

Tennessee Comprehensive Assessment Program / Mathematics

TCAP/CRA PILOT 2012



Task 2 : Cutting Ribbon

Scoring Guide

Task 2. Cutting Ribbon Task

Amber has a 12-foot-long piece of ribbon. She wants pieces that are each $\frac{3}{4}$ of a foot long.

How many $\frac{3}{4}$ -foot-long pieces can Amber cut from the ribbon? Use words, diagrams, and multiplication or division equations to explain your thinking.



2. Cutting Ribbon Task Scoring Guide

The CCSS for Mathematical Content (1 point)

5.NF.4a Finding the number of $\frac{3}{4}$ -foot-long pieces in 12 feet of ribbon via the use of multiplication. The student may: _____

- Show 16 iterations of $\frac{3}{4}$ and write $16 \times \frac{3}{4} = 12$ feet.
- Draw the ribbon, divide each foot into fourths, and count the number of $\frac{3}{4}$ -foot-long pieces.
- Convert the lengths given in feet to inches: 12 feet of ribbon \times 12 inches per foot = 144 inches of ribbon; $\frac{3}{4}$ -foot pieces are each $\frac{3}{4} \times 12$ inches = 9 inches. Divide $144 \div 9 = 16$ to determine there are 16 pieces.

OR

5.NF.7b Finding the number of $\frac{3}{4}$ -foot-long pieces in 12 feet of ribbon, the student may: _____

- Determine each foot contains one $\frac{3}{4}$ -foot piece and one $\frac{1}{4}$ -foot piece. Twelve $\frac{3}{4}$ -foot pieces are therefore identified; three $\frac{1}{4}$ -foot pieces can be combined into $\frac{3}{4}$ -foot pieces and there will be $12 \div 3 = 4$ of those, for a total of 16 pieces.
- Write a division equation; invert and multiply. $\frac{12}{1} \div \frac{3}{4} = \frac{12}{1} \times \frac{4}{3} = \frac{48}{3} = 16$
- Change both numbers to fractions with like denominators. Then, simply divide the numerators, $12 = \frac{48}{4}$. $\frac{48}{4} \div \frac{3}{4} = 48 \div 3 = 16$. (Student reasoning: "There are 4 fourths in 1 foot, so there are $12 \times 4 = 48$ fourths in 12 feet. Divide 48 by 3 to find the number of $\frac{3}{4}$ -foot-long pieces in 48 fourths.")

Total Content Points _____

The CCSS for Mathematical Practices (3 points)

MP2 Writes either a multiplication or division equation and labels the quantities appropriately. *(See possible examples of solution paths listed above.)* _____
(MP2: Reason abstractly and quantitatively.)

MP4 Provides diagrams, equations and/or words that demonstrate reasoning related to partitive division involving a fractional amount. _____
(MP4: Model with mathematics.)

MP7 Work indicates that the whole can be partitioned into fractional parts and the parts are smaller than the whole, but, if recombined, equal the whole. _____
(MP7: Look for and make use of structure.)

Total Practice Points _____

Total Awarded Points _____

The CCSS for Mathematical Content Addressed in This Task

Apply and extend previous understandings of multiplication and division to multiply and divide fractions.

Apply and extend previous understandings of multiplication to multiply a fraction or whole number by a fraction.

5.NF.4a Interpret the product $(a/b) \times q$ as a parts of a partition of q into b equal parts; equivalently, as the result of a sequence of operations $a \times q \div b$. *For example, use a visual fraction model to show $(2/3) \times 4 = 8/3$, and create a story context for this equation. Do the same with $(2/3) \times (4/5) = 8/15$. (In general, $(a/b) \times (c/d) = ac/bd$.)*

Apply and extend previous understandings of multiplication and division to multiply and divide fractions.

Apply and extend previous understandings of division to divide unit fractions by whole numbers and whole numbers by unit fractions.

5.NF.7b Interpret division of a whole number by a unit fraction, and compute such quotients. *For example, create a story context for $4 \div (1/5)$, and use a visual fraction model to show the quotient. Use the relationship between multiplication and division to explain that $4 \div (1/5) = 20$ because $20 \times (1/5) = 4$.*

The CCSS for Mathematical Practices*

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

*Gray text indicates Mathematical Practices not addressed in this task.


Students' responses to a mathematical task provide evidence of what they understand and are able to do in relation to the standards and practices. Across tasks, this cumulative evidence shows students' understanding and abilities within a domain. When students do not respond completely to all parts of a task, they provide insufficient evidence of their mathematical understanding and abilities and therefore do not fully demonstrate the expectations of the standards and practices aligned with that task.

Task 2. Cutting Ribbon Task

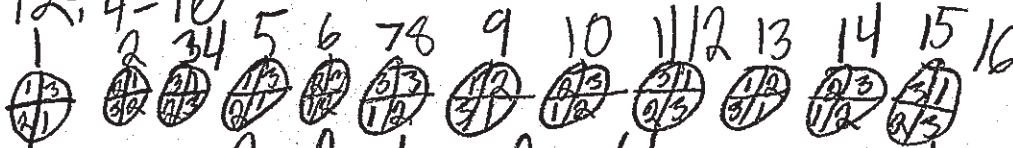
Guide 1

Amber has a 12-foot-long piece of ribbon. She wants pieces that are each $\frac{3}{4}$ of a foot long.

How many $\frac{3}{4}$ -foot-long pieces can Amber cut from the ribbon? Use words, diagrams, and multiplication or division equations to explain your thinking.

 $12 \div \frac{3}{4} = 16$

1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16



When 12 feet of ribbon is cut into 16 pieces they are $\frac{3}{4}$ of a foot long.

Guide 1

Litho 50235

Total Content Points: 1 (5.NF.7b)

Total Practice Points: 3 (MP2, MP4, MP7)

The student solves for the correct answer of 16 using an appropriate division equation (5.NF.7b). The student correctly labels the quantities in both the diagram and in the explanation that follows (MP2). The student's equation demonstrates reasoning related to partitive division involving a fractional amount. The student constructs a diagram in which 12 circles are divided into fourths. Above each circle, the student counts the number of $\frac{3}{4}$ -foot-long pieces formed by the parts within the circle while also noting

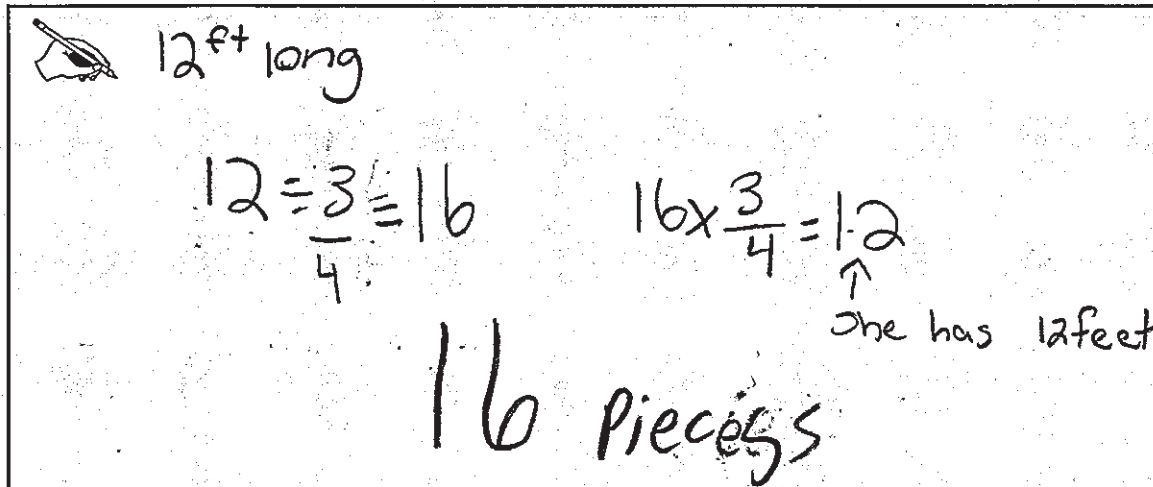
where the remaining $\frac{1}{4}$ -foot-long pieces can be combined into another $\frac{3}{4}$ -foot-long piece (MP4). In doing so, the student demonstrates proficiency at recognizing a pattern or structure, as the work indicates that the whole can be partitioned into fractional parts and the parts are smaller than the whole, but, if recombined, equal the whole (MP7).

Total Awarded Points: 4 out of 4

Task 2. Cutting Ribbon Task

Amber has a 12-foot-long piece of ribbon. She wants pieces that are each $\frac{3}{4}$ of a foot long.

How many $\frac{3}{4}$ -foot-long pieces can Amber cut from the ribbon? Use words, diagrams, and multiplication or division equations to explain your thinking.



Handwritten student work showing a ribbon being cut and mathematical calculations:

12^{ft} long

$12 = 3 \frac{1}{4} = 16$

$16 \times \frac{3}{4} = 12$

↑
she has 12 feet

16 pieces

Guide 2

Litho 50028

Total Content Points: 1 (5.NF.7b)

Total Practice Points: 2 (MP4, MP7)

The student solves for the correct answer of 16 using an appropriate division equation $\left(12 \div \frac{3}{4} = 16\right)$ and accurately labels 12 as the number of feet and 16 as the number of

pieces (5.NF.7b). However, the student does not provide the unit of measure for $\frac{3}{4}$,

which shows an incomplete contextualization of the data from the task (no credit for MP2). The student's equation demonstrates reasoning related to partitive division involving a fractional amount (MP4). By first solving for 16 using a division equation

and then proving that the quotient of that equation multiplied by $\frac{3}{4}$ equals the original

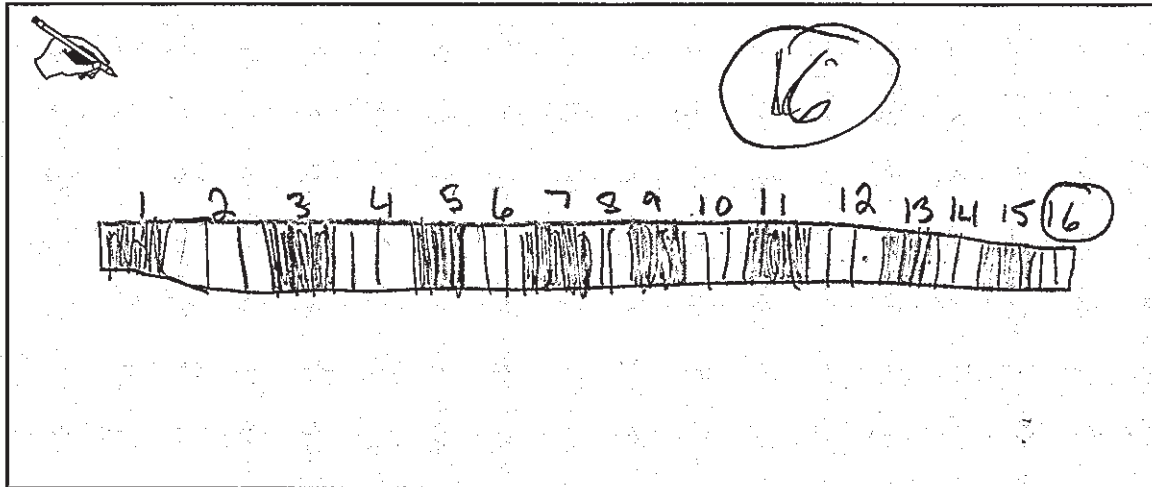
size of 12 feet, the work indicates that the whole can be partitioned into fractional parts and the parts are smaller than the whole, but, if recombined, equal the whole (MP7).

Total Awarded Points: 3 out of 4

Task 2. Cutting Ribbon Task

Amber has a 12-foot-long piece of ribbon. She wants pieces that are each $\frac{3}{4}$ of a foot long.

How many $\frac{3}{4}$ -foot-long pieces can Amber cut from the ribbon? Use words, diagrams, and multiplication or division equations to explain your thinking.



Guide 3

Litho 50146

Total Content Points: 1 (5.NF.4a)

Total Practice Points: 2 (MP4, MP7)

The student demonstrates the use of multiplication by drawing a diagram of 48 sections, indicating 12 feet divided in fourths, and counts the number of $\frac{3}{4}$ -foot-long pieces

(5.NF.4a). The student fails to provide an equation as required by the task, and does not label the quantities appropriately (no credit for MP2). By providing a diagram that shows the ribbon divided into fourths of a foot and counting each $\frac{3}{4}$ -foot-long piece, the student clearly demonstrates reasoning related to partitive division involving a fractional amount (MP4). This diagram also demonstrates that the student has discerned an appropriate pattern, and it indicates that the whole can be partitioned into fractional parts and the parts are smaller than the whole, but, if recombined, equal the whole (MP7).

Total Awarded Points: 3 out of 4

Task 2. Cutting Ribbon Task

Amber has a 12-foot-long piece of ribbon. She wants pieces that are each $\frac{3}{4}$ of a foot long.

How many $\frac{3}{4}$ -foot-long pieces can Amber cut from the ribbon? Use words, diagrams, and multiplication or division equations to explain your thinking.

Handwritten work showing the calculation: $12 \div \frac{3}{4} = 16$. The work includes a drawing of a ribbon being cut, the equation $12 \div \frac{3}{4} = 16$, and the final answer 16 circled.

Guide 4

Litho 50003

Total Content Points: 1 (5.NF.7b)

Total Practice Points: 2 (MP4, MP7)

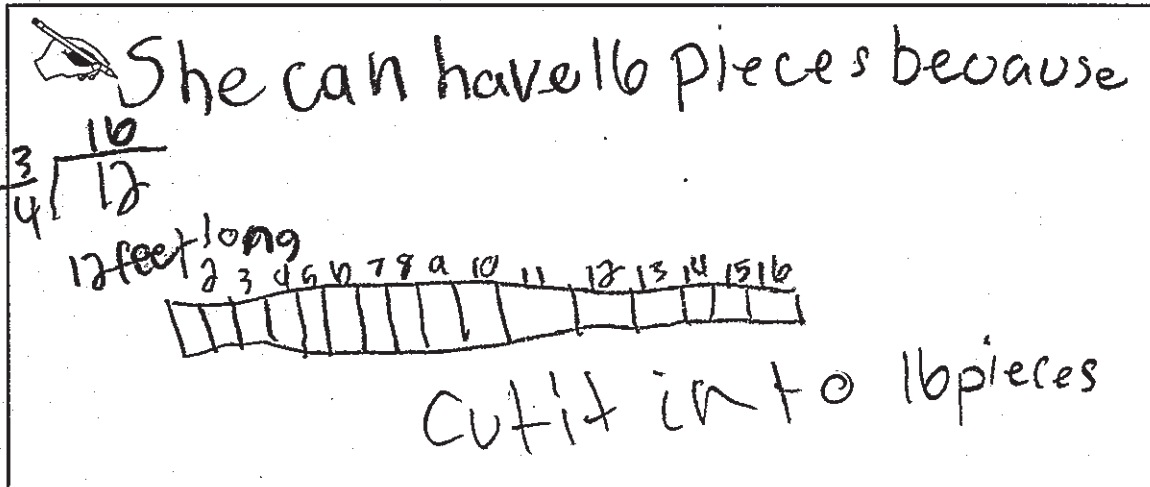
The student writes an appropriate division equation, inverts, and multiplies to solve for the correct product (5.NF.7b). However, the student fails to label the quantities appropriately, which demonstrates incomplete attention to the context of the task (no credit for MP2). The student's correct equation demonstrates reasoning related to partitive division involving a fractional amount (MP4). By showing the division of 12 by a fraction to solve for 16, the student also demonstrates understanding that the whole can be partitioned into fractional parts and the parts are smaller than the whole (MP7).

Total Awarded Points: 3 out of 4

Task 2. Cutting Ribbon Task

Amber has a 12-foot-long piece of ribbon. She wants pieces that are each $\frac{3}{4}$ of a foot long.

How many $\frac{3}{4}$ -foot-long pieces can Amber cut from the ribbon? Use words, diagrams, and multiplication or division equations to explain your thinking.



She can have 16 pieces because

$$\frac{3}{4} \overline{) 12} \begin{array}{r} 16 \\ 12 \\ \hline \end{array}$$

12 feet long

Cut it into 16 pieces



Guide 5

Litho 50045

Total Content Points: 1 (5.NF.7b)

Total Practice Points: 1 (MP7)

The student appropriately divides 12 by $\frac{3}{4}$ and solves for the correct answer (5.NF.7b).

The student does not represent the relationship between pieces and feet in an equation format, and although it is noted that the ribbon is 12 feet long and will be divided into 16 pieces, there is no labeling of the length of the pieces (no credit for MP2). The student does not provide an equation and the diagram is simply a ribbon labeled as “12 feet

long,” which does not model the pieces as being $\frac{3}{4}$ -foot-long, and therefore is ineffective

in modeling the situation (no credit for MP4). Though incorrectly representing the problem, the diagram does indicate that the whole can be partitioned into fractional parts and the parts are smaller than the whole, but, if recombined, equal the whole (MP7).

Total Awarded Points: 2 out of 4

Task 2. Cutting Ribbon Task

Amber has a 12-foot-long piece of ribbon. She wants pieces that are each $\frac{3}{4}$ of a foot long.

How many $\frac{3}{4}$ -foot-long pieces can Amber cut from the ribbon? Use words, diagrams, and multiplication or division equations to explain your thinking.

$12 \div \frac{3}{4} = 9$ foot long =

if you multiply the $\frac{3}{4}$ x the number of boxes it will be 9

$\frac{3}{4}$	$\frac{3}{4}$	$\frac{3}{4}$	$\frac{3}{4}$	$\frac{3}{4}$	$\frac{3}{4}$	$\frac{3}{4}$	$\frac{3}{4}$	$\frac{3}{4}$	$\frac{3}{4}$
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Total Content Points: 0

Total Practice Points: 2 (MP4, MP7)

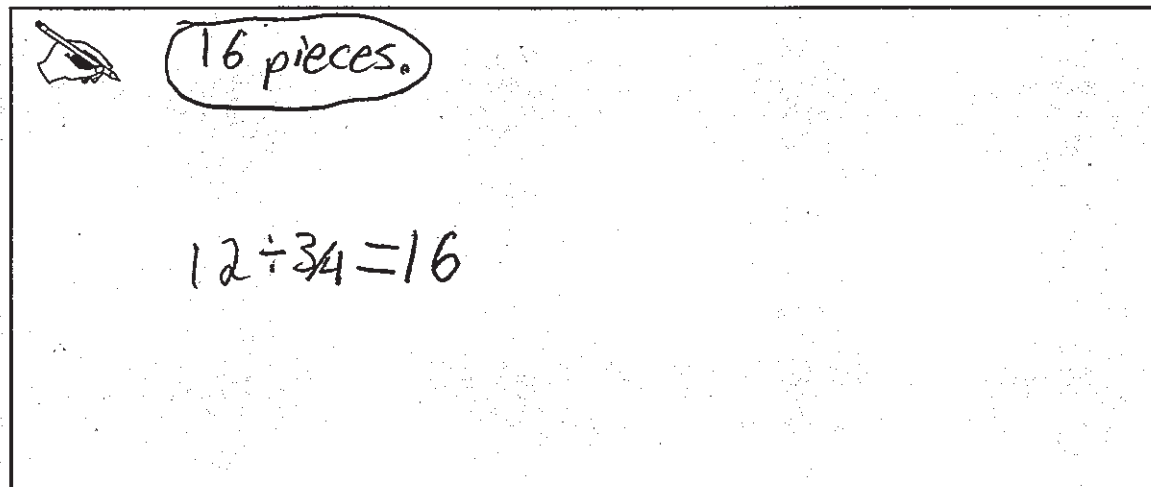
Although the student sets up a partially correct equation, the solution and the written explanation indicate insufficient understanding of how to divide fractions (no credit for 5.NF.7b) or how to use multiplication to find the number of $\frac{3}{4}$ -foot-long pieces in a ribbon 12 feet long (no credit for 5.NF.4a). The equation indicates misunderstanding of the relation between the given values, and the quantities are not labeled appropriately (no credit for MP2). Working with the incorrect solution of 9 feet, the student constructs a diagram modeling how a 9-foot ribbon would be partitioned into twelve $\frac{3}{4}$ -foot pieces, thereby demonstrating reasoning related to partitive division involving a fractional amount (MP4). The student's diagram and written explanation, although incorrect in regard to the task itself, do also demonstrate that the whole can be partitioned into fractional parts and the parts are smaller than the whole, but, if recombined, equal the whole (MP7).

Total Awarded Points: 2 out of 4

Task 2. Cutting Ribbon Task

Amber has a 12-foot-long piece of ribbon. She wants pieces that are each $\frac{3}{4}$ of a foot long.

How many $\frac{3}{4}$ -foot-long pieces can Amber cut from the ribbon? Use words, diagrams, and multiplication or division equations to explain your thinking.



A hand-drawn response box containing a pencil icon, the text "16 pieces." circled in a hand-drawn oval, and the equation $12 \div \frac{3}{4} = 16$.



Guide 7

Litho 50121

Total Content Points: 1 (5.NF.7b)

Total Practice Points: 1 (MP4)


The student constructs an appropriate division equation and solves for the correct answer of 16 (5.NF.7b). Though the equation is correct, the quantities are not labeled correctly (no credit for MP2). The student's equation does demonstrate reasoning related to partitive division involving a fractional amount by finding the correct number of pieces in a ribbon 12 feet long (MP4). Since the student only provides an equation with no additional explanation of the answer, the response does not demonstrate clear understanding that a whole can be partitioned into fractional parts which are smaller than the whole (no credit for MP7).

Total Awarded Points: 2 out of 4

Task 2. Cutting Ribbon Task

Amber has a 12-foot-long piece of ribbon. She wants pieces that are each $\frac{3}{4}$ of a foot long.

How many $\frac{3}{4}$ -foot-long pieces can Amber cut from the ribbon? Use words, diagrams, and multiplication or division equations to explain your thinking.

 16 cause I just added $\frac{3}{4} + \frac{3}{4}$ until I got to 12.



Total Content Points: 0

Total Practice Points: 1 (MP7)


The student explains that the correct answer of 16 was derived by adding $\frac{3}{4}$ until 12 was reached. However, the student does not follow through with a multiplicative equation showing that $16 \times \frac{3}{4} = 12$ feet (no credit for 5.NF.4a), or explain the next step of the given solution (counting the number of iterations of $\frac{3}{4}$ needed to reach 12), so the explanation of process remains incomplete (no credit for 5.NF.7b). The student does not provide a correct equation, and there is no apparent attempt to label or contextualize the numbers used in the student's approach (no credit for MP2). The student does not provide a diagram, equation, or words that model an appropriate solution to this task (no credit for MP4). By stating that 16 was derived by adding $\frac{3}{4}$ until 12 was reached, the student does indicate an understanding that the whole can be partitioned into fractional parts and the parts are smaller than the whole, and also shows recognition of an appropriate pattern related to the task (MP7).

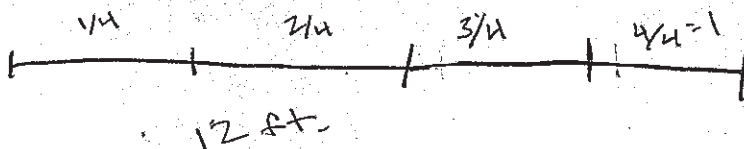
Total Awarded Points: 1 out of 4

Task 2. Cutting Ribbon Task

Amber has a 12-foot-long piece of ribbon. She wants pieces that are each $\frac{3}{4}$ of a foot long.

How many $\frac{3}{4}$ -foot-long pieces can Amber cut from the ribbon? Use words, diagrams, and multiplication or division equations to explain your thinking.

 Say you cut the rope into $\frac{1}{4}$, you would want to mark the $\frac{1}{4}$ spots to see where you would cut the one, two, three fourths, then there's only 1 fourth left.



Total Content Points: 0

Total Practice Points: 1 (MP7)


The student does not utilize an appropriate multiplicative or partitive approach to solve this problem (no credit for 5.NF.4a). The student divides the ribbon into fourths as opposed to dividing it into $\frac{1}{4}$ - foot-long pieces and solving (no credit for 5.NF.7b). The student does not provide a correct equation, and the lack of appropriate labeling and context reflects a lack of understanding of the task (no credit for MP2). The student apparently misinterprets the prompt to mean that a single piece would be $\frac{3}{4}$ of the entire ribbon, so the student divides the ribbon into fourths and states that “ $\frac{4}{4} = 1$.” There is no evidence of an appropriate model to find the solution to this task (no credit for MP4); however, the diagram does indicate that the whole can be partitioned into fractional parts and the parts are smaller than the whole, but, if recombined, equal the whole (MP7).

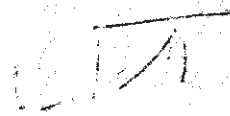
Total Awarded Points: 1 out of 4

Task 2. Cutting Ribbon Task

Amber has a 12-foot-long piece of ribbon. She wants pieces that are each $\frac{3}{4}$ of a foot long.

How many $\frac{3}{4}$ -foot-long pieces can Amber cut from the ribbon? Use words, diagrams, and multiplication or division equations to explain your thinking.

 18, because $\frac{3}{4} \times 12 = 18.$



Total Content Points: 0

Total Practice Points: 0


Although the student provides the correct solution of 16, the equation given is incorrect, and there is no evidence of an appropriate multiplicative or partitive strategy to solve the problem (no credit for either 5.NF.4a or 5.NF.7b). The inappropriate equation also lacks labeling (no credit for MP2). The student does not provide a diagram, equation, or words that model an appropriate solution to this task (no credit for MP4). Despite providing a correct answer, the lack of a correct process or explanation means that the student has failed to demonstrate recognition of any discernible pattern or structure related to this task (no credit for MP7).

Total Awarded Points: 0 out of 4

Task 2. Cutting Ribbon Task

Amber has a 12-foot-long piece of ribbon. She wants pieces that are each $\frac{3}{4}$ of a foot long.

How many $\frac{3}{4}$ -foot-long pieces can Amber cut from the ribbon? Use words, diagrams, and multiplication or division equations to explain your thinking.



and $\frac{3}{4} \div 12 = \frac{1}{16}$ $\frac{3}{4} \times 12 = 9$ You can multiply
and divide the problem.



Total Content Points: 0

Total Practice Points: 0

The student constructs two equations $\left(\frac{3}{4} \div 12 = \frac{1}{16} \text{ and } \frac{3}{4} \times 12 = 9\right)$ that both represent inappropriate strategies for this task (no credit for either 5.NF.4a or 5.NF.7b). These two inappropriate equations also lack labeling (no credit for MP2). The student does not provide a diagram, equation, or words that demonstrate correct reasoning related to partitive division (no credit for MP4). This student also fails to demonstrate recognition of any discernible pattern or structure related to this task (no credit for MP7).

Total Awarded Points: 0 out of 4

Task 2. Cutting Ribbon Task

Amber has a 12-foot-long piece of ribbon. She wants pieces that are each $\frac{3}{4}$ of a foot long.

How many $\frac{3}{4}$ -foot-long pieces can Amber cut from the ribbon? Use words, diagrams, and multiplication or division equations to explain your thinking.



$$\frac{3}{4} \times \frac{12}{1} = 9 \text{ pieces}$$

I multiplied $\frac{3}{4}$ and $\frac{12}{1}$ to
equal 9 pieces.



Total Content Points: 0

Total Practice Points: 0

Instead of dividing 12 by $\frac{3}{4}$, the student uses an inappropriate strategy (multiplying $\frac{3}{4}$ by 12), which demonstrates a lack of understanding of the content standards (no credit for either 5.NF.4a or 5.NF.7b). The student provides an inappropriate equation with no quantities labeled (no credit for MP2). The student does not provide a diagram, equation, or words that model an appropriate solution to this task (no credit for MP4). This student also fails to demonstrate recognition of any discernible pattern or structure related to this task (no credit for MP7).

Total Awarded Points: 0 out of 4