the boxcar is wasted with airspace (depending on the size and shape of the jelly bean used to pack). This is addressed as an "extension" at the end of the task.

To compute the inside volume of the boxcar, the following interior measurements were used: Length: 15.4 m (~50.52 feet); Width: 3.98 m (~13.06 feet); and Height: 2.90 m (~9.51 feet). To compute the volume of the jelly bean used in this problem, the jelly bean was modeled with a cylinder and estimates of the dimensions were: Height: 2 cm; Diameter: 1.5 cm. Note that both jelly beans and boxcars are not standard and sizes will vary.

Common Core State Standards for Mathematical Content	Common Core State Standards for Mathematical Practice
7.RP.A.3 Use proportional relationships to solve multistep ratio and percent problems. Examples: simple interest, tax, markups and markdowns, gratuities and commissions, fees, percent increase and decrease, percent error. **7.G.B.6** Solve real-world and mathematical problems involving area, volume and surface area of two- and three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes, and right prisms.	Mathematical Practices 1. Make sense of problems and persevere in solving them. 2. Reason abstractly and quantitatively. 3. Construct viable arguments and critique the reasoning of others. 4. Model with mathematics. 5. Use appropriate tools strategically. 6. Attend to precision. 7. Look for and make use of structure. 8. Look for and express regularity in repeated reasoning.

Essential Understandings

- Reasoning with ratios involves attending to and coordinating two quantities.
- Any number, measure, numerical expression, algebraic expression, or equation can be represented in an infinite number of ways that have the same value.
- COMPARISON: Numbers, expressions, and measures can be compared by their relative values.
- ESTIMATION: Numerical calculations can be approximated by replacing numbers with other numbers that are close and easy to compute with mentally. Measurements can be approximated using known referents as the unit in the measurement process.
- MEASUREMENT: Some attributes of objects are measurable and can be quantified using unit amounts.

Explore Phase

Possible Solution Paths	Assessing and Advancing Questions
a) Here are several ways that the volume can be estimated. Students may identify others. 1) Treat a jelly bean as a cylinder. Measure the length and diameter of the jelly bean, take half of the diameter to find the radius, and then use the formula for the volume of a cylinder. 2) Measure several samples of jelly beans to find an average height and diameter, and then use the formula for the volume of a cylinder. 3) Compare a jelly bean with known objects, such as centimeter	**Advancing Question:** What does a jelly bean look like? Does it resemble any shapes we have studied? **Assessing Question:** Explain to me how your estimation strategy would work.

cubes. 4) Use a graduated cylinder partially filled with water, drop some number of jelly beans into the water, calculate the displacement, and then divide by the number of jelly beans to find an average volume. 5) Measure 50 mL of jelly beans into a graduated cylinder, count the number of jelly beans in the cylinder, and then divide 50 mL by the number of jelly beans to find the volume of one jelly bean in mL. *(Note: This will require an extra conversion between mL and cm^3.)*	
b) Answers will vary widely. It is likely that students will have no concept of the relative size of the number associated with the national debt, so expect fairly small numbers of boxcars to be chosen.	
c) *(Note: Here, we rounded the national debt to the nearest whole dollar.)* **Approach 1:** Convert the volume of the boxcar to cubic centimeters. Find the number of jelly beans per boxcar. Then find the number of boxcars you will need. • The volume of the boxcar is 177.75 m^3 = (177.75 m^3) (1,000,000 cm^3) = 177,750,000 cm^3 • The number of jelly beans per boxcar can be found by: (177,750,000 cm^3 / boxcar) / (3.53 cm^3 / jelly bean) = 50,354,107.65 jelly beans / boxcar • The national debt is being represented by 16,743,576,637,803 jelly beans. (16,743,576,637,803 jelly beans) / (50,354,107.65 jelly beans / boxcar) = 332,516.60 boxcars. *(Note: Students may round the number of jelly beans per car to 50,354,108. This will not impact the number of boxcars needed (to*	**Advancing Question:** What units do your measurements have? Can you work with two different units? What do you need to do next? **Assessing Questions:** Explain your calculations to me. Why did you convert your units? What process did you use? How do you know your answer is reasonable?

the nearest hundredth).) **Approach 2:** Convert the volume of the jelly bean to cubic meters. Find the number of jelly beans per boxcar. Then find the number of boxcars you will need. • The volume of the jelly bean is 3.53 cm^3 = (3.53 cm^3) (0.000001 m^3 / cm^3) = 0.00000353 m^3 (or 3.53 x 10^{-6} m^3) • The number of jelly beans per boxcar can be found by: (177.75 m^3 / boxcar) / (0.00000353 m^3 / jelly bean) = 50,354,107.65 jelly beans / boxcar • The national debt is being represented by 16,743,576,637,803 jelly beans. (16,743,576,637,803 jelly beans) / (50,354,107.65 jelly beans / boxcar) = 332,516.60 boxcars.	
Possible Student Misconceptions	
Students may have trouble with metric conversions, such as: • Converting volume units using only a linear conversion (forgetting to cube the linear conversion) • Inverting their conversion factor (e.g., to convert from cubic centimeters to cubic meters, they will multiply by 1,000,000 instead of dividing by 1,000,000)	**Advancing Questions:** What strategy do you have for converting units? What happens if you have squared or cubic units? Do you need to adjust your strategy? **Assessing Questions:** Explain your strategy for converting your units. Why did you cube your conversion factor?
Because of the size of the numbers involved throughout the calculations, it will be easy to miss a calculation because of a dropped digit (especially leaving out a 0 in some of the calculations).	**Advancing Questions:** Compare your answer with your partner (or with someone else in class). Are your answers different? Should the answers be different?
Students may have trouble believing that it will take that many boxcars since they are not used to working a problem with numbers as large as the national debt.	**Assessing Questions:** Why does it take so many boxcars? Is this a reasonable answer? Why?
Entry/Extensions	**Assessing and Advancing Questions**
If students can't get started…	**Advancing Questions:**

<table>
<tr><td>Talk about using concrete representations as a way to understand abstract thoughts. In this case, we are trying to determine what the large number that represents the national debt really means. Students rarely have any experience with comprehending numbers that large.</td><td>How do you say the number that represents the national debt?

What is the biggest number of items you have ever had to count?

Suppose you represented this amount with $100 bills. How many $100 bills would you need? How high would this stack be?</td></tr>
<tr><td>If students finish early….

A good estimate of the length of one of your boxcars is 52 feet. If we count only the boxcars holding jelly beans (and ignore the engines we would need to pull the train), how long will the train be in feet? In miles?

(Note: To compute the length of the train, the students will need to know how many feet are in one mile. Note also that the system of units has changed in this question (from metric to U.S. standard). The inside boxcar dimensions used for this problem are: Length: 15.4 m (~50.52 feet); Width: 3.98 m (~13.06 feet); and Height: 2.90 m (~9.51 feet). The 52 feet outside length is an estimate for this car. Using the numbers calculated in this problem, the train would be approximately 3,275 miles long.)

When we worked the problem, we assumed the jelly beans were packed tightly with no air space between them. Typically, between 20% and 30% of the volume is lost to air space between the jelly beans. If we take this into account, how will this affect your answer for the number of boxcars needed?</td><td>Advancing Questions:

How would you calculate the length of your train?

How do you convert from feet to miles?

Why are you losing volume to air? How would you calculate how much of your volume is taken by air instead of jelly beans?

Assessing Questions:

How long is your train in miles? Can you describe a good way to think about how long this train really is?

How did you take the air space into account? What calculations did you use? Why did you choose that strategy?</td></tr>
<tr><td colspan="2">Discuss/Analyze</td></tr>
<tr><td colspan="2">Whole Group Questions</td></tr>
<tr><td colspan="2">How large is the national debt? Does the number of boxcars full of jelly beans give an accurate picture of the size of this number? Why?
Did you have to approximate anything? How did approximations affect your answers?
Why did you need to use conversions from one unit to another?
How accurate was your initial guess in part (b)? Were you surprised by the actual answer? Why?</td></tr>
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