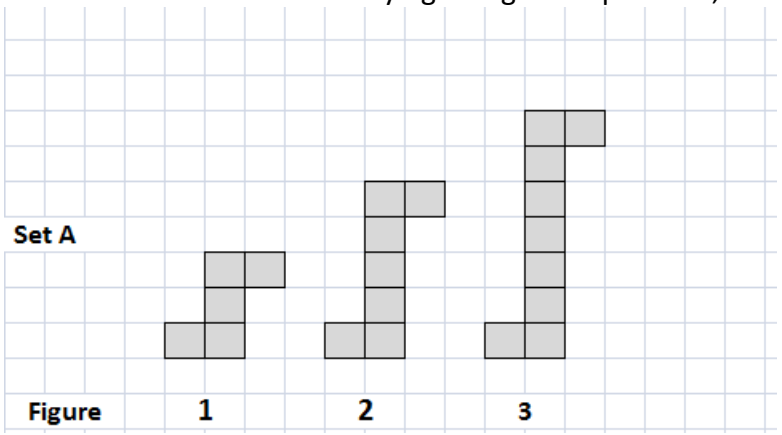
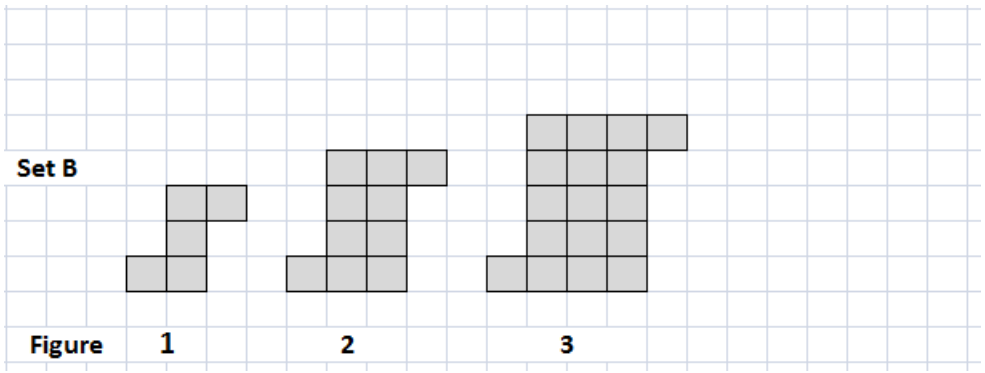
Task: Growth Pattern Task	**8th Grade**

Michael and Marissa are studying two growth patterns, Set A and Set B.





A. Draw the fourth figure in each pattern. For each set, create a table of values showing the number of the figure and how many square tiles are needed to make it.
B. Draw a graph showing the figure number and the number of square tiles needed to make it. If you graph the points on the same set of coordinate axes, use different symbol marks for the ordered pairs in Set A and in Set B.
C. Describe the patterns you see.
D. Can each set of figures be described using a linear equation? If so, write the linear equation. If not, explain why the pattern is not linear.

Teacher Notes:

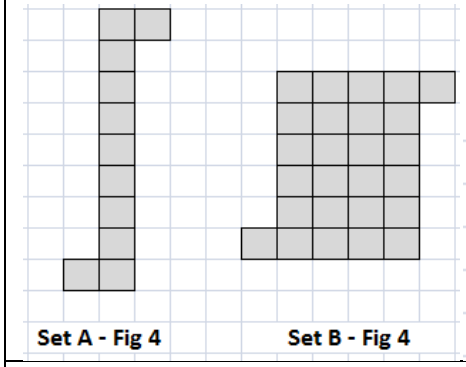
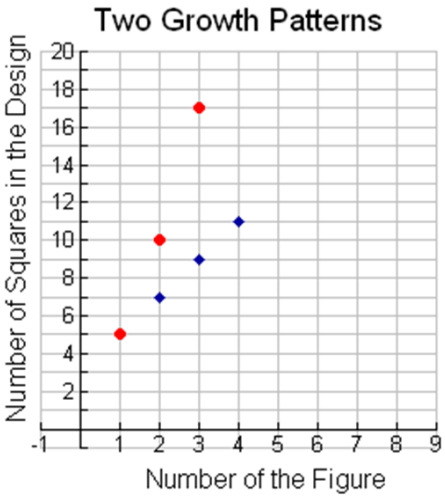
Students may benefit from building the figures in Set A and Set B with color tiles or another square manipulative. When graphing in Part B, the math talk surrounding the choice of which quantity to put on the x-axis and y-axis will be beneficial for students and informative for the teacher.

Common Core State Standards for Mathematical Content	Common Core State Standards for Mathematical Practice
8.F.A.3 Interpret the equation y = mx + b as defining a linear function, whose graph is a straight line; give examples of functions that are and are not linear.	1. Make sense of problems and persevere in solving them 2. Reason abstractly and quantitatively 3. Construct viable arguments and critique the reasoning of others 4. Model with mathematics 5. Use appropriate tools strategically 6. Attend to precision 7. Look for and make use of structure 8. Look for and express regularity in repeated reasoning

Essential Understandings

- Functions provide a tool for describing how variables change together. Using a function in this way is called *modeling*, and the function is called a *model*.
- Functions can be represented in multiple ways—in algebraic symbols, situations, graphs, verbal descriptions, tables, and so on—and these representations, and the links among them, are useful in analyzing patterns of change.
- Some representations of a function may be more useful than others, depending on how they are used.

Explore Phase

Possible Solution Paths	**Assessing and Advancing Questions**
Part A  Set A - Fig 4 Set B - Fig 4 Figure (A) Squares: 1 5; 2 7; 3 9; 4 11 Figure (B) Squares: 1 5; 2 10; 3 17; 4 26	Assessing Questions • What stays the same from Figure 1 to Figure 2 in Set A? (Set B?) • What stays the same from Figure 2 to Figure 3 in Set A? (Set B?) • What is changing from Figure 1 to Figure 2 in Set A? (Set B?) • What is changing from Figure 2 to Figure 3 in Set A? (Set B?) Advancing Questions • Compare and contrast the parts that stay the same (and the parts that change) in Sets A and B.
Part B  BLUE: Figure Number / Number of Squares: 1 5; 2 7; 3 9; 4 11; 5 13 RED: Figure Number / Number of Squares: 1 5; 2 10; 3 17; 4 26; 5 37	Assessing Questions • What quantity's values should be assigned to the horizontal axis? (vertical axis?) • What scale would be appropriate for each axis? Advancing Questions • What patterns do these relationships suggest graphically? (numerically?) • How could you go about predicting the number of squares necessary to create the 10th figure in Set A? (Set B?)
Part C Numerically, the number of squares in the Set A / Blue pattern increase by two with each new figure. The number of squares in the Set B / Red pattern increase by consecutive odd numbers starting with five.	Assessing Questions • Describe the figures in Set A. (Set B) • What stays the same and what is changing in the first three figures of Set A? (Set B?)

Graphically, the Set A figures have 2 squares sticking out that remain constant and vertical rectangle that begins with dimensions 1x3 in the first figure but grows to 1x5, 1x7, 1x9, … 1x(2n+1). The total number of squares would be 2+1x(2n+1) where n is the figure number. The Set B figures have 2 squares sticking out that remain constant and a rectangle of squares whose dimensions change with each figure from 1x3 to 2x4 to 3x5 to … n(n+2). The total number of squares for any figure would be 2+n(n+2) where n is the figure number. Alternately, the Set B figures may be seen as an inner square with an identical rectangle above and below the square. Inner squares are 1x1, 2x2, 3x3, … n x n while the identical rectangles have dimensions 1x2, 1x3, 1x4, … 1x(n+1). This gives total squares of n x n + 2[1 x (n+1)] where n is the figure number.	<u>Advancing Questions</u> • Compare and contrast the first three figures of Sets A and B graphically. (numerically.) • Create an algebraic model that will allow you to find the total number of tiles in any figure without finding all the figures that precede it.
Part D The Set A figures can be modeled using a linear equation because the total number of squares pattern exhibits a constant rate of change. S = 2n+3 where S is the total number of squares and n is the figure number. The Set B figures cannot be described by a linear equation because although the figures exhibit a predictable growth pattern the total number of squares does not have a constant rate of change. Since the second differences are constant for the total number of squares, this pattern is quadratic in nature and can be described by the equation S = n^2 + 2n +2 which may be found by students using a variety of methods.	<u>Assessing Questions</u> • What patterns do you see graphically? (numerically?) • Compare the change in total number of square tiles needed for Figures 1, 2, 3, and 4 for Set A. (Set B.) • What characterizes patterns that are linear? (nonlinear?) <u>Advancing Questions</u> • How many square tiles are needed to make any figure? (or the n^{th} figure) • Create a set of growth figures that illustrate a pattern. What characterizes the pattern you chose? Can you write an algebraic model (expression or equation) for the number of tiles it will take to create the n^{th} figure in your pattern?
Possible Student Misconceptions	
A student indicates that the figure number and total number of square tiles from Set A illustrate a proportional relationship by stating that the pattern increases by 2 each time.	<u>Assessing Questions</u> • What characterizes a proportional relationship? • Can you show how the total number of square tiles needed and the figure number represent a proportional relationship? <u>Advancing Questions</u> • In the figures of Set A, how can the elements of the geometric figure that stay the same be related to the expression for the total number of tiles? • In the figures of Set A, how can the elements of the geometric

	figure that change be related to the expression for the total number of tiles? • Create a set of figures made up of square tiles whose total number of tiles and figure number exhibit a proportional relationship.
A student indicates that Set B illustrates a linear relationship because the width and length of the figure increase by 1 each time.	<u>Assessing Questions</u> • What are the characteristics of a linear relationship? • Explain how the elements of the graphical (or numerical) relationship illustrate a linear pattern. <u>Advancing Questions</u> • Compare and contrast the graphs you created in Part C. What do you notice about the patterns? • Compare and contrast the data tables you created in Part C. What do you notice about the patterns?
Entry/Extensions	**Assessing and Advancing Questions**
If students can't get started….	<u>Assessing Questions</u> • What kind of pattern is illustrated by the numbers 1, 2, 3, 4, 5? • If the top and bottom rows of the figures in Set A that each contain two square tiles were removed, describe the shapes that remain. What kind of pattern do these figures form? • If the two tiles that stick out were removed from the first three figures of Set B, describe the shapes that remain. What kind of pattern do these figures form? <u>Advancing Questions</u> • Write a predictable pattern you are familiar with that contains numbers. Draw a graphical pattern to illustrate the numerical pattern you chose. • Draw a set of rectangles that illustrate a pattern. Create a numerical sequence that illustrates the same pattern.
If students finish early….	<u>Assessing Questions</u> • Create a set of growth figures that illustrate a pattern. What characterizes the pattern you chose? Can you write an expression for number of tiles it will take to create the n^{th} figure in your pattern? <u>Advancing Questions</u> • Given the expression $n^2 – 2n + 3$ as representing the number of

	square tiles necessary to build the n^{th} figure. Create the first four figures in a growth pattern that satisfies the given information.
Discuss/Analyze	
Whole Group Questions	
• Why are the figures in Sets A and B called "Growth Patterns?" How do the figures in Sets A and B demonstrate growth? • Which representation was more helpful in illustrating the nature of the pattern for the figures in Set A? (Set B?) • Was the same representation more helpful when creating the expression for the n^{th} figure in the pattern?	