

Task: Reading Rates Task 6th Grade	
<p>At the beginning of 6th grade, friends Jennifer and Susan decided to read the books within the Series of Unfortunate Events. Jennifer read a total of 6 books over 8 months. Susan read a total of 8 books over 10 months.</p> <ol style="list-style-type: none"> a. Which girl is reading at a faster rate? Make a table or graph to justify your answer. b. At this rate, how many books will Jennifer read after one year? How many books will Susan read? Explain how you arrived at your answers. 	
<p>Teacher Notes:</p> <ul style="list-style-type: none"> • This is an instructional task where the teacher should be looking for multiple solution paths from the students and expecting to hear rate language as the students are ask to explain their thinking and/or reasoning. • Care should be taken to focus on unit rate, and a good discussion could be built around the do they know how to determine it from a table, graph or equation. 	
Common Core State Standards for Mathematical Content	Common Core State Standards for Mathematical Practice
<p>6.RP.1 Understand the concept of a ratio and use ratio language to describe a ratio relationship between two quantities.</p> <p>6.RP.2 Understand the concept of the unit rate a/b associated with a ratio $a:b$ with $b \neq 0$, and use rate language in the context of a ratio relationship.</p> <p>6.RP.3 Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations.</p> <p>6.RP.3a. Make tables of equivalent ratios relating quantities with whole number measurements, find missing values in the tables, and plot the pairs of values on the coordinate plane. Use tables to compare ratios.</p>	<ol style="list-style-type: none"> 1. Make sense of problems and persevere in solving them. 2. Reason abstractly and quantitatively. 3. Construct viable arguments and critique the reasoning of others. 4. Model with mathematics. 5. Use appropriate tools strategically. 6. Attend to precision. 7. Look for and make use of structure. 8. Look for and express regularity in repeated reasoning.

Essential Understandings

- A ratio can compare a part of a quantity to the whole (part-whole), or to another part (part-part). A third type of ratio compares two different things, e.g., miles/hour.
- Reasoning with ratios involves attending to and coordinating two quantities. (*NCTM EU #1*)
- A ratio is a multiplicative comparison of two quantities, or it is a joining of two quantities in a composed unit (*NCTM EU #2*)

Explore Phase

Possible Solution Paths

a.

CONVERTING TO UNIT RATES AND COMPARING

Jennifer

$$\frac{6 \text{ books}}{8 \text{ months}} = \frac{3 \text{ books}}{4 \text{ months}} = \frac{.75 \text{ book}}{1 \text{ month}} \text{ or } \frac{3}{4} \text{ book}$$

Susan

$$\frac{8 \text{ books}}{10 \text{ months}} = \frac{4 \text{ books}}{5 \text{ months}} = \frac{.8 \text{ book}}{1 \text{ month}} \text{ or } \frac{4}{5} \text{ book}$$

Susan is reading at a faster rate because $\frac{4}{5} > \frac{3}{4}$

Assessing and Advancing Questions

Assessing:

How can you explain your work?

Why did you choose to set your problem up as a ratio?

How did you get from this ratio to the next?

How did you get a decimal answer?

How do you know Susan is reading at a faster rate?

Advancing:

How are you going to use this and put it into a table or a graph?

How do you know $\frac{4}{5} > \frac{3}{4}$? Can you prove it using a picture?

b.

MAKING TABLES

Jennifer

Books	Months
3	4
6	8
9	12
12	16
15	20

Susan

Books	Months
4	5
8	10
12	15
16	20

If both girls continue at a constant rate, then after 20 months Jennifer will have only read 15 books and Susan has read 16 books. Therefore, Susan is reading at a faster rate.

Scaling: Decreasing or increasing both sides by the same amount (Divide each side by two, or double each side)

Jennifer

Books	Months
6	8
3	4
9	12
12	16
15	20

Susan

Books	Months
8	10
4	5
12	15
16	20

Assessing:

How did you get to 3 books?

Why did you go all the way to 15 books? Did you need to?

How do you know Susan is reading at faster rate?

Advancing:

Does Susan read a book every month?

Can you determine how many months it takes Susan to read one book?

What would the graph of each girl's line look like?

Assessing:

Can you explain to me how you got these numbers on your table?

How did you know when you had enough data in your table to derive at your answer?

Advancing:

Could you continue this pattern forever?

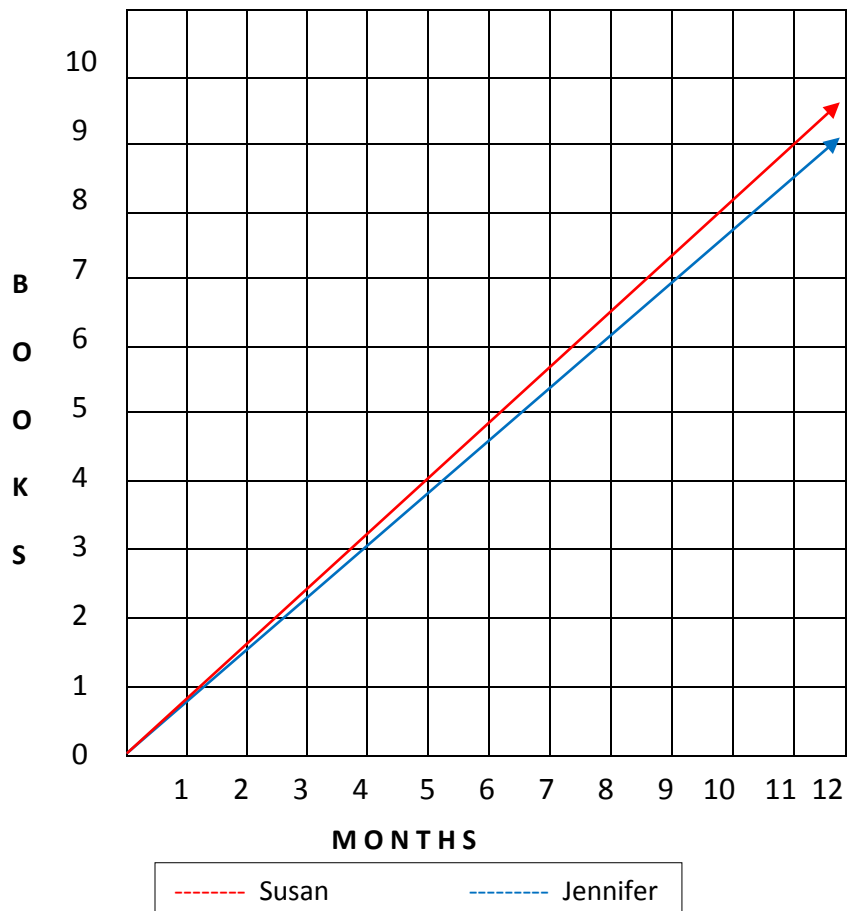
Can you reduce these to a unit rate? How?

It takes Jennifer 16 months to read 12 books and takes Susan 15 months to read 12 books, therefore Susan is reading at a faster rate.

Or, Jennifer can read 15 books in 20 months, where Susan will read 16 books in 20 months, therefore Susan is reading at a faster rate.

GRAPHING

b.



Susan, shown as the red line, is reading at a faster rate because her line is steeper.

Assessing:

What do these lines represent?

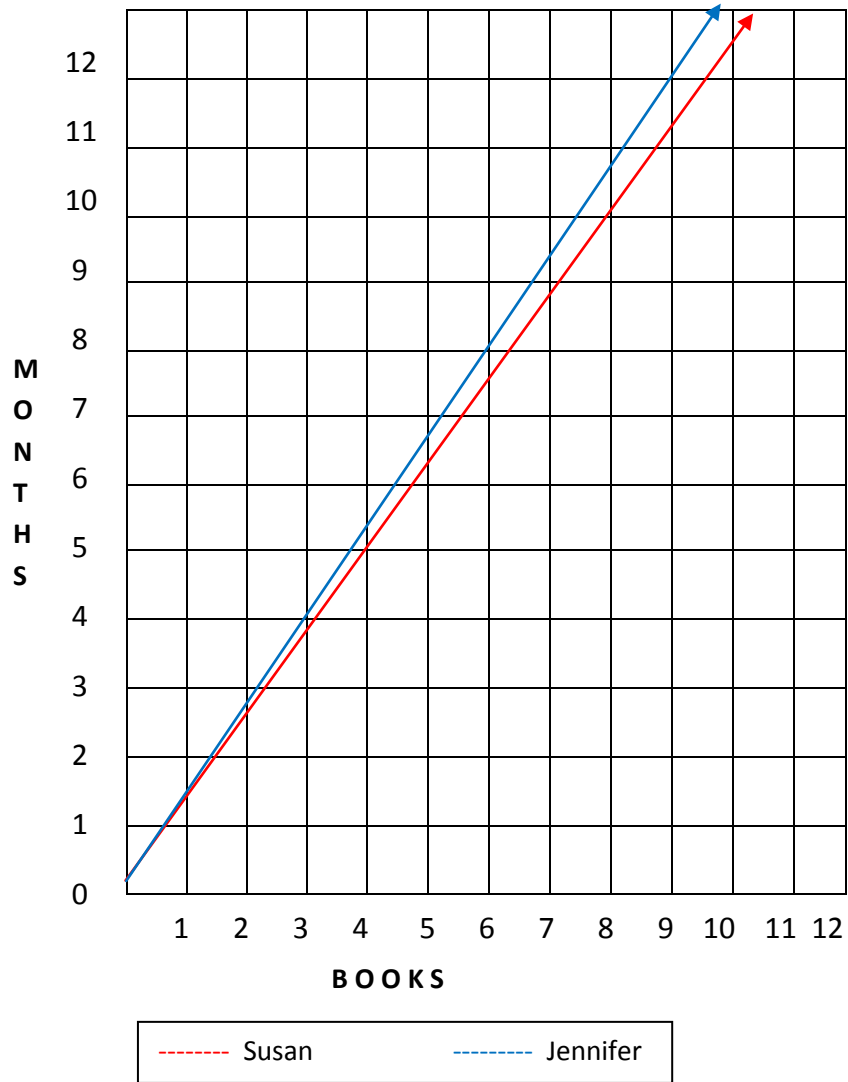
How did you know where to draw the lines?

Advancing:

If you reversed the axis and put books on the x-axis and months on the y-axis, would you still derived at the same answer? Justify your response.

How would that have changed the graph of the lines? Would Susan's still be steeper?

How can we determine the unit rate for the lines that you graphed?



Susan (red line) is reading more books than Jennifer (blue line) each month, therefore Susan is reading at a faster rate.

Assessing:

How do you read this graph?

How can you tell that Susan is reading more books than Jennifer?

Advancing:

Could you have made your chart and used the months for the x-axis and books for the y-axis?

How would that change how the lines graphed would look?

How can you use the graph to help you determine the unit rate?

c.

PROPORTIONS

$$\frac{.75 \text{ books}}{1 \text{ month}} = \frac{x}{12 \text{ months}}$$

$$X = 9 \text{ books}$$

$$\frac{\text{Susan } .8}{1 \text{ month}} = \frac{x}{12 \text{ months}}$$

$$x = 9.6 \text{ books}$$

RATIO BOXES

	Jennifer	
	given rate	desired rate
Books	6	x
Months	8	12

$$\frac{6}{8} = \frac{x}{12}$$

$$8x = 72$$

$$x = 9 \text{ books}$$

	Susan	
	given rate	desired rate
Books	8	x
Months	10	12

$$\frac{8}{10} = \frac{x}{12}$$

$$10x = 96$$

$$x = 9.6 \text{ books}$$

Assessing:

Can you explain to me why you set up your problem this way?

What does the "x" represent?

How does your answer relate back to the question?

Advancing:

Did you have to use the unit rate to solve this problem?

Can you tell me how many books each girl would read in 2 years? 3 ½ years?

Assessing:

Why did you choose to use a ratio box?

What ratio was given in the problem?

Can you explain to me how you came up with your ratio box?

What does the "x" represent in your problem?

How did you use your ratio box to set up your proportion?

How does your answer relate back to the question?

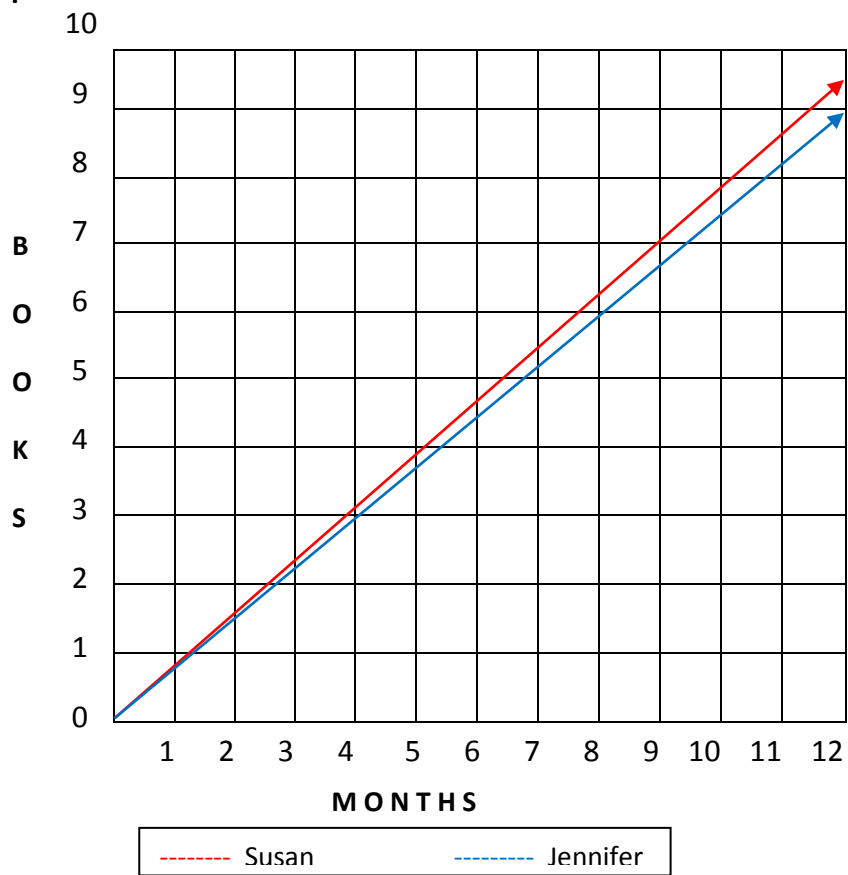
Advancing:

Could you have set up your ratio box differently?

Can you tell me how many books each girl will read in 2 years? 3 ½ years?

Can you make a graph to illustrate this information?

Graph



Reading the graph at 12 months, Jennifer will be at 9 books, and Susan will be a little more, but it does not give exact amounts. Students should be pushed to precision for this answer using another method.

Assessing:

What does your graph tell you?

According to the graph, can you tell me exactly how many books Jennifer will read in 12 months? Susan?

Can you estimate how many books it appears that Susan can read in 12 months? Explain.

Advancing:

How can you determine exactly how many books that Susan read in 12 months?

Explain how you can determine the unit rate from graph?

Is this the best method to use for precision?

How can you use your information from the graph to make a table?

Possible Student Misconceptions	
<p>Students may have a tendency to confuse the books and the months. As a teacher, make sure that the student is comparing the same unit rates.</p> <p>Rather than scaling accurately, students may think there is a constant difference of two between the books and the months.</p>	<p>Assessing: Can you explain to me how you set up your problem? What is the ratio that is given in the problem?</p> <p>Advancing: Can you tell me the unit rate in books per month? What about months per book?</p> <p>Assessing: Can you explain to me what your numbers are referring to?</p> <p>Advancing: If Jennifer is reading 6 books in 8 months, how can you figure out how many books she is reading in one month? What do we call it when you can tell me how many books she reads in one month?</p>
Entry/Extensions	Assessing and Advancing Questions
<p>If students can't get started....</p>	<p>Assessing: What does the problem say? What information is the problem giving me? What could I do to help me organize or set up the problem?</p> <p>Advancing: Can you make a table organize the information? Which tool would you like to use to organize this information to compare the rates, a table or graph? Why?</p>
<p>If students finish early....</p>	<p>Advancing: Can you determine how many books each girl will read in 2 years? $3\frac{1}{2}$ years? Susan's mom will give her \$50.00 if she has reads 20 books in 2 years. At this rate, will she earn the \$50.00?</p>

Discuss/Analyze

Whole Group Questions

What was the ratio in this problem?

How can I use a ratio to find unknown quantity?

Who can explain to the class how we would find the unit rate in this problem on the graph?

How could we find the unit rate from the table?

Is there more than one unit rate?

What does the unit rate mean?

Why would finding the unit rate help us answer the question?

What is the relationship between the table, graph and the equation?

Can someone pose another problem for the class by providing us a rate to use? (*ie. Johnny drove 30 miles in 20 minutes and Stacy drove 24 miles in 15 minutes to get to the the movie theater. Which person was driving faster to get to the movies?*)