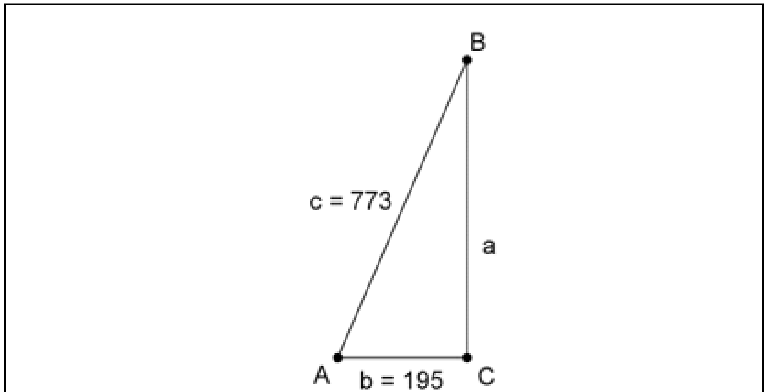
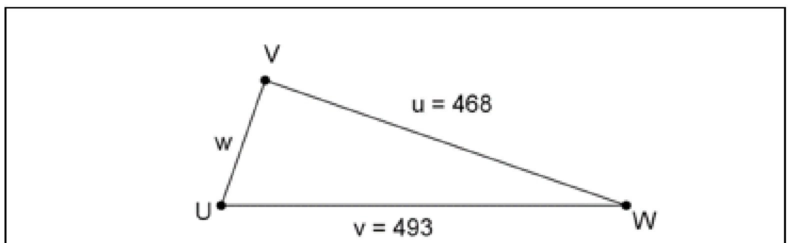
Task: Relating Trig Functions **Geometry/Core Math II**

a) In the tables below, you are given two right triangles. In triangle ABC, angle C is the right angle and in triangle UVW, angle V is the right angle. Fill in the missing entries for each table.



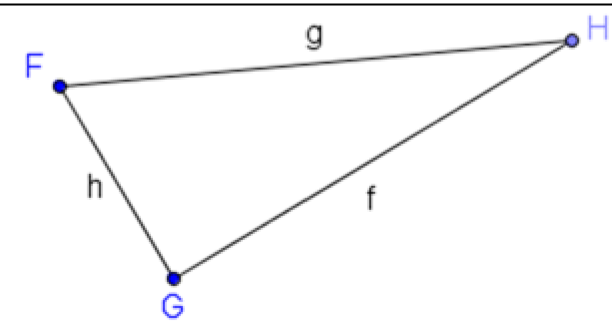
sin A =	cos A =	tan A =
sin B =	cos B =	tan B =



sin U =	cos U =	tan U =
sin W =	cos W =	tan W =

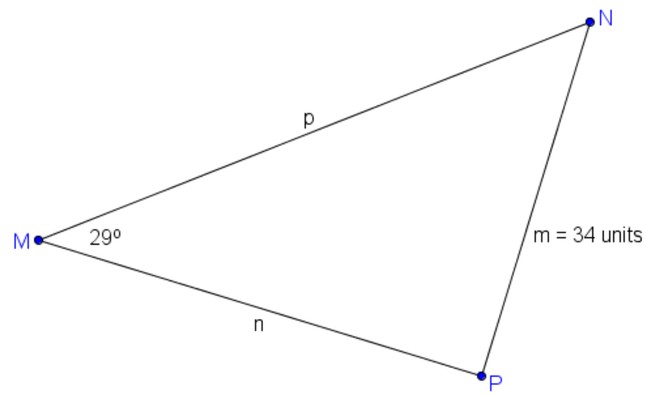
b) Look at the entries you have completed. What relationships do you notice between the trigonometric functions of the acute angles in your tables? Make conjectures about these relationships.

c) In triangle FGH below, angle G is a right angle. Use the diagram to answer the questions. Do the conjectures you made in part (b) hold for your answers in this table? Explain.

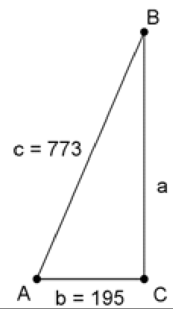


Name the trigonometric function(s) of an angle with ratio $\frac{f}{g}$.	
Name the trigonometric function(s) of an angle with ratio $\frac{h}{g}$.	
Name the trigonometric function(s) of an angle with ratio $\frac{h}{f}$.	
Name the trigonometric function(s) of an angle with ratio $\frac{f}{h}$.	

d) In the triangle below, angle P is a right angle, and the measures of angle M and side m are given. Use this information to find the length of sides n and p.



Teacher Notes:	
See the teacher notes in part (b) below.	
Common Core State Standards for Mathematical Content	**Common Core State Standards for Mathematical Practice**
G-SRT.C.7 Explain and use the relationship between the sine and cosine of complementary angles.	Mathematical Practices 1. Make sense of problems and persevere in solving them. 2. Reason abstractly and quantitatively. 3. Construct viable arguments and critique the reasoning of others. 4. Model with mathematics. 5. Use appropriate tools strategically. 6. Attend to precision. 7. Look for and make use of structure. 8. Look for and express regularity in repeated reasoning.
Essential Understandings	
• Empirical verification is an important part of the process of proving, but it can never, by itself, constitute a proof. • Behind every proof is a proof idea.	
Explore Phase	
Possible Solution Paths	**Assessing and Advancing Questions**
Part (a): Students must identify the lengths of all three sides of the right triangle in each table. Since only two sides are given, students must use the Pythagorean Theorem to find the third side. Students will then use the definitions of the trigonometric functions to determine the sine, cosine, and tangent of each acute angle in the triangle.	**Assessing Questions:** How did you determine the values of the trigonometric functions? Did you need any information not given in the diagram? If so, how did you find that information? **Advancing Questions:**



sin A = $\frac{748}{773}$	cos A = $\frac{195}{773}$	tan A = $\frac{748}{195}$
sin B = $\frac{195}{773}$	cos B = $\frac{748}{773}$	tan B = $\frac{195}{748}$

Work to support the computations:

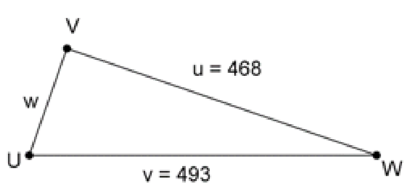
$a^2 + b^2 = c^2$

$a^2 + (195)^2 = (773)^2$

$a^2 + 38025 = 597529$

$a^2 = 559504$

$a = 748$



sin U = $\frac{468}{493}$	cos U = $\frac{155}{493}$	tan U = $\frac{468}{155}$
sin W = $\frac{155}{493}$	cos W = $\frac{468}{493}$	tan W = $\frac{155}{468}$

Work to support the computations:

$u^2 + w^2 = v^2$

$(468)^2 + w^2 = (493)^2$

$219024 + w^2 = 243049$

$w^2 = 24025$

$w = 155$

Do you know the lengths of all three sides of the triangle? If not, can you find the length of the third side?

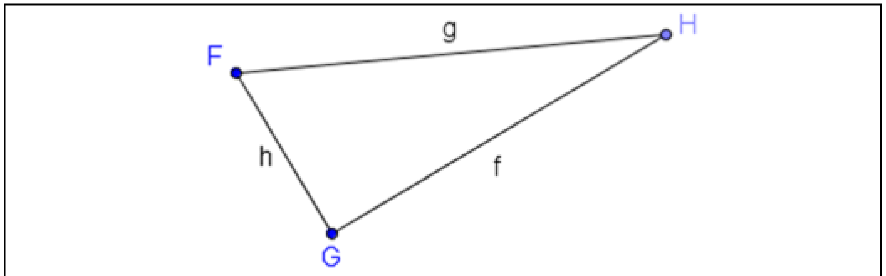
Which side is the hypotenuse? How do you know that side is the hypotenuse?

For angle A, which side should be labeled the "opposite" and which side should be labeled "adjacent? How do you know?

(These same questions can be asked for any other acute angles in the diagrams.)

Part (b): There are two "obvious" conjectures and at least two "not-so-obvious" conjectures that can be made. Student notation should be checked here to ensure that the conjectures are stated in a generic way. "obvious" conjectures: (i) In a right triangle ABC, where C is the right angle, sin A = cos B. (This could also be written as sin A = cos (90 – A).) (ii) In a right triangle ABC, where C is the right angle, tan A = $\frac{1}{\tan B}$. (This could also be written as tan A = $\frac{1}{\tan(90 - A)}$.) "not-so-obvious" conjectures: (iii) In a right triangle ABC, where C is the right angle, $(\sin A)^2 + (\cos A)^2 = 1$. (Students may also refer to angle B instead of angle A.) (iv) In a right triangle ABC, where C is the right angle, tan A = $\frac{\sin A}{\cos A}$. (Students may also refer to angle B instead of angle A.) ***Teacher notes: Other trigonometric relationships may be identified. Relationship (i) is the main purpose/focus of this task, but other identified relationships should be acknowledged and may be explored as time permits.*** ***Teachers may have to draw the connection to complementary angles out in the course of the class discussion. Students will most likely see the relationship between sin A and cos B, but may not consciously realize that A and B are complementary angles.***	**Assessing Questions:** What relationships did you notice? How did you write down the relationships you noticed as conjectures? Explain your notation to me. **Advancing Questions:** Let's focus on one table. Do you notice anything about the values in your table? [after the student responds] Does this work in the other tables? Do you think it would work for every right triangle?

Part (c): Use the definitions of the trigonometric functions to find the answers. Note that the first two questions each have two answers.



Name the trigonometric function(s) of an angle with ratio $\frac{f}{g}$.	cos H or sin F
Name the trigonometric function(s) of an angle with ratio $\frac{h}{g}$.	cos F or sin H
Name the trigonometric function(s) of an angle with ratio $\frac{h}{f}$.	tan H
Name the trigonometric function(s) of an angle with ratio $\frac{f}{h}$.	tan F

Assessing Questions:

How did you find your answers?

Advancing Questions:

Which letter represents the hypotenuse of the triangle? What trigonometric functions require the use of the hypotenuse?

What do "h" and "f" represent in your triangle? How are these "pieces" used in the definitions of the trigonometric functions?

Part (d): To find n, students may use angle M or angle N (note that students will have to find angle N in order to use it):

$\tan 29^\circ = \frac{34}{n}$, so n = $\frac{34}{\tan 29^\circ}$ = 61.34 (rounded to the nearest hundredth)

or

$\tan 61^\circ = \frac{n}{34}$, so n = $34 \tan 61^\circ$ =61.34 (rounded to the nearest hundredth)

Assessing Question:

How did you decide which trigonometric function to use?

Advancing Questions:

What pieces of information do you know? What piece are you trying to find? Is there a trigonometric function that relates all of these pieces of information?

To find p, students may use either angle or side, or they may use the Pythagorean Theorem. To avoid issues with rounding, they will be better served to use angle M and side m. (It may be a good idea to use the Pythagorean Theorem as a way to "check" their answers." $\sin 29^\circ = \frac{34}{p}$, so $p = \frac{34}{\sin 29^\circ}$ = 70.13 (rounded to the nearest hundredth) or $\cos 61^\circ = \frac{34}{p}$, so $p = \frac{34}{\cos 61^\circ}$ = 70.13 (rounded to the nearest hundredth)	
Possible Student Misconceptions	
Students may confuse which side of the triangle is the hypotenuse.	Where is your right angle in the triangle? How does this help you identify the hypotenuse?
In using the Pythagorean Theorem with triangles labeled with letters other than ABC, students may have difficulty identifying the two legs and the hypotenuse.	Where is your right angle in the triangle? How does this help you identify the hypotenuse? Where does the hypotenuse "go" in the conclusion of the Pythagorean Theorem?
Entry/Extensions	**Assessing and Advancing Questions**
If students can't get started….	Draw triangle ABC, and label the right angle. What information is "missing" from your triangle? How would you find that missing information? What are your definitions of the trigonometric functions? For angle A, where is the opposite side? The adjacent side? The hypotenuse?
If students finish early….	Are there other conjectures you can make about your right triangles? Can you prove these other conjectures?

Discuss/Analyze

Whole Group Questions

Key idea: In a right triangle ABC, where C is the right angle, sin A = cos (90 – A).

Questions: If students identify the conjecture as: *In a right triangle ABC, where C is the right angle, sin A = cos B.*
How are angle A and angle B related? Can you rewrite the conjecture completely in terms of angle A?

In part (d), is there more than one way to find sides n and p? How do you know all of those ways will give you the same answer?

Key idea: In a right triangle ABC, where C is the right angle, tan A = $\frac{1}{\tan(90 - A)}$.

Questions: If students identify the conjecture as: *In a right triangle ABC, where C is the right angle, tan A =* $\frac{1}{\tan B}$.
How are angle A and angle B related? Can you rewrite the conjecture completely in terms of angle A?

Relating Trig Functions

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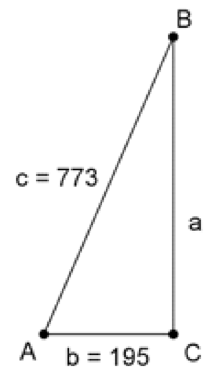

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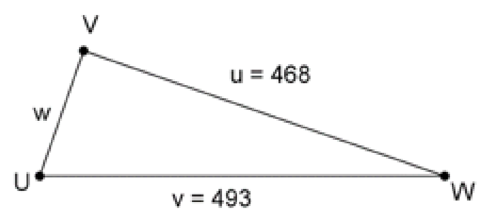

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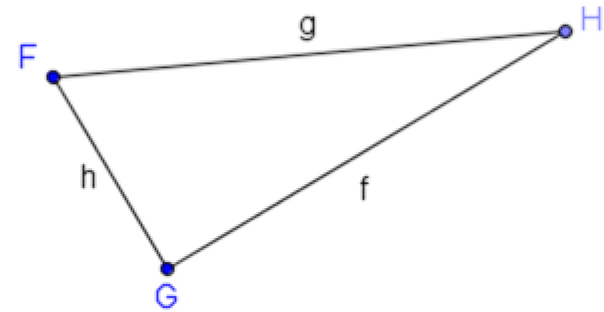

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