

**Task: Coupon Book Sales**

**7<sup>th</sup> Grade**

In the table below, Mikayla recorded the number of students selling coupon books and the number of coupon books sold for the first four weeks of the sale.

Number of Students Selling Books	Number of Coupon Books Sold
20	60
45	135
60	180
85	255



- Her friend, Nora, says that the numbers in Mikayla’s table represent a proportional relationship, but Mikayla disagrees. Explain to Mikayla and Nora who is correct. Show your mathematical thinking.
- Represent the data in the table using a graph or equation.
- Suppose sales continued at the same rate. How many students would have sold 405 books? Explain your reasoning.

**Teacher Notes:**

- This is an instructional task where the teacher should be looking for multiple solution paths from the students and expecting to hear rate language as the students are ask to explain their thinking and/or reasoning.
- Care should be taken to focus on unit rate, and a good discussion could be built around understanding how to determine it from a table, graph or equation.

**Common Core State Standards for Mathematical Content**

**Common Core State Standards for Mathematical Practice**

7.RP.A.2. Recognize and represent proportional relationships between quantities.

- Decide whether two quantities are in a proportional relationship, e.g., by testing for equivalent ratios in a table or graphing on a coordinate plane and observing whether the graph is a straight line through the origin.
- Identify the constant of proportionality (unit rate) in tables, graphs, equations, diagrams, and verbal descriptions of proportional relationships.
- Represent proportional relationships by equations. *For example, if total cost  $t$  is proportional to the number  $n$  of items purchased at a constant price  $p$ , the relationship between the total cost and the number of items can be expressed as  $t = pn$ .*
- Explain what a point  $(x, y)$  on the graph of a proportional relationship means in terms of the situation, with special attention to the points  $(0, 0)$  and  $(1, r)$  where  $r$  is the unit rate.

7.EE.B.4 Use variables to represent quantities in a real-world or mathematical problem, and construct simple equations and inequalities to solve problems by reasoning about the quantities.

- Solve word problems leading to equations of the form  $px + q = r$  and  $p(x + q) = r$ , where  $p, q,$  and  $r$  are specific rational numbers. Solve equations of these forms fluently. Compare an algebraic solution to an arithmetic solution, identifying the sequence of the operations used in each approach. *For*

- Make sense of problems and persevere in solving them.
- Reason abstractly and quantitatively.
- Construct viable arguments and critique the reasoning of others.
- Model with mathematics.
- Use appropriate tools strategically.
- Attend to precision.
- Look for and make use of structure.
- Look for and express regularity in repeated reasoning.

example, the perimeter of a rectangle is 54 cm. Its length is 6 cm. What is its width?

**Essential Understandings**

- A rate is a set of infinitely many equivalent ratios.
- A proportion is a relationship of equality between two ratios. In a proportion, the ratio of two quantities remains constant as the corresponding values of the quantities change.
- Proportional reasoning is complex and involves understanding that if one quantity in a ratio is multiplied or divided by a particular factor, then the other quantity must be multiplied or divided by the same factor to maintain the proportional relationship.
- A ratio is a multiplicative comparison of two quantities, or it is a joining of two quantities in a composed unit.

**Explore Phase**

**Possible Solution Paths**

**PART A:**

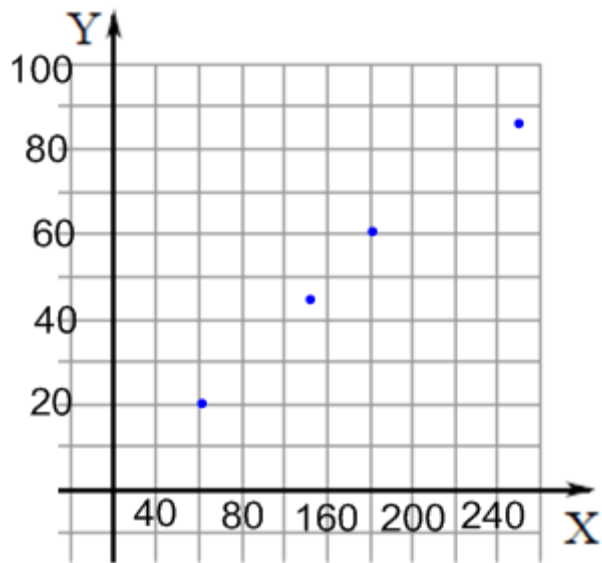
Shows mathematical reasoning/explanation such as

- Every ratio 20/60, 45/135, 60/180, 85/255 simplifies to 1/3
- By showing the scale factor is the same for each line of the table
- By dividing the Number of Coupon Books by the Number of Students in each line to see that quotient is the same.

**PART B:**

Equation:  $y = 3x$ , where  $x$  = Number of Students Selling Books  
 $y$  = Number of Coupon Books Sold

Graph:



**PART C:**

- Demonstrates an understanding of how to form the equation

**Assessing and Advancing Questions**

Assessing

What is the unit rate are we interested in for this task?

Advancing

Create another combination of Students Selling Books and Number of Books Sold that would represent the proportional relationship.

Assessing

How do you determine the unit rate for the equation? The graph?

Advancing

How many coupon books would have been sold if 40 students were selling books? How does your equation or graph help you find the answer?

Assessing

Tell me about your equation. What the constant of proportionality in your

<p><math>3x = 315</math></p> <ul style="list-style-type: none"> <li>The number of coupon books sold is three times the number of students, so they divided by three.</li> <li>Student uses one of the lines to set up a proportion to solve (e.g. <math>255/85 = 315/x</math>)</li> </ul>	<p>equation? Why did you set it equal to 315?</p> <p><u>Advancing</u></p> <p>If your constant of proportionality in your equation had been larger how would that have changed your answer?</p> <p>If your constant of proportionality in your equation had been smaller how would that have changed your answer?</p>
<p><b>Possible Student Misconceptions</b></p>	
<p>The student may not test all the ratios in the table to show proportionality.</p>	<p><u>Assessing</u></p> <p>Compare of all the ratios for <b>Number of Coupon Books Sold: Number of Students Selling Books.</b></p> <p><u>Advancing</u></p> <p>How can you use the unit rate in the first entry to determine/verify the number of coupon books sold in the remaining lines of the table ?</p>
<p><b>Entry/Extensions</b></p>	
<p>If students can't get started....</p>	<p><u>Assessing</u></p> <p>Can you explain your work?</p> <p>How do you compute the unit rate between two quantities? What is the unit rate for this task?</p> <p><u>Advancing</u></p> <p>How many coupon books would 21 students have sold?</p>
<p>If students finish early....</p>	<p><u>Assessing</u></p> <p>How can you use the equation/reasoning you used to derive you answer in part B to verify each line in the table?</p> <p>Why did you choose to represent the data with a graph (or table)?</p> <p>How could you use your graph or equation to answer part c.</p> <p><u>Advancing</u></p> <p>If coupon books cost \$10, how much money would 85 students earn?</p> <p>If you needed to raise \$4500, at this rate how many students would you need selling coupon books?</p>
<p><b>Discuss/Analyze</b></p>	
<p><b>Whole Group Questions</b></p>	
<p><b><u>PART A:</u></b></p> <p>How do you decide whether two quantities form a proportional relationship?</p> <p>How do you identify the constant of proportionality in a table of values?</p>	
<p><b><u>PARTB:</u></b></p> <p>Why did you make <b>x = Number of Students Selling Books</b> and <b>y = Number of Coupon Books Sold</b> in the equation or graph?</p> <p>How would switching the labels for the x and y axes change the graph?</p>	
<p><b><u>PART C:</u></b></p> <p>Why did some of you create an equation where you multiplied by 3?</p>	

Why did we get the same answer when some of you multiplied by 3 and others divided by 3?