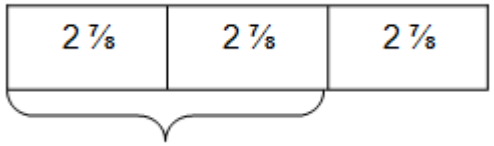
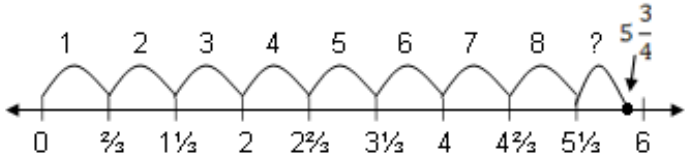
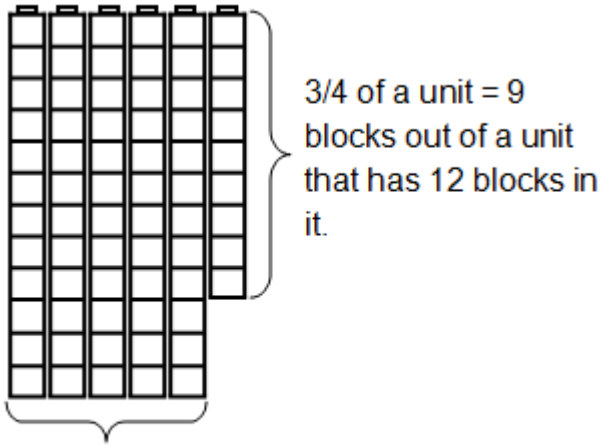
Task: Bicycle Ride to School	**6th Grade**
Yesterday Josh rode his bicycle from home to school. He had given himself 10 minutes to get to school so he wouldn't be late. After he had ridden for 5 ¾ minutes, he was two-thirds of the way to school. Draw a model and write an equation to help you determine how long it took Josh to get from home to school and if he will be late or not.	
Common Core State Standards for Mathematical Content	**Common Core State Standards for Mathematical Practice**
FOCUS **CCSS.Math.Content.6.RP.A.3d.** Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations. d. Use ratio reasoning to convert measurement units; manipulate and transform units appropriately when multiplying or dividing quantities. OTHER 6.NS.A.1 Interpret and compute quotients of fractions, and solve word problems involving division of fractions by fractions, e.g., by using visual fraction models and equations to represent the problem. *For example, create a story context for (2/3) ÷ (3/4) and use a visual fraction model to show the quotient; use the relationship between multiplication and division to explain that (2/3) ÷ (3/4) = 8/9 because 3/4 of 8/9 is 2/3. (In general, (a/b) ÷ (c/d) = ad/bc.) How much chocolate will each person get if 3 people share 1/2 lb of chocolate equally? How many 3/4-cup servings are in 2/3 of a cup of yogurt? How wide is a rectangular strip of land with length 3/4 mi and area 1/2 square mi?*.	1. Make sense of problems and persevere in solving them. 2. Reason abstractly and quantitatively. 3. Construct viable arguments and critique the reasoning of others. 4. Model with mathematics. 5. Use appropriate tools strategically. 6. Attend to precision. 7. Look for and make use of structure. 8. Look for and express regularity in repeated reasoning.
Essential Understandings	
-Division by a fraction means that we know a fractional part and want to find a number the represents the whole -Dividing a mixed number by a fraction is the same as multiplying by the reciprocal of the fraction	
Explore Phase	
Possible Solution Paths	Assessing and Advancing Questions
⅔ of the ride takes 5 ¾ minutes so we can divide 5 ¾ by two to determine how	**Assessing Questions:** -What does your drawing show?

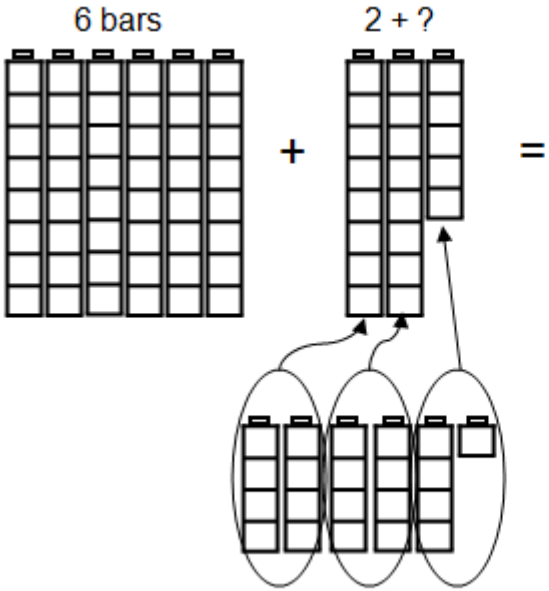
many minutes it takes to make ⅓ of the ride. Then we multiply that value by 3(or use repeated addition) to get the time for the whole ride. $\frac{5\ ¾}{2} = (5 \div 2) + (¾ \div 2) = 2\ ½ + ⅜ = 2\ ⅞$ Whole Ride broken into thirds  ⅔ of the ride = 5¾ 3 (2 ⅞) = (3 x 2) + (3 x ⅞) = 6 + 21/8 = 6 + 2 ⅝ = 8 ⅝ It will take Josh 8 ⅝ minutes to ride his bike to schools. Since he alowed 10 minutes for the ride, he will NOT be late.	-When you divide 5 ¾ by 2, what is it going to tell you? -Once you know how much is in each third, how did you find your total number of minutes? **Advancing Questions:** - I see you used division, could you use multiplication? -Why when you divided, did you get a larger answer than what you started with? -How long would it take to make a round trip?
Divide 5 ¾ by ⅔ to get the time for the whole journey. We can change 5 ¾ to an improper fraction and multiply that by the reciprocal of 2/3. 5 ¾ = 23/4 the reciprocal of ⅔ is 3/2. 23/4 x 3/2 = 69/8. We can then divide 69 by 8 to find the actual number of whole and partial minutes the ride took Josh. 69 ÷ 8 = 8 with 5 left over. Since we were dividing by 8, the left over represents so many eighthts of a minute. It took Josh 8 whole minutes and 5/8 of another minute to ride his bike to school. Since he alowed 10 minutes for the ride, he will NOT be late.	**Assessing Questions:** -Why does this method work? -What does the reciprocal mean in the context of this situation? -How does your diagram match your equation? **Advancing Questions:** You divided here, how could you solve this problem using mulitplication? -What other solution method could you use to confirm your results?
Break up the number line to show the number of ⅔ unit pieces that are in 5 ¾	**Assessing Questions:**

units and count.  The ride will take Josh 8 full minutes and a fraction of another minute. Since he allotted 10 minutes, he will not be late. Now, to find out exactly how much more than 8 minutes find the distance from 5 ⅓ to 5 ¾ by subtracting. $5\frac{3}{4} - 5\frac{1}{3} = \frac{5}{12}$ Next determine the fraction of a ⅔ unit piece that is in 5/12 by dividing: $\frac{5/12}{2/3} = \frac{5}{\cancel{12}_{4}} \circ \frac{\cancel{3}^{1}}{2} = \frac{5}{8}$ So it will take Josh 8 ⅝ minutes to get to school by riding his bike.	-Why does your method work? -How does your diagram match your equation? **Advancing Questions:** You divided here, how could you solve this problem using mulitplication? -What other solution method could you use to confirm your results?
We can use unifix cubes to represent this because I am using units that need to be broken into thirds and fourths. I can use 12/12 for 1 unit because I can break 12/12 into thirds and fourths. 12/12 can be broken into four groups of 3 (fourths) and into 3 groups of 4 (thirds). We can use unifix cubes with 12 pieces in a bar to represent the 5 units. Then we can break the 6th unit of 12 cubes into 4 groups of 3 and use 3 of those groups. (This will be a bar that has 9 cubes in it because ¾ of 12 = 9)	**Assessing Questions:** -Why does this method work? -Explain how your diagram match your equation. **Advancing Questions:** You divided here, how could you solve this using mulitplication? -What other solution method could you use to confirm your results?



We want to know how many bars that are ⅔ of a unit (1 bar of 12 blocks) long can be made from the 5¾ units we have.

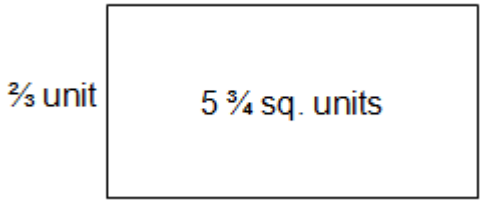
⅔ of a bar with 12 blocks in it has 8 blocks in it. So I break all the bars so each only has 8 blocks.



Next I use the blocks left over to form as many additional bars of 8 blocks as I can.

Each bar that is ⅜ of a unit long represents a minute. So it takes Josh 8⅝ minutes to ride his bike to school. Since he allowed 10 minutes for the ride, he will not be late.

Area model- 5 ¾ sq. units represents area of a rectagle. ⅔ unit represents length of that rectangle. What is the width of the rectangle?



As with all area models, the values for the length and width of the rectangle are factors of the value for the area of the rectangle. Since we know that we can multiply the values for the length and width of a rectangle to find its area,

Assessing Questions:

-Why does your method work?

-How does your diagram match your equation?

Advancing Questions:
-You divided here, how could you solve this problem using mulitplication?

-What other solution method could you use to confirm your results?

we can use the inverse of multiplication to find the value of the width (missing factor) by dividing the value of the area by the value of the length.

$$\frac{5\frac{3}{4} = 5\frac{3}{4} \circ \frac{3}{2}}{{}^{2}/_{3}}$$

We can find this value in one of 2 ways:

1. Rewrite the mixed numeral as an improper fraction and multiply by 3/2, then rewrite as a mixed numeral.

$$5\frac{3}{4} = \frac{23}{4} \circ \frac{3}{2} = \frac{69}{8} = 8\frac{5}{8} \text{ minutes}$$

2. Use distributive property to multiply $5\frac{3}{4}$ by the reciprocal of ⅔. The reciprocal of ⅔ is 3/2.

$$\frac{3}{2}\left(5+\frac{3}{4}\right) = \frac{3}{2}(5) + \frac{3}{2}\left(\frac{3}{4}\right) = \frac{15}{2} + \frac{9}{8}$$

We can only add if the wholes are broken into the same size pieces so we use a common demominator to add halves with eighths.

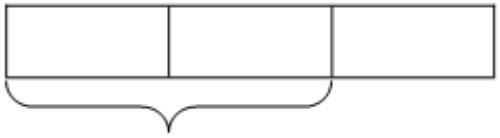
$$\frac{15}{2} = \frac{60}{8} + \frac{9}{8} = \frac{69}{8}$$

Divide 69 by 8 to find the number of minutes this represents.

$$\begin{array}{r} 8 \\ 8\overline{)69} \\ \underline{-\ 64} \\ 5 \end{array} \longrightarrow 8\ ⅝ \text{ minutes}$$

Rounding and Compensating:

Whole Ride broken into thirds



5 ¾ minutes = ⅔ of the whole ride.

Round 5 ¾ to be 6. Divide 6 by 2 to find the approximate length of one third of the ride.

6 ÷ 2 = 3 →

≈ 3 min	≈ 3 min	≈ 3 min

Compensate for roundi ng by calculating the difference between the actual value (5 ¾) and the rounded value (6) → 6 - 5 ¾ = ¼

That difference occurs in all pieces of the ride. So, to find out how much we should subtract from each piece, we take half of ¼ and get ½ ⁰ ¼ = ⅛ because we only know the number of the minutes it takes for 2 pieces of the ride. Now we should subtract ⅛ from all 3 pieces of the whole ride.

$3 - \frac{1}{8} + 3 - \frac{1}{8} + 3 - \frac{1}{8}$

$2\frac{7}{8} + 2\frac{7}{8} + 2\frac{7}{8}$

Now we add $2\frac{7}{8} + 2\frac{7}{8} + 2\frac{7}{8}$ by adding whole numbers then by adding fractions:

2 + 2 + 2 = 6

⅞ + ⅞ + ⅞ = (⅞ + ⅞) + ⅞ in the group we can take ⅛ from one fraction and add it to the other:

Assessing Questions:

-Why does your method work?

-How does your diagram match your equation?

Advancing Questions:

-You divided here, how could you solve this using mulitplication?

-What other solution method could you use to confirm your results?

$7/8 + (1/8 + 6/8) = (7/8 + 1/8) + 6/8 = 1\,6/8$ Then we can add the last $7/8$ to $1\,6/8$. $1 + 6/8 + 7/8 = 1 + 6/8 + (2/8 + 5/8) =$ $1 + (6/8 + 2/8) + 5/8 = 1 + 1 + 5/8 = 2\,5/8$ Now we can add this to the other wholes and get $6 + 2 + 5/8 = 8\,5/8$ minutes It will take Josh 8⅝ minutes to ride his bike to school since he allowed 10 minutes for the ride, he will NOT be late.	
Possible Student Misconceptions	
Students might not understand that when dividing a fraction by a fraction, you get a larger number.	-So are you saying that the total amount of length of time for the whole journey is less than the amount of time for 2/3 of the journey?
Students might see the word “of” in the question and assume they must multiply.	-So are you saying that the total amount of length of time for the whole journey is less than the amount of time for 2/3 of the journey?
Entry/Extensions	Assessing and Advancing Questions
If students can’t get started….	-What are you trying to find out? -What do you know? -Can you show me what the whole bike ride would look like? -Could you solve it using 5 minutes instead of 5 3/4 minutes? -Would it help you to find how long it takes for Josh to ride 1/3 of the way? -If you know how long it takes to make one third of the ride, how can you use that to find the total amount of time for the

	whole journey?
If students finish early….	-Can you solve the problem a different way? -Can you represent the problem using an area model?
Discuss/Analyze	
Whole Group Questions	
-Why can we use both multiplication and division to solve this problem? -How is mulitplying fractions the same or different than multiplying whole numbers? -What happens when you divide any number by a fraction? Why? -What happens when you divide any number by a whole number? Why?	
Models: -How many ways can we model this problem? -Why do all of our different types of models work?	