Task: A Hexagonal Puzzle **8th Grade**

The task is for each student/group to explain how the equation(s) expresses the relationship between the number of hexagons (n) and the perimeter (p) of the total figure. Students are also to give their interpretation of the meaning of each term in the equations as supported by the geometry of the figures.

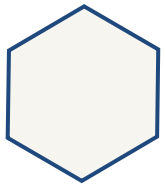


Figure 1

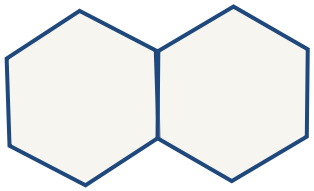


Figure 2

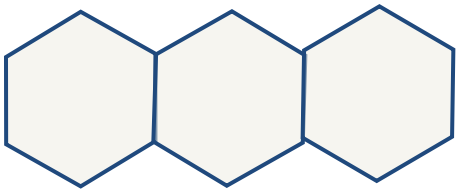


Figure 3

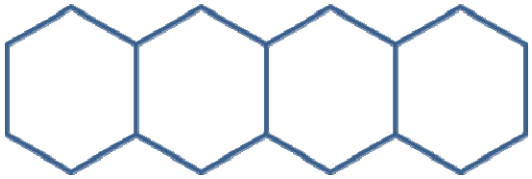


Figure 4

a. p = 4n + 2
b. p = 6n – 2(n – 1)
c. p = 10 + 4(n-2)
d. p = 2(2n + 1)

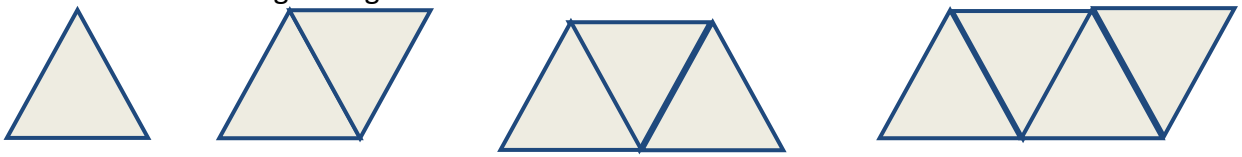
Each side represents one unit

Teacher Notes:

Each side of the hexagon represents one unit. If you have access to pattern blocks the students can use those to model perimeter with one, two, three and four hexagonal shapes. This will help them to have some understanding of the task.

To increase the demands of the task, the task could be for students to find a model for the relationship between the number of hexagons and the perimeter as many ways as possible (parts a, b, c, and d not given).

A possible extension (noted later in this template) is for students to come up with different ways to express the relationship between the number of triangles and perimeter of the total figure. After working through the task with hexagons and the formulas given, working to model the formulas with the triangles might be more accessible.



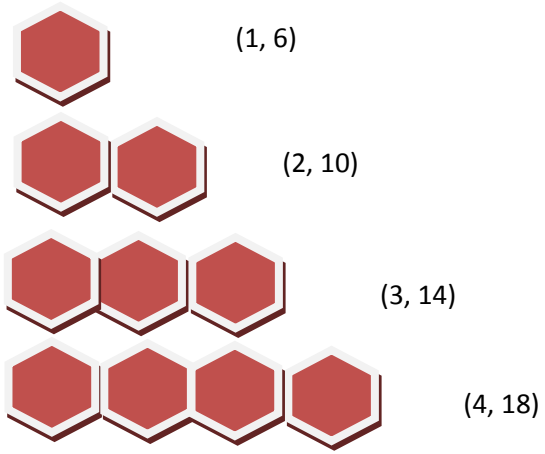
Common Core State Standards for Mathematical Content	Common Core State Standards for Mathematical Practice
8.F.A.1. Understand that a function is a rule that assigns to each input exactly one output. The graph of a function is the set of ordered pairs consisting of an input and	**1. Make sense of problems and persevere in solving them.**

the corresponding output. 8.F.B.4. Construct a function to model a linear relationship between two quantities. Determine the rate of change and initial value of the function from a description of a relationship or from two (x, y) values, including reading these from a table or from a graph. Interpret the rate of change and initial value of a linear function in terms of the situation it models, and in terms of its graph or a table of values. 8.EE.B.6 Use similar triangles to explain why the slope m is the same between any two distinct points on a non-vertical line in the coordinate plane; derive the equation y = mx for a line through the origin and the equation y = mx + b for a line intercepting the vertical axis at b.	**2. Reason abstractly and quantitatively.** **3. Construct viable arguments and critique the reasoning of others.** **4. Model with mathematics.** 5. Use appropriate tools strategically. **6. Attend to precision/accuracy.** **7. Look for and make use of structure.** **8. Look for and express regularity in repeated reasoning.**

Essential Understandings

- Variables have many different meanings, depending on context and purpose.
- Using variables permits writing expressions whose values are not known or vary under different circumstances.
- Using variables permits representing varying quantities. This use of variables is particularly important in studying relationships between varying quantities.

Explore Phase

Possible Solution Paths	**Assessing and Advancing Questions**
 **a. 4n+2=p**	Assessing questions: How does each term in the equation relate to the total perimeter? How does the value of “b” (y-intercept) relate to the problem? What is the relation of 4n to the problem? Advancing questions: How can you find the rate of change that expresses the progression in perimeter as you add another shape to your total shape?

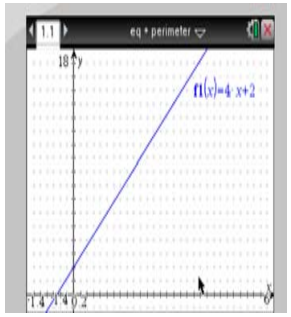
Method one: Find slope then calculate the y intercept.

Shape	Perimeter
1	6
2	10
3	14
4	18

$$\frac{10-6}{2-1}=4$$

$$y=mx+b$$
$$10=4(2)+b$$
$$2=b$$
$$y=4x+2$$

Method two: Graph and find "b"



Graph and find rise over run

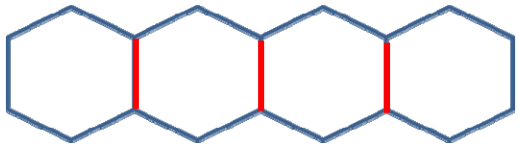
Find the intercept

Y=4x+2

Contextual Meaning:

***4n represents the rate of change resulting from the adding of each new shape. Each hexagon has 4 sides that are part of the perimeter on the top and bottom.**

***The 2 represents the first and last side which are always present.**



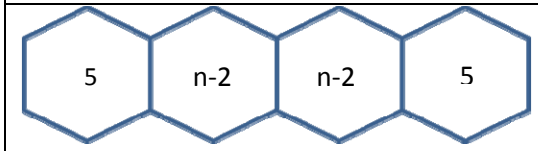
6n-2(n-1)=p

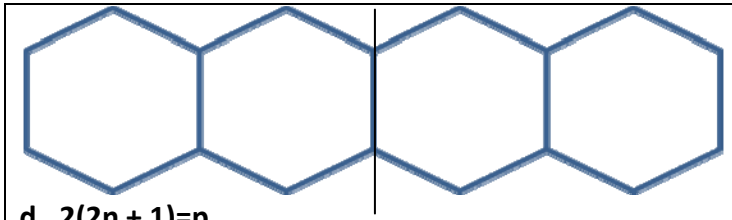
Assessing questions:

- How can you justify the use of the term (n-1) in the question?
- Explain how this equation would work for 1, 2 and 3 hexagons?

Advancing questions:

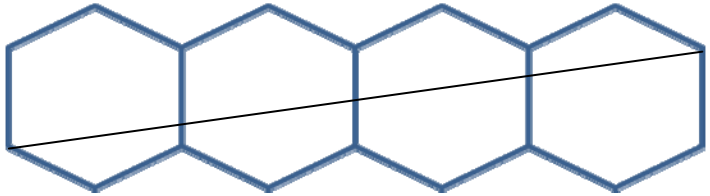
- What property can you use to simplify this equation?

***6n represents the perimeter of one hexagon** ***(n-1) represents the number of "shared" sides there are each time a shape is added** ***2 represents that the "shared" sides would be counted twice.**	• What are your first steps in simplifying an equation?
 **c. 10+4(n-2)=p** ***10 represents the exposed sides on the first and last shape.** ***4 represents the number of sides exposed in the middle two shapes** ***(n-2) represents the number of hexagons not exposing all 6 sides but are only exposing 4 sides.**	Assessing questions: • How can you justify the use of the value 10 in this equation? • How could you provide a proof showing if this equation would work for both 1 and 3 hexagons? Give examples of your proof. Advancing questions: • How does the perimeter of each shape help to form the given equation? • How does each shape contribute to the total perimeter?

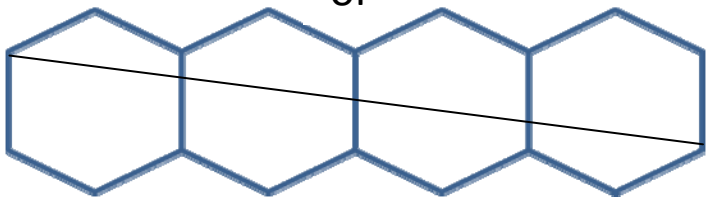


d. 2(2n + 1)=p

Contextual Meaning:
***The first "2" means the figure has been split into 2 parts.**
***(2n+1) is the represents the value of the left or right part. Each hexagon exposes 2 sides to the top and bottom of the perimeter plus there is one side exposed on the first or last hexagon.**



or



This is another way of splitting the figure. Here the value of the 1 is more easily depicted.

Assessing Question:

- How does each term contribute to the formation of the equation?

Advancing Question:

- With the 2 starting the equation how could you split the problem into two parts to make sense of the equation?

Possible Student Misconceptions	
Students may include sides of hexagons in the interior of the figure that aren't part of the perimeter.	Not all equations would represent any number of shapes. The equation **10+4(n-2)=p** would only apply to this shape**.** Another miscomception could be that perimeter only involves the out side distance around the figure.
Entry/Extensions	**Assessing and Advancing Questions**
If students can't get started….	*Could you write a table of values of number of shapes compared to the perimeter of the figure? This may give you insight into the equation. *How does equation A relate to the linear model of y=mx+b?

If students finish early….	*If I added a fifth shape would any of the equations model that relationship. *What if I put together triangles with each side being one unit. How many different equations could you write to model that relationship 
Discuss/Analyze	
Whole Group Questions	
The key understandings that students should come to in the discussion of this task and questions you can ask in the whole group setting to support arrival at these key understandings. Key understanding: *Are all of these expressions equivalent? How do we know? What would be the perimeter of the 9^{th} figure? 100^{th} figure? *Discuss different students' approaches to solving the different models. *There may be several paths to form a relationship but if they all simplify to the same equation then they must be equal. *What property can we use to justify our findings? Reflexive property of equality y=y 4n+2=4n+2	