Task: Capture-Recapture Problem **7th Grade**





1. Cynthia learns that a proportional relationship exists between the number of deciduous trees and the amount of land in acres at the Alex Haley National Forest.
   a. If the constant of proportionality is 25 deciduous trees per acre, write an equation for the number of trees in the Alex Haley National Forest, *N*, in terms of the number of acres, *A*, in the forest.
   b. Estimate the number of deciduous trees in the Alex Haley National Forest if the land in the forest totals 14,500 acres. Show your work and explain your reasoning.

2. Max takes one liter samples from the shallow pond near his house. The number of tadpoles per sample is given in the table below.

Sample Number	**1**	**2**	**3**	**4**	**5**
Tadpoles	**124**	**117**	**99**	**118**	**107**

   a. Estimate the number of tadpoles per liter for his pond. Show your work and explain your reasoning.
   b. If the pond holds 30,000 liters of water, how many tadpoles do you estimate are in the pond? Show your work and explain your reasoning.

3. Reesa is monitoring the snail population in the garden of her home. She captures 250 snails and marks their shells. Afterwards she releases the snails back into the garden to go about their work. Two days later she returns to the garden to capture 50 snails. Of those, 17 snails have marked shells.
   a. Reesa writes the equation $\frac{250}{x} = \frac{17}{50}$ to estimate the number of snails in her garden, *x*. Explain whether or not this is a reasonable equation.
   b. Estimate the number of snails in Reesa's garden. Explain your reasoning.

Teacher Notes:

Population sizes for living things can be found using a variety of methods depending on the circumstances.

- The African elephants of the savannah live in stable herds that are easily seen so that each individual can be counted to find the number of elephants in the total population.
- Redwood trees, barnacles, and tadpoles can have their population size estimated using a density approach. If a population has a uniform distribution across a specific area or volume then by counting the number of individuals from a specific portion of that area or volume, the population density (constant of proportionality) is found and can be used to extrapolate the size of the total population.
- Other species such as bears, fish, birds, banded snails, or woodland ants have stable but mobile populations living in a specific habitat. These animals may have their population estimated by a Capture – Mark – Release – Recapture model sometimes called the Lincoln-Peterson Index.

Common Core State Standards for Mathematical Content	Common Core State Standards for Mathematical Practice
7.RP.A.2 Recognize and represent proportional relationships between quantities. **7.RP.A.2c** Represent proportional relationships by equations. For exam ple, if total cost t is proportional to the number n of items purchased at a constant price p, the relationship between the total cost and the number of items can be expressed as t = pn. **7.EE.B.3** Solve multi-step real-life and mathematical problems posed with positive and negative rational numbers in any form (whole numbers, fractions, and decimals), using tools strategically. Apply properties of operations to calculate with numbers in any form; convert between forms as appropriate; and assess the reasonableness of answers using mental computation and estimation strategies. For example: If a woman making $25 an hour gets a 10% raise, she will make an additional 1/10 of her salary an hour, or $2.50, for a new salary of $27.50. If you want to place a towel bar 9 3/4 inches long in the center of a door that is 27 1/2 inches wide, you will need to place the bar about 9 inches from each edge; this estimate can be used as a check on the exact computation.	1. Make sense of problems and persevere in solving them. 2. Reason abstractly and quantitatively. 3. Construct viable arguments and critique the reasoning of others. 4. Model with mathematics. 5. Use appropriate tools strategically. 6. Attend to precision. 7. Look for and make use of structure. 8. Look for and express regularity in repeated reasoning.

Essential Understandings

- Mathematical situations and structures can be translated and represented abstractly using variables, expressions, and equations.
- If two quantities vary proportionally that relationship can be represented as a linear function.
- Mathematical rules can be used to assign members of one set of members to another set. A special rule assigns each member of one set to a unique number of the other set.

Explore Phase

Possible Solution Paths	Assessing and Advancing Questions
1.a. N = 25 A or 25/1 = N/A **1.b.** To estimate the number of trees in the Alex Haley National Forest, substitute 14,500 acres into the rule N = 25 A for the variable A to calculate N = 25 (14,500) = 362,500 deciduous trees.	Assessing Questions • Can the constant of proportionality also be a unit rate? Explain. • Explain the constant of proportionality as ratio. Advancing Questions • Can you write the equation for 1.a in another way? • If 1 acre = 4046.86 square meters, then how many square meters are available for each deciduous tree?
2.a. To estimate the number of tadpoles per liter of water, students may use the median of the data (117) or find the mean of the five values (124+117+99+118+107)/5 = 113 tadpoles per liter of water **2.b.** Depending on the student's answer to part 2.a, an appropriate rule for the proportion is T = 117 W or T = 113 W where T = number of tadpoles and W = liters of water. To calculate the number of tadpoles,	Assessing Questions • What value is representative of the number of tadpoles in each of the five samples taken? • Could you locate the number of tadpoles per sample on a number line to visualize what a representative number might be? Advancing Questions • How is your equation in 2.b similar or different from your answer to 1.a?

substitute 30,000 liters of water for the variable W to find that T is either 3,510,000 or 3,390,000 tadpoles.	• How could you go about making a mental estimate of the number of tadpoles in the pond?
3a. Reesa has set up a reasonable proportion because the first ratio 250/x represents the number of snails with marked shells to the entire population of snails while 17/50 represents the number of snails with marked shells compared to the number of snails in the sample population. Since both ratios compare the marked snail shells to the total in the group, this is a correctly written equation. **3.b.** Using this proportion to multiply the means and extremes produces a new equation 17x = 50 (250) which when solved for x gives 735.29 or 735 snails in Reesa's garden.	<u>Assessing Questions</u> • Can you think of two ways to create ratios using the snails in the garden? • Why does Reesa compare snails with marked shells to the total number of snails instead of snails with marked shells to snails with unmarked shells? <u>Advancing Questions</u> • Can you calculate and state the constant of proportionality for the relationship between the quantity of snails with marked shells and the total population of snails in the garden? • Can you create an equation for Reesa that is of the form $y = mx$ that uses the constant of proportionality for the snails with marked shells and the total population of snails in the garden?
Possible Student Misconceptions	
One possible misunderstanding could be that students write the equation with the constant of proportionality on the incorrect side, e.g. A = 25 N instead of N = 25 A or W = 117 T instead of T = 117 W.	• If the constant of proportionality represents trees per one acre, then what type of variable should be its multiplier and why? • If the constant of proportionality is multiplied by acres to find trees, then the constant of proportionality should be written as (trees or acres) per (acre or tree)?
Students may conclude in part 3.a that Reesa's equation is incorrect.	• In Reesa's equation, 17/50 represents snails in the sample with marked shells to what? • In Reesa's equation, the 250 represents what? • In a correctly written proportion should each ratio have similar quantities for numerator and denominator or different ones?
Entry/Extensions	**Assessing and Advancing Questions**
If students can't get started….	• What does the 25 tell about the forest? (Question 1a) • What is a constant of proportionality? • What does the constant of proportionality indicate about a relationship between two quantities?
If students finish early….	• If all the bears in the Smokies are represented by the bowl of black beans in the bowl, describe how to create a capture – recapture situation similar to Question 3 to estimate the bear population. (This would require preparation by placing two one-pound bags of black beans in a bowl.)
Discuss/Analyze	
Whole Group Questions	

- What role does the constant of proportionality play in helping to relate two quantities?
- When selecting a representative value for a data set what are some ideas to keep in mind?
- What key ideas should be remembered when creating a proportion?
- How can a proportion be rewritten as an equation of the form $y = mx$ (and vice versa)?