Tennessee Department of Education:

TNCore

Task: Matthew's Dilemma	**3rd Grade**

Matthew did not know the answer to 72 divided by 8. Are each of the following an appropriate way for Matthew to think about the problem? Explain why or why not by drawing a picture and writing an explanation for each one.

1) **"I know 8 x 9 = 72, so 72 divided by 8 must be 9."**
2) **"I know 8 x 10 = 80. If I take away a group of 8, that means I have 8 x 9 = 72. So 72 divided by 8 is 9."**
3) **"I know that 8 x 5 = 40. 72 – 40 = 32. I know that 8 x 4 = 32. So if I add 8 x 5 and 8 x 4, I get 72. That means that 8 x 9 is 72 or 72 ÷ 8 = 9."**

Teacher Notes

The focus of three key strategies in this task involve multiplication and division as inverse operations, using a known fact to solve an unknown fact, and using a distributive property to partition an area in solving the problem. All three strategies used in the problem are correct.

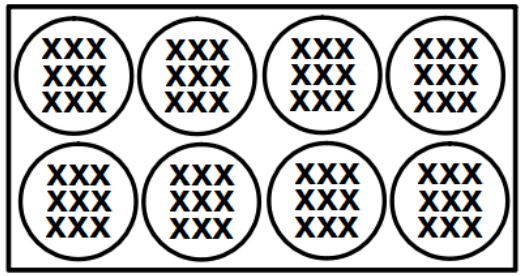
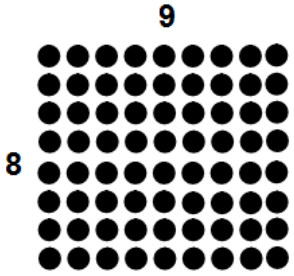
Models used in solution paths show three key strategies for multiplication and division problems. It is possible for a student to use any combination of these strategies shown below.

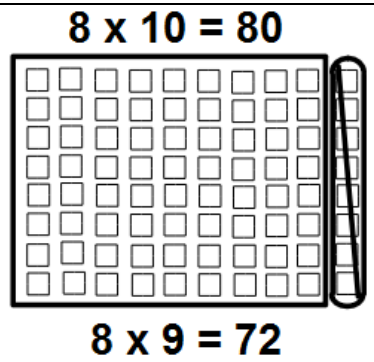
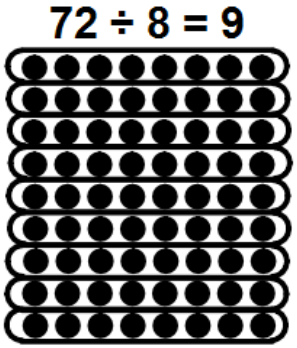
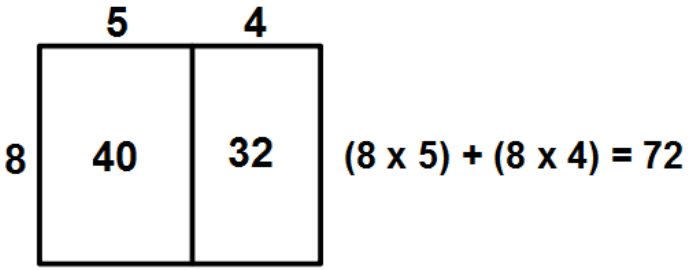
Common Core State Standards for Mathematical Content	**Common Core State Standards for Mathematical Practice**
3.OA.A.1 Interpret products of whole numbers, e.g., interpret 5 × 7 as the total number of objects in 5 groups of 7 objects each. *For example, describe a context in which a total number of objects can be expressed as 5 × 7.* 3.OA.A.2 Interpret whole-number quotients of whole numbers, e.g., interpret 56 ÷ 8 as the number of objects in each share when 56 objects are partitioned equally into 8 shares, or as a number of shares when 56 objects are partitioned into equal shares of 8 objects each. *For example, describe a context in which a number of shares or a number of groups can be expressed as 56 ÷ 8.* 3.OA.A.3 Use multiplication and division within 100 to solve word problems in situations involving equal groups, arrays, and measurement quantities, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem. 3.OA.B.5. Apply properties of operations as strategies to multiply and divide.2 *Examples: If 6 × 4 = 24 is known, then 4 × 6 = 24 is also known. (Commutative property of multiplication.) Knowing that 8 × 5 = 40 and 8 × 2 = 16, one can find 8 × 7 as 8 × (5 + 2) = (8 × 5) + (8 × 2) = 40 + 16 = 56. (Distributive property.)* 3.OA.B.6 Understand division as an unknown-factor problem. For example, find 32 ÷ 8 *by finding the number that makes 32 when multiplied by 8.*	1. Make sense of problems and persevere in solving them 2. Reason abstractly and quantitatively 3. Construct viable arguments and critique the reasoning of others 4. Model with mathematics. 5. Use appropriate tools strategically 6. Attend to precision 7. Look for and make use of structure 8. Look for and express regularity in repeated reasoning

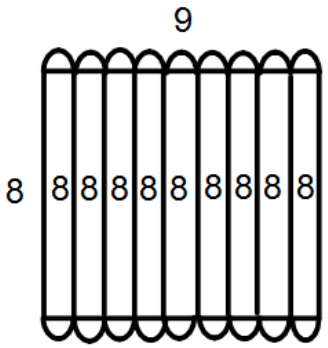
Essential Understandings

- Multiplication can be used to find the total number of objects when there are a specific number of groups with the same number of objects.
- When multiplying two factors, either factor can be partitioned or both. Example: 4 x 16 = 4 x (10 + 6) or (2 + 2) x 16
- Division can be used to find how many equal groups (measurement - repeated subtraction) or how many are in each group (partitive - sharing).
- Multiplication and division have an inverse relationship and can be used to find division or multiplication facts.

Explore Phase

Possible Solution Paths	**Assessing and Advancing Questions**
<u>Equal Groups or Arrays</u> 1) Matthew knows 8 x 9 = 72 by thinking about 8 groups of 9.  So, 72 divided into groups of 8 or 72 ÷ 8 = 9. Therefore, this is a correct way for Matthew to think about the problem.  	Assessing Questions: - Why did you choose 8 groups? - Why did you choose 9 in each group? - What does each number represent in your equation? (Ask students to relate numbers back to the model.) - How does an array show equal groups? Advancing Questions: - What is the relationship between multiplication and division? - What do you notice about the product and the dividend? Why are they the same? - What would happen if you turned your array sideways? - How can you make a connection between the two models (equal sized groups and arrays)?
<u>Equal Groups or Arrays</u> 2) Matthew knows 8 x 10 = 80 by thinking about 8 longs in base ten blocks. If one column of 8 is taken away, that leaves 9 columns of 8 or 9 x 8 = 72.	Assessing Questions: - Why did you multiply 8 x 10? - Why did you cross out a group of 8? - What does each number represent in your equation? (Ask students to relate numbers back to the model.)

 So, 72 divided into groups of 8 or 72 ÷ 8 = 9. Therefore, this is a correct way for Matthew to think about the problem. 	- How does an array show equal groups? Advancing Questions: - What is the relationship between multiplication and division? - What do you notice about the product and the dividend? Why are they the same? - What would happen if you turned your array sideways? - How can you make a connection between the two models? - Will this strategy work on all multiplication/division facts? Explain.
<u>Area Model</u> 3) Matthew may think about the distributive property to solve 8 x 9 by thinking about how 9 can be broken apart into 5 and 4. Therefore, (8 x 5) + (8 x 4) = 72.  So, 72 divided into groups of 8 or 72 ÷ 8 = 9. Therefore, this is a	Assessing Questions: - How is breaking the rectangle apart helpful in solving the problem? - How does your equation relate to the model? Advancing Questions: - Is it possible to partition (break apart) either factor or both? Explain your thinking. - Will this strategy work on all multiplication problems? Explain. - How can you make a connection between the two models (area model and base-ten model)?

correct way for Matthew to think about the problem.  	
Possible Student Misconceptions	
Students may think that if they get stuck on a multiplication fact, the problem can't be solved. Students may not see that multiplication and division are inverse operations and that connection can be used to find the unknown.	Assessing Questions: - What is the question you are trying to answer? - What multiplication/division fact do you know that is related to this question? - Will this strategy work? Explain. Advancing Questions: - Is it possible to use a multiplication fact to solve an unknown fact in division? Explain. - What do you notice about the product and the dividend? Why are they the same? - Does the strategy make sense? How? - Will this strategy work on other multiplication/division problems? Explain.
Entry/Extensions	**Assessing and Advancing Questions**
If students can't get started….	

	Assessing Questions: - What are you trying to find or do? - What model could you draw to help you understand these statements? Advancing Questions: - What multiplication/division fact do you know that is related to this question? - Using the fact you know, how can you figure out if these statements are true?
If students finish early….	Assessing Questions: - Does your solution make sense when you look at the original problem? Explain. - Is it reasonable? Explain your thinking. Advancing Questions: - How could you work the problem differently? - Write another strategy that is true and/or false.

Discuss/Analyze

Whole Group Questions

Select and sequence refers to when a teacher anticipates possible student strategies ahead of time and then selects and determines the order in which the math ideas/strategies that students will share during the whole group discussion. The purpose of this is to determine which ideas will be most likely to leverage and advance student thinking about the core math idea(s) of the lesson.

During a whole group discussion, students are sharing their strategies that have been pre-selected and sequenced by the teacher. Strategies to consider sharing in order to justify student thinking on three statements: Equal Groups, Arrays, and an Area Model.

- How are these strategies similar and different? (Use Accountable Talk to ask students to compare strategies.)

- Where do you see the numbers in your equation in the model?

- The first strategy used the relationship between multiplication and division to find the solution. What is the relationship between multiplication and division? Will this strategy work all the time? Explain.

- What do you notice about the product and the dividend? Why are they the same in this problem?

- The second strategy started with 8 x 10 in order to figure out 72 divided by 8. Will this strategy work on all multiplication/division problems?

- The third strategy used the partition model to solve the problem. How is breaking the rectangle apart helpful in solving the problem? Is it possible to partition (break apart) either factor or both? Explain your thinking.

- Which of these strategies work on all multiplication/division problems?

- When do you use multiplication to solve a problem? When do you use division to solve a problem?