Task: Collecting Plant Species	**7th Grade**

Darien is working on a science project involving the diversity of plants found in a field near his home. To do his science project, he has measured a 12 foot by 15 foot plot of land and has divided this plot into 1 foot by 1 foot squares. He is listing the plants in each square and collecting samples of the plants for his project. After 2 ½ hours, Darien has collected samples from 2/9 of the field.

a) How many 1 foot by 1 foot squares has he completed in 2 ½ hours? Explain your reasoning.

b) If Darien worked at a constant rate, what fraction of the field would he have completed in 1 hour? Draw a picture to support your calculation.

c) If Darien continues to work at the same constant rate, how long will it take him to collect the samples from his entire plot of land? Explain how you know.

d) Darien's partner LiliAndra is doing a similar investigation in a field near her home. This field is long and narrow, so she has measured a 6 foot by 30 foot plot of land to use and has divided her plot into 1 foot by 1 foot squares. In 3 hours and 20 minutes, LiliAndra has collected samples from 60 of her 1 foot by 1 foot squares. At what rate is LiliAndra collecting samples? How long will it take her to complete her field?

e) Draw a graph to compare Darien's progress to LiliAndra's. Does the shape of the field make a difference in your graph? Why or why not?

Teacher Notes:

Some parts of this task require students to report a rate. Note that rates can be given either as the number of squares per hour or as the fraction of the field completed per hour.

Some of the diagrams may be difficult to draw in order to show equal parts. Students may need additional guidance in order to make sense of the diagrams.

Parts (d) and (e) can be omitted in the interest of time if necessary. However, the question regarding the impact of the shape of the field on calculations and mathematical thinking should be considered at some point in the discussions.

Common Core State Standards for Mathematical Content	**Common Core State Standards for Mathematical Practice**
7.RP.A.1. Compute unit rates associated with ratios of fractions, including ratios of lengths, areas and other quantities measured in like or different units. *For example, if a person walks 1/2 mile in each 1/4 hour, compute the unit rate as the complex fraction 1/2/1/4 miles per hour, equivalently 2 miles per hour.*	Mathematical Practices 1. Make sense of problems and persevere in solving them. 2. Reason abstractly and quantitatively. 3. Construct viable arguments and critique the reasoning of others. 4. Model with mathematics.

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7.RP.A.2. Recognize and represent proportional relationships between quantities.
a. Decide whether two quantities are in a proportional relationship, e.g., by testing for equivalent ratios in a table or graphing on a coordinate plane and observing whether the graph is a straight line through the origin.
b. Identify the constant of proportionality (unit rate) in tables, graphs, equations, diagrams, and verbal descriptions of proportional relationships.
c. Represent proportional relationships by equations. *For example, if total cost t is proportional to the number n of items purchased at a constant price p, the relationship between the total cost and the number of items can be expressed as t = pn*.
d. Explain what a point *(x, y)* on the graph of a proportional relationship means in terms of the situation, with special attention to the points (0, 0) and (1, *r)* where *r* is the unit rate.

7.RP.A.3. Use proportional relationships to solve multistep ratio and percent problems. *Examples: simple interest, tax, markups and markdowns, gratuities and commissions, fees, percent increase and decrease, percent error.*
</td><td>5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.</td></tr>
<tr><td colspan="2">Essential Understandings</td></tr>
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- Reasoning with ratios involves attending to and coordinating two quantities.
- Forming a ratio as a measure of a real-world attribute involves isolating that attribute from other attributes and understanding the effect of changing each quantity on the attribute of interest.
- Several ways of reasoning, all grounded in sense making, can be generalized into algorithms for solving proportion problems.
- Linear functions have constant rates of change.
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<tr><td colspan="2">Explore Phase</td></tr>
<tr><td>Possible Solution Paths</td><td>Assessing and Advancing Questions</td></tr>
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a) Students can use some calculations to determine how many squares have been completed:

- Total number of squares: 12 x 15 = 180 squares.
- 2/9 of the field has been completed, so 2/9 x 180 squares = 40 squares have been completed.
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Advancing Questions:
How many 1 foot by 1 foot squares are in Darien's plot? How can this help you answer the question?

Can you draw a picture to help you with this problem?
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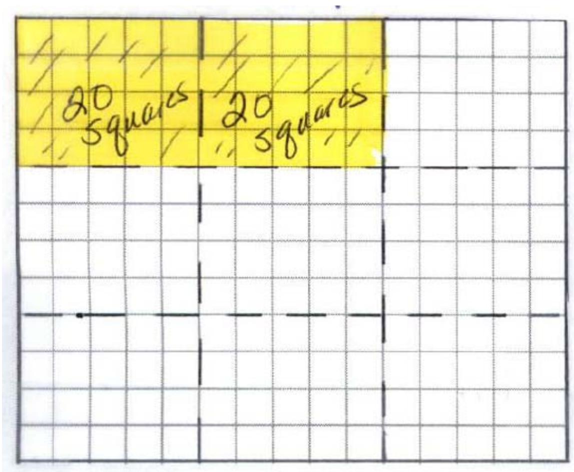
<td>Students can also draw a picture to determine how many squares have been completed. The original 12 foot by 15 foot field can be drawn as a rectangle. The rectangle can be divided into 9 equal parts, with 2 of those 9 parts shaded. The student can then count the number of squares in those 2 parts.



Note that the "2 ½ hours" information is not needed to complete part (a).</td>
<td>Assessing Questions:
How did you know how many squares were in Darien's plot of land?

How did you figure out how many squares had been completed?

How did you use the 2 ½ hours in your calculations?</td>
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<td>b) Students can calculate the number of squares completed per hour using their information from part (a):

(40 squares) / (2 ½ hours) = (40 squares) / (5/2 hours)
= 16 squares/hour,
and (16 squares per hour)/(180 squares in the field)
= 4/45 of the field per hour.

Students can also calculate the fraction of the field completed per hour:

(2/9 field) / (2 ½ hours) = (2/9 field) / (5/2 hours)
= 4/45 of the field per hour.</td>
<td>Advancing Questions:
What information do you need to build your fraction?

If you know how much is completed in 2 ½ hours, how can you use this information to figure out how much is completed in 1 hour?

How can a picture of the field help you with your calculations?

Assessing Questions:
How did you figure out your fraction? What information did you use to complete your calculation?

Explain your diagram to me. How does your diagram support your calculations?</td>
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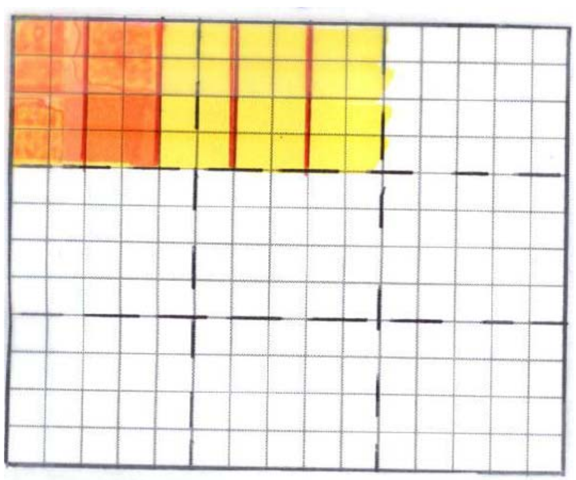
For the diagram, we will go back to the graph in part (a). Note that the portion highlighted in yellow represents the part of the field Darien has completed.

We need to figure out how much Darien has completed in one hour. Since Darien has worked for 2 ½ hours, we need a way to divide our "completed" portion into equal parts that can be connected in some way to the 2 ½ hours. We know that 2 ½ hours is equal to 5/2 hours, or 5 "1/2 hour" pieces. We begin by dividing the yellow portion into 5 equal parts; these divisions are noted in red.

Since 1 full hour is made up of 2 of the "1/2 hour" pieces, we will shade 2 of the 5 "pieces" in our picture:



Next, we try to extend our "divisions" to the entire field (denoted in red below).

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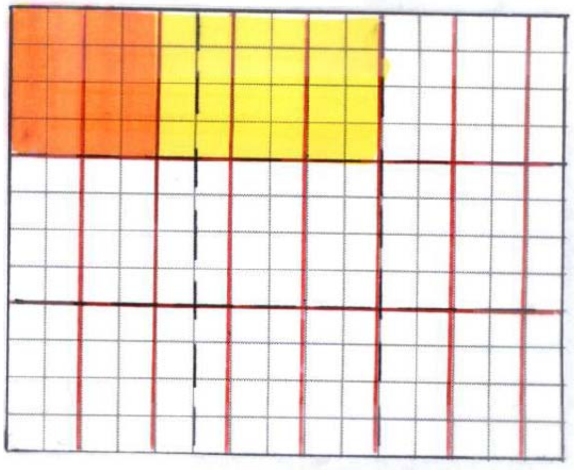
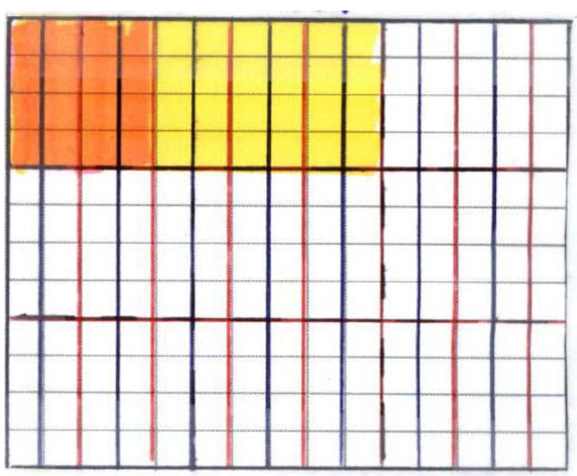


We note that this does not divide the field into equal parts, so we will have to add some subdivisions (in blue on the next graph). Our region shaded in orange, which represents the part of the field completed in 1 hour, is now 4/45 of the entire field.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c) Using calculations:

- 1 field / (4/45 of the field per hour) = 45/4 hours.
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Advancing Questions:
If you know how much of the field you can complete in one hour, how can you use this to figure out how long it takes to complete the field?
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= 11 ¼ hours to complete the field

OR, using the number of squares:

- 1 field = 180 squares, and 180 squares / (16 squares per hour) = 11 ¼ hours to complete the field.

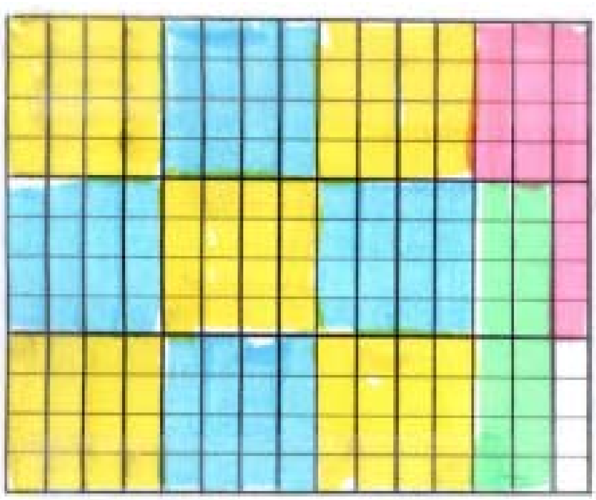
OR, using a proportion:

- (2/9 of the field) / (5/2 hours) = (1 field) / (x hours).

  So 2/9 times x = 5/2 times 1.

  So x = 5/2 divided by 2/9 = 45/4 = 11 ¼ hours.

OR, using a diagram similar to the last one in part (b), we divide the field into 45 equal parts (since we know Darien completes 4 / 45 of the field per hour), and we circle groups of 4 parts (since 4 "parts" equals 1 hour of work). There are 11 groups of 4 parts (so we have 11 whole hours of work), with 1 part "left over", and this 1 part represents ¼ of an hour. (Note: In the diagram below, groups of "4 parts" are represented with different colors.)



Can you use the number of completed squares to do your calculation?

Can you use your picture do your calculation?

Assessing Questions:
How did you do your calculation?

Are there any other ways to do the problem? If you use a different method, will you get a different answer?

d) Using calculations:

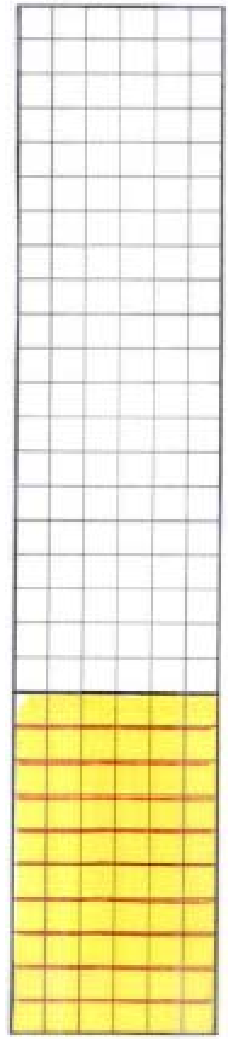
3 hours and 20 minutes = 3 1/3 hours = 10/3 hours, so
(60 squares) / (10/3 hours) = 18 squares/hour.

OR, using fractions:

(60 squares) / (180 squares/field) = 1/3 of the field completed in 3 1/3 hours, so (1/3 field) / (10/3 hours) = 1/10 field/hour.

OR, using the diagram:

60 squares are shaded yellow and are completed in 10/3 hours. Dividing the 60 squares into 10 “1/3 hour” segments (using the red divisions) gives 6 squares per “1/3 hour”, or 18 squares/hour.



To complete the field:

(180 squares) / (18 squares/hour) = 10 hours.

OR, using the rate of 1/10 field/hour:

In 1 hour, 1/10 of the field is completed, so since the entire

Advancing Questions:

How can you describe the rate at which LiliAndra is completing her work?

What fraction of the field is completed each hour?

Would a diagram be helpful in solving the problem?

Assessing Questions:

How did you calculate your rate?

Is there more than one way to describe LiliAndra’s rate?

Explain your calculations to me.

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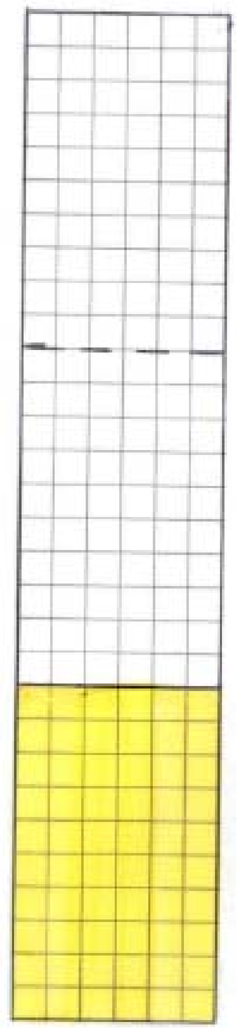
field = 10/10 of the field, it will take 10 hours to complete the field.

OR, using the diagram:
In 3 hours and 20 minutes, 60 squares (or 1/3 of the field) is completed, so it will take 3 x 3 hours and 20 minutes = 10 hours to complete the field.

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e) Darien completes 16 squares per hour; after 0 hours he has completed 0 squares and after 1 hour he has completed 16 squares, so we graph the line that goes through the points (0,0) and (1,16) (where the x-coordinate represents the number of hours and the y-coordinate represents the number of squares completed). Darien's graph is the red line below.

Similarly, LiliAndra completes 18 squares per hour. LiliAndra's
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Advancing Questions:
In your graph, what will x represent? What will y represent?

Will your graph representing Darien's progress be a line or curved? Why? How will you determine what the graph should look like?

(Similar questions can be asked for LiliAndra's progress.)
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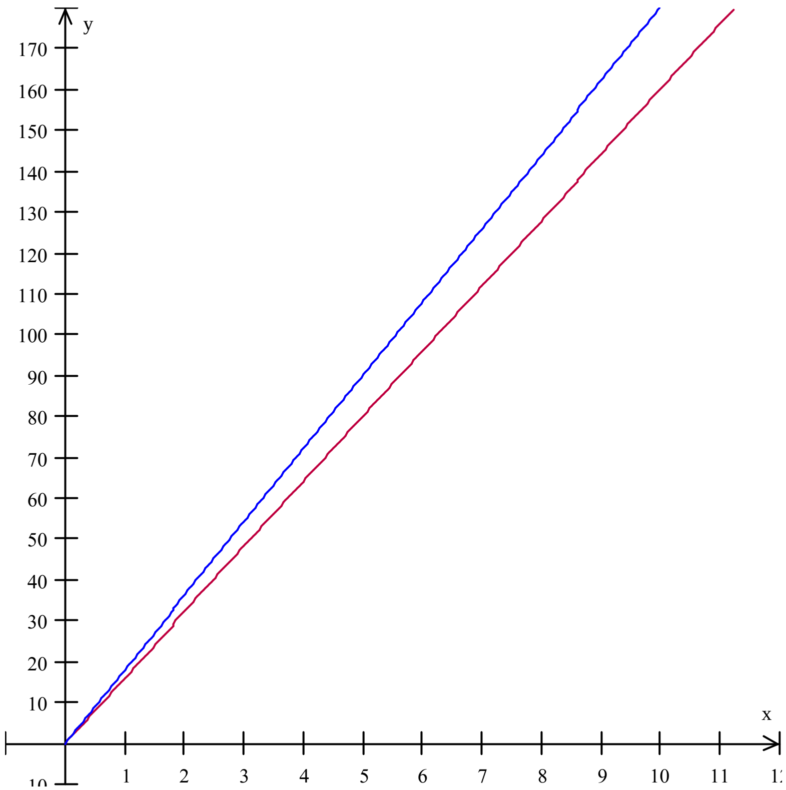
graph is in blue.



The shape of the field does not affect the graph since, in both cases, we are counting the number of 1 foot by 1 foot squares completed in each hour.

Assessing Questions:

What does the slope of each line represent?

How can you use the graph to determine who finishes first?

What point on the graph shows how long it takes each person to complete the graph?

Possible Student Misconceptions	
In part (a), students may try to multiply 2/9 by 2 ½ in order to use all of the information given in the problem.	What does the 2 ½ represent? Does this help you answer the question?
In part (b), students may find it difficult to draw the picture.	What fraction of the field is completed in 2 ½ hours? Can you draw a picture of this? Can you use your drawing to figure out how much of the field is completed in 1 hour?
In part (c), students can work using either the number of squares completed in one hour or the fraction of the field completed in one hour.	Can you use a diagram with the number of squares completed every hour to help you determine how long it will take to complete the field?
In part (d), the rate can be given either using the number of squares	What does the rate measure in this case? How can you use this to

per hour or the fraction of the field completed in one hour. Students may be confused regarding which should be used.	determine LiliAndra's rate of completion?
Entry/Extensions	**Assessing and Advancing Questions**
One of the key pieces of information students need to know is how many 1 foot by 1 foot squares are marked off.	How many 1 foot by 1 foot squares are in Darien's plot of land? How do you know? Can you draw a picture to show me how you know?
Several parts of the problem can be approached in different ways.	Compare your work to someone else's (who worked the problems in a different way). Explain your approaches to each other. Do you both get the same answers? Why or why not?

Discuss/Analyze

Whole Group Questions

- How does the use of a diagram help you with your problems?
- Compare the different solution paths. Some of you gave me rates as the number of squares per hour and others gave me rates as the fraction of the field completed per hour. Are these equivalent? Why or why not?
- Did your different rates result in different graphs in part (e)? Why or why not?