Grade 7: Reasoning with Ratios and Rates

A Set of Related Tasks and Lesson Guides

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Introduction

Tasks and Lesson Guides

ARC OVERVIEW

In this set of related tasks, 7th Grade students will develop and solidify knowledge of language, notation, representations, and general skills related to using ratios and proportions to solve problems. Through individual and group work followed up by rich discussions, students will not only practice these skills but also gain the ability to reason about ratio and proportion problems on a higher-order thinking level.

The Arc Preview table on page 5 provides all of the task questions contained in this arc. The tasks are aligned to standards 7.RP.A.1, 7.RP.A.2 and 7.RP.A.3.

- Task 1 develops students' ability to convert complex fractions to unit rates.
- Tasks 2 and 3 continue to develop skills with ratios and proportions, adding the ability to recognize when two ratios are proportional given specific ratios, a table, or a graph.
- Task 4 *solidifies* the concepts developed in Tasks 1 through 3.
- Task 5 continues to develop student understanding of unit rates and representations of ratios and proportions.
- Task 6 develops develops understanding of percentages as ratios.
- Task 7 develops understanding of unit conversions through representing conversion factors as ratios.
- Task 8 solidifies student understanding of all of the concepts outlined in Tasks 1 through 7.

Before starting this task arc, students should be familiar with tables, equations, and graphs of linear relationships and be able to simplify complex fractions. They should also be capable of finding and recognizing equivalent ratios and proportional relationships with whole number values in order to extend this idea to other rational numbers. The teacher may wish to review diagrams that can be used to represent ratios, such as tape diagrams and double number line diagrams, in order to give students visual ways to reason about proportional relationships, although the arc will focus on graphs as the visual representation.

Note that the some of the Essential Understandings listed in each task were modified from those contained in Pearson's EnVision Math series. Others were taken from NCTM's Developing Essential Understanding series. Tennessee State Mathematics Standards were retrieved from http://www.tn.gov/education/standards/math.shtml.

By the end of these eight tasks, students will be able to answer the following overarching questions:

- What is a unit rate? How do you calculate the unit rates?
- What unit of measure do you use to describe a unit rate?
- What does *proportional* mean?
- How can proportional relationships be represented?
- How can you tell if a relationship is proportional given a statement? Equation? Table? Graph?
- What is the constant of proportionality?
- What types of ratios can be used to represent percentages?
- How are unit conversion factors written as ratios?

The assessing questions, advancing questions, and whole group questions provided in this guide will ensure that students are working in ways aligned to the Standards for Mathematical Practice. Although the students will not be aware that this is occurring, the teacher can guide the process so that each MP (Mathematical Practice) is covered through good explanations, understanding of context, and clarification of reasoning behind solutions.

Arc Preview

Task 1: Running Buddies

Josiah and Elena are planning to start running together a couple of days per week. In order to estimate how far they could run together in a certain amount of time, they each timed a ½ mile solo run. Josiah ran the ½ mile in 5 minutes, 30 seconds, while Elena ran the ½ mile in 5 minutes, 15 seconds.

a) What is the unit rate for each runner? Explain what the unit rate means.
b) What is the maximum number of miles Josiah and Elena can run together in an hour if they run at Josiah's pace?

Goals for Task 1:

- Convert a complex fraction to a unit rate
- Compare unit rates to solve problems

Standards for Task 1:

7.RP.A.1 Compute unit rates associated with ratios of fractions, including ratios of lengths, areas and other quantities measured in like or different units. *For example, if a person walks ½ miles in each ¼ hour, compute the unit rate as the complex fraction ½/¼ miles per hour, equivalently 2 miles per hour.*

Task 2: Running Buddies, Part 2

Josiah and Elena's running group has gained popularity, and a few of their friends want to join them. They are concerned about the compatibility of running times, so they've required everyone to submit their best times. If a runner's time matches either Josiah's or Elena's, they will be permitted to join. The problem is, everyone has measured his or her running times for different distances!

Can you compare the times and decide who should join the running group?

Name	Time	Distance
Josiah	5 minutes, 30 seconds	½ mile
Elena	5 minutes, 15 seconds	½ mile
Simon	22 minutes, 5 seconds	2 miles
Oceana	16 minutes, 30 seconds	1 ½ miles
Delia	4 minutes, 12 seconds	2/5 miles
George	9 minutes	¾ mile

Goals for Task 2:

- Convert a complex fraction to a unit rate
- Compare unit rates to solve problems
- Recognize when two ratios are proportional

Standards for Task 2

7.RP.A.1 Compute unit rates associated with ratios of fractions, including ratios of lengths, areas and other quantities measured in like or different units. *For example, if a person walks ½ miles in each ¼ hour, compute the unit rate as the complex fraction ½/¼ miles per hour, equivalently 2 miles per hour.*

7.RP.A.2 Recognize and represent proportional relationships between quantities.
a. Decide whether two quantities are in a proportional relationship, e.g., by testing for equivalent ratios in a table or graphing on a coordinate plane and observing whether the graph is a straight line through the origin.

Task 3: What's the Catch?

Your parents have agreed to let you get a cell phone! Finally! The catch is that you have to research the plans available and decide which one would give you the best value.

Phone Co.	Monthly	Cost / # of Messages
Snappy & Co.	$0	$0.25/message
Venti Mobile	$20	$0.15/message
Cheshire Wireless	$0	$0.50/2 messages
Ritzy Cellular	$30	$0.10/message

a) Which plan would fit your needs best if you send between 0 and 100 messages each month?

b) Which plan would fit your needs best if you send over 200 messages each month?

c) Make 3 observations about the plans, using math vocabulary, graphs, or tables.

Goals for Task 3:

- Recognize when two ratios are proportional
- Recognize proportional relationships in a table or graph

Standards for Task 3:

7.RP.A.2 Recognize and represent proportional relationships between quantities.
a. Decide whether two quantities are in a proportional relationship, e.g., by testing for equivalent ratios in a table or graphing on a coordinate plane and observing whether the graph is a straight line through the origin.
b. Identify the constant of proportionality (unit rate) in tables, graphs, equations, diagrams, and verbal descriptions of proportional relationships.

Task 4: The Veggie Dispute

The teacher asked each member of your class to measure the growth of a vegetable in the school garden every day and record its changes over the course of a week. Your classmates have differing opinions about whether the growth of the vegetables is proportional or not, and they've come to you to settle the disagreement.

Review the data given and determine whether the relationships are proportional or not. Justify your answers using tables, graphs, or equations.

Veggie	Day 1	Day 2	Day 3	Day 4	Day 5
Tomato	0.5 in	1.0 in	1.5 in	2.0 in	2.5 in
Carrot	3.9 cm	4.5 cm	5.1 cm	5.7 cm	6.3 cm
Onion	4.1 cm	5.2 cm	6.4 cm	7 cm	8.5 cm
Zucchini	2.25 cm	4.5 cm	6.75 cm	9.00 cm	11.25 cm

Goals for Task 4:

- Convert a complex fraction to a unit rate
- Compare unit rates to solve problems
- Recognize when two ratios are proportional
- Recognize proportional relationships in a table or graph

Standards for Task 4:

7.RP.A.1 Compute unit rates associated with ratios of fractions, including ratios of lengths, areas and other quantities measured in like or different units. *For example, if a person walks ½ miles in each ¼ hour, compute the unit rate as the complex fraction ½/¼ miles per hour, equivalently 2 miles per hour.*

7.RP.A.2 Recognize and represent proportional relationships between quantities.
a. Decide whether two quantities are in a proportional relationship, e.g., by testing for equivalent ratios in a table or graphing on a coordinate plane and observing whether the graph is a straight line through the origin.
b. Identify the constant of proportionality (unit rate) in tables, graphs, equations, diagrams, and verbal descriptions of proportional relationships.

Task 5: Charlotte's Chocolates

Charlotte makes truffles for special occasions in her community. Her recipe uses 1 cup of cream and 2 cups of dark chocolate to make 20 truffles. Charlotte's granny is famous for her chocolatier skills, and she has offered to share her coveted chocolate truffle recipe with Charlotte! Granny's recipe uses 2 cups of cream to 5 cups of dark chocolate and makes 50 truffles.

a) Using math symbols, words, tables or diagrams, represent each of the ratios in these recipes in at least 2 ways. Are the recipes the same?
b) Graph the ratios and provide equations to match the graphs. Are the ratios the proportional? How can you tell? Are the recipes the same?
c) Charlotte needs to whip up a batch of truffles and has 10 cups of chocolate and 4 cups of cream on hand. Which recipe should she use to make the maximum number of truffles?
d) Which representation of the recipes seems the most useful to you for truffle-making? Why?

Goals for Task 5:

- Recognize when two ratios are proportional
- Recognize proportional relationships in a table or graph
- Express proportional relationships as equations
- Move easily between multiple representations of proportions, including graphs, tables, and equations

Standards for Task 5:

7.RP.A.1 Compute unit rates associated with ratios of fractions, including ratios of lengths, areas and other quantities measured in like or different units. *For example, if a person walks ½ miles in each ¼ hour, compute the unit rate as the complex fraction ½/¼ miles per hour, equivalently 2 miles per hour.*

7.RP.A.2 Recognize and represent proportional relationships between quantities.
a. Decide whether two quantities are in a proportional relationship, e.g., by testing for equivalent ratios in a table or graphing on a coordinate plane and observing whether the graph is a straight line through the origin.
b. Identify the constant of proportionality (unit rate) in tables, graphs, equations, diagrams, and verbal descriptions of proportional relationships.
c. Represent proportional relationships by equations. *For example, if total cost t is proportional to the number n of items purchased at a constant price p, the relationship between the total cost and the number of items can be expressed as* $t = pn$.
d. Explain what a point (x, y) on the graph of a proportional relationship means in terms of the situation, with special attention to the points $(0,0)$ and $(1,r)$, where *r* is the unit rate.

Task 6: Mental Math – Mall Edition!

Your favorite store is having a 25% off *everything* sale! You try on everything that catches your eye, love it all, and then realize that 25% off is still far from free.

a) If you buy something 25% off, what percentage of the original price do you pay? Can you represent these percentages in several equivalent ways?
b) If your mom offers to buy you $200 worth of clothes, which items from the table below can you buy? Come up with a plan based on the 25% discount and sales tax of 10%. Note that you may buy more than one of each should spend as close to $200 as possible. Describe how you would do the math mentally, in order to avoid getting caught doing math at the mall.

Item	Unit Price
Graphic Tee	$3.99
Blue Jeans	$43.99
Shoes	$31.99
Watch	$19.99
Cologne	$23.99
Jacket	$47.99
Socks	$7.99

Goals for Task 6:

- Recognize percentages as ratios
- Use ratios to solve percent problems

Standards for Task 6:

7.RP.A.3 Use proportional relationships to solve multistep ratio and percent problems. *Examples: simple interest, tax, markups and markdowns, gratuities and commissions, fees, percent increase and decrease, percent error.*

Task 7: Empire State Building Run-Up

The Empire State Building in New York City has 86 flights of stairs, covering a distance of 1,050 feet. Every year, 100 lucky people get to race to the top in the annual Empire State Building Run-Up.

The fastest recorded time was 9 minutes and 33 seconds by Australian runner Paul Crake in 2003. What is Crake's average speed in meters per second?

Goals for Task 7:

- Recognize unit conversions as multistep ratio problems
- Use ratios to convert units

Standards for Task 7:

7.RP.A.3 Use proportional relationships to solve multistep ratio and percent problems. *Examples: simple interest, tax, markups and markdowns, gratuities and commissions, fees, percent increase and decrease, percent error.*

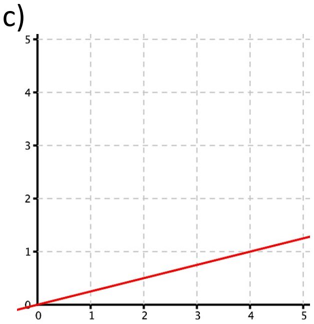
Task 8: Working Backward

For each of the following mathematical representations, come up with a real-life situation that it could represent. Express your real-life situation in multiple ways, using mathematical symbols and words. Why would this situation need to be represented using math symbols?

a) $\frac{25}{100}$

b) $c = 30n$

c)



d) $\frac{1}{4}$ mile, 154.25 seconds

e)

m	0	4	7	8	11
s	0	2	$\frac{7}{2}$	4	$\frac{11}{2}$

Goals for Task 8:

- Convert a complex fraction to a unit rate
- Compare unit rates to solve problems
- Recognize proportional relationships
- Recognize when two ratios are proportional
- Recognize proportional relationships in a table or graph
- Express proportional relationships as equations
- Move easily between multiple representations of proportions, including graphs, tables, and equations

Standards for Task 8:

7.RP.A.1 Compute unit rates associated with ratios of fractions, including ratios of lengths, areas and other quantities measured in like or different units. *For example, if a person walks ½ miles in each ¼ hour, compute the unit rate as the complex fraction ½/¼ miles per hour, equivalently 2 miles per hour.*

7.RP.A.2 Recognize and represent proportional relationships between quantities.

a. Decide whether two quantities are in a proportional relationship, e.g., by testing for equivalent ratios in a table or graphing on a coordinate plane and observing whether the graph is a straight line through the origin.

b. Identify the constant of proportionality (unit rate) in tables, graphs, equations, diagrams, and verbal descriptions of proportional relationships.

c. Represent proportional relationships by equations. *For example, if total cost t is proportional to the number n of items purchased at a constant price p, the relationship between the total cost and the number of items can be expressed as* $t = pn$.

d. Explain what a point (x, y) on the graph of a proportional relationship means in terms of the situation, with special attention to the points $(0,0)$ and $(1, r)$, where *r* is the unit rate.

7.RP.A.3 Use proportional relationships to solve multistep ratio and percent problems. *Examples: simple interest, tax, markups and markdowns, gratuities and commissions, fees, percent increase and decrease, percent error.*

Tasks' Standards Alignment

Task	7.RP.A.1	7.RP.A.2.a	7.RP.A.2.b	7.RP.A.2.c	7.RP.A.2.d	7.RP.A.3	MP 1	MP 2	MP 3	MP 4	MP 5	MP 6	MP 7	MP 8
Task 1 **Running Buddies**	✔						✔		✔			✔	✔	✔
Task 2 **Running Buddies, Part 2**	✔	✔					✔	✔			✔	✔	✔	✔
Task 3 **What's the Catch?**		✔	✔				✔	✔		✔	✔	✔		✔
Task 4 **The Veggie Dispute** *Solidifying Understanding*	✔	✔	✔				✔		✔	✔		✔	✔	
Task 5 **Charlotte's Chocolates**	✔	✔	✔	✔	✔		✔	✔	✔	✔	✔		✔	✔
Task 6 **Mental Math – Mall Edition!**						✔	✔	✔			✔		✔	✔
Task 7 **Race to the Top**						✔	✔	✔		✔	✔	✔	✔	✔
Task 8 **Working Backward** *Solidifying Understanding*	✔	✔	✔	✔	✔	✔	✔	✔		✔	✔	✔	✔	✔

The Standards for Mathematical Practice
1. Make sense of problems and persevere in solving them. 2. Reason abstractly and quantitatively. 3. Construct viable arguments and critique the reasoning of others. 4. Model with mathematics. 5. Use appropriate tools strategically. 6. Attend to precision. 7. Look for and make use of structure. 8. Look for and express regularity in repeated reasoning.

Name ______________________________

Task 1: Running Buddies

Josiah and Elena are planning to start running together a couple of days per week. In order to estimate how far they could run together in a certain amount of time, they each timed a ½ mile solo run. Josiah ran the ½ mile in 5 minutes, 30 seconds, while Elena ran the ½ mile in 5 minutes, 15 seconds.

a) What is the unit rate for each runner? Explain what the unit rate means.

b) What is the maximum number of miles Josiah and Elena can run together in an hour if they run at Josiah's pace?

Tennessee Department of Education: Lesson Guide 1

Task 1: Running Buddies — **7th Grade**

Josiah and Elena are planning to start running together a couple of days per week. In order to estimate how far they could run together in a certain amount of time, they each timed a ½ mile solo run. Josiah ran the ½ mile in 5 minutes, 30 seconds, while Elena ran the ½ mile in 5 minutes, 15 seconds.

a) What is the unit rate for each runner? Explain what the unit rate means.
b) What is the maximum number of miles Josiah and Elena can run together in an hour if they run at Josiah's pace?

Teacher Notes:

Since this task is meant to develop students' skills with ratios and unit rates, they should have a basic understanding of setting up ratios and calculating unit rates with whole numbers prior to beginning. Note that there are multiple acceptable answers for the first part of the problem, as the structure of the unit rate is not specified. This lends itself toward a rich whole class discussion, in which students can see how different ratios can be derived from the same information.

Tennessee State Standards for Mathematical Content	Tennessee State Standards for Mathematical Practice
7.RP.A.1 Compute unit rates associated with ratios of fractions, including ratios of lengths, areas and other quantities measured in like or different units. *For example, if a person walks ½ miles in each ¼ hour, compute the unit rate as the complex fraction ½/¼ miles per hour, equivalently 2 miles per hour.*	1. Make sense of problems and persevere in solving them. 2. Reason abstractly and quantitatively. 3. Construct viable arguments and critique the reasoning of others. 4. Model with mathematics. 5. Use appropriate tools strategically. 6. Attend to precision. 7. Look for and make use of structure. 8. Look for and express regularity in repeated reasoning.

Essential Understandings:

- A rate is a ratio that compares two quantities that are measured in different units.
- A unit rate is the comparison of two measurements in which the denominator has a value of one unit.

Explore Phase

Possible Solution Paths	Assessing and Advancing Questions
a) Josiah's unit rate is one of the following: 0.0015 mi/sec, 0.09 mi/min, 660 sec/mi, or 11 min/mi. Elena's unit rate one of the following: 0.0016 mi/sec, 0.095 mi/min, 630 sec/mi, or $10\frac{1}{2}$ min/mi. **The unit rates tell how many miles each runner can run in 1 second or minute or how many seconds or minutes each runner takes to run 1 mile. (Specifically, any of the following are acceptable:** • **Josiah can run 0.0015 miles in 1 second.** • **Josiah can run 0.09 miles in 1 minute.**	**Assessing Questions:** • How did you change the runners' times into a form that was easier to use to calculate a unit rate? • What value did you use for the distance each runner covered? • Can you express the distance in a different way and still solve the problem? • How did you decide whether to divide the distance by the time or the time by the distance? • Why does dividing the time by the distance (or distance by time) make it a unit rate?

- **Josiah can run 1 mile in 660 seconds.**
- **Josiah can run 1 mile in 11 minutes.**
- **Elena can run 0.0016 miles in 1 second.**
- **Elena can run 0.095 miles in 1 minute.**
- **Elena can run 1 mile in 630 seconds.**
- **Elena can run 1 mile in 10 and one half minutes.)**

Note: Answer should include what the unit rate means. Students should use language that shows an understanding that a unit rate is a ratio that tells how many parts of the first thing correspond with one part of the second. For example, *Josiah can run about 0.0015 miles in one second*, or *In 660 seconds, Josiah can run one mile*.

Students should recognize that the runners' times should to be converted to a single unit of measurement. They may choose to convert the time to seconds,

Josiah:

$$\begin{aligned} 5\,\text{min} + 30\,\text{sec} &= 5(60)\,\text{sec} + 30\,\text{sec} \\ &= (300+30)\,\text{sec} \\ &= 330\,\text{sec} \end{aligned}$$

Elena:

$$\begin{aligned} 5\,\text{min} + 15\,\text{sec} &= 5(60)\,\text{sec} + 15\,\text{sec} \\ &= (300+15)\,\text{sec} \\ &= 315\,\text{sec} \end{aligned},$$

or to minutes,

Josiah:

$$\begin{aligned} 5\,\text{min} + 30\,\text{sec} &= 5\,\text{min} + \frac{30}{60}\,\text{min} \\ &= (5+\frac{1}{2})\,\text{min} \\ &= 5\frac{1}{2}\,\text{min} \end{aligned}$$

Elena:

$$\begin{aligned} 5\,\text{min} + 15\,\text{sec} &= 5\,\text{min} + \frac{15}{60}\,\text{min} \\ &= (5+\frac{1}{4})\,\text{min} \\ &= 5\frac{1}{4}\,\text{min} \end{aligned}.$$

Depending on whether they converted to seconds or minutes, they will do one of two things:

- Do you think the unit rate is more useful in seconds or minutes?

Advancing Questions:

- What two measurements are we comparing in this problem?
- Can you write some ratios to compare Josiah's time and distance? Elena's?
- What does unit rate mean?
- If you want a denominator of 1, what operation would you need to perform?
- Does it matter if you use the runners' times in seconds or minutes? Do they need to be written in the same unit?
- In what types of situations would these unit rates be useful?

1) Students will find Josiah's unit rate by either dividing the number of miles by the number of seconds or by dividing the number of seconds by the number of miles. Since we give speeds in distance per time (such as, *miles per hour*) or in time per distance (such as, *minutes per mile*), either of the following is acceptable:

$$\begin{aligned}\frac{\frac{1}{2}mi}{330\sec} &= \frac{1}{2}mi \div 330\sec \\ &= \left(\frac{1}{2}\times\frac{1}{330}\right)mi/\sec \\ &= \frac{1}{660}mi/\sec \\ &\approx 0.0015 mi/\sec\end{aligned}$$

OR

$$\begin{aligned}\frac{330\sec}{\frac{1}{2}mi} &= 330\sec \div \frac{1}{2}mi \\ &= \left(330\times\frac{2}{1}\right)\sec/mi \\ &= 660\sec/mi\end{aligned}$$

Students will find Elena's unit rate in a similar manner, as below:

$$\begin{aligned}\frac{\frac{1}{2}mi}{315\sec} &= \frac{1}{2}mi \div 315\sec \\ &= \left(\frac{1}{2}\times\frac{1}{315}\right)mi/\sec \\ &= \frac{1}{630}mi/\sec \\ &\approx 0.0016 mi/\sec\end{aligned}$$

OR

$$\begin{aligned}\frac{315\sec}{\frac{1}{2}mi} &= 315\sec \div \frac{1}{2}mi \\ &= \left(315\times\frac{2}{1}\right)\sec/mi \\ &= 630\sec/mi\end{aligned}$$

Note: If students use the decimal 0.5 instead of the fraction $\frac{1}{2}$, they should be encouraged to consider how this problem would be done without decimals. This will help them to notice that the numbers are easier to calculate by hand if the fraction is used. 2) If students converted 5 minutes, 30 seconds to minutes, Josiah's unit rate will be found through the following steps: $\frac{\frac{1}{2}mi}{5\frac{1}{2}\text{min}} = \frac{1}{2}mi \div 5\frac{1}{2}\text{min}$ $= \left(\frac{1}{2} \div \frac{11}{2}\right) mi/\text{min}$ $= \left(\frac{1}{2} \times \frac{2}{11}\right) mi/\text{min}$ $= \frac{1}{11} mi/\text{min}$ $\approx 0.09 mi/\text{min}$ OR $\frac{5\frac{1}{2}\text{min}}{\frac{1}{2}mi} = 5\frac{1}{2}\text{min} \div \frac{1}{2}mi$ $= \left(\frac{11}{2} \div \frac{1}{2}\right)\text{min}/mi$ $= \left(\frac{11}{2} \times \frac{2}{1}\right)\text{min}/mi$ $= 11\text{min}/mi$ Elena's unit rate would be found in one of the following ways:	

$$\frac{\frac{1}{2}mi}{5\frac{1}{4}\text{min}} = \frac{1}{2}mi \div 5\frac{1}{4}\text{min}$$ $$= \left(\frac{1}{2} \div \frac{21}{4}\right) mi/\text{min}$$ $$= \left(\frac{1}{2} \times \frac{4}{21}\right) mi/\text{min}$$ $$= \frac{2}{21} mi/\text{min}$$ $$\approx 0.095 mi/\text{min}$$ OR $$\frac{5\frac{1}{4}\text{min}}{\frac{1}{2}mi} = 5\frac{1}{4}\text{min} \div \frac{1}{2}mi$$ $$= \left(\frac{21}{4} \div \frac{1}{2}\right)\text{min}/mi$$ $$= \left(\frac{21}{4} \times \frac{2}{1}\right)\text{min}/mi$$ $$= \frac{21}{2}\text{min}/mi$$ $$= 10\frac{1}{2}\text{min}/mi$$ As in 1), either form of the unit rate is acceptable, and students should be encouraged to use fractions, rather than decimals.	
b) Josiah and Elena could run about $5\frac{1}{2}$ miles together in an hour. In order to find the number of miles the runners could run in an hour if they ran at Josiah's pace, the students need to multiply the amount of time, 1 hour, by the number of miles Josiah can run in an hour. The exact method for doing this depends on how they found the unit rate in part a). 1) If students found Josiah's unit rate to be 0.0015 mi/sec or 660 sec/mi, then they need to find the number of seconds in an hour, which may be done as follows:	**Assessing Questions:** • How did you find the number of seconds (or minutes) in one hour? • How did you find the number of miles run in one hour? • How did you know to multiply (or divide)? **Advancing Questions:** • How long will Josiah and Elena be running? • Could you solve this problem without finding the unit rate first? • Whose unit rate should you use? • What was Josiah's unit rate in part a)? • If you want to know how many miles Josiah can run in one hour, how can you use the unit rate you found? Would a different unit rate be easier

$$1hr \times \frac{60\min}{1hr} \times \frac{60\sec}{1\min} = 3{,}600\sec .$$ Rather than using unit multipliers, they may also simply do the multiplication, $1 \times 60 \times 60 = 3{,}600$, to arrive at the same answer. Next, students must multiply the amount of time in seconds by the unit rate in order to find the number of miles run. $$3{,}600\sec \times \frac{0.0015mi}{1\sec} = 5.4mi$$ OR $$3{,}600\sec \times \frac{1mi}{660\sec} \approx 5.5mi$$ 2) If students found Josiah's unit rate to be 0.09 mi/min or 11 min/mi, then they need to know there are 60 minutes in 1 hour. Next, they must multiply the amount of time in minutes by the unit rate in order to find the number of miles run. $$60\min \times \frac{0.09mi}{1\min} = 5.4mi$$ OR $$60\min \times \frac{1mi}{11\min} \approx 5.5mi .$$	to work with? • How many seconds (or minutes) are in an hour? • What is Josiah's unit rate in miles per hour?
Possible Student Misconceptions	**Assessing and Advancing Questions**
Students think that unit rates are always multiplied (i.e. If they found Josiah's rate to be 660 sec/mi, they may think multiplying this by 3,600 seconds would give the number of miles run in an hour, as opposed to dividing by 660 sec/mi.)	• Can you write out the math you are doing, including the units? • What does *seconds per mile* (alternately, *minutes per mile*, *miles per second*, or *miles per minute*) mean? • What is the number in the denominator of the unit rate when you write it as a fraction? • How can you multiply these fractions to get the final units to be miles?
Students do not know that complex fractions can be simplified using division.	• Is $\left("\frac{\frac{1}{2}}{330} mi/\sec" , "\frac{330}{\frac{1}{2}} \sec/mi" , "\frac{\frac{1}{2}}{5\frac{1}{2}} mi/\min" , \right.$

	or "$\frac{5\frac{1}{2}}{\frac{1}{2}}$ min/mi") a unit rate? • What operation is represented by a fraction bar? • Can you write ("$\frac{1}{2}$ divided by 330", "330 divided by $\frac{1}{2}$", "$\frac{1}{2}$ divided by $5\frac{1}{2}$", or "$5\frac{1}{2}$ divided by $\frac{1}{2}$") without using a complex fraction? • What process can be used to divide ("a fraction and a whole number" or "a fraction and a mixed number")?
Entry/Extensions	**Assessing and Advancing Questions**
If students can't get started….	• What measurements are mentioned in this problem? • Can you write some ratios with the measurements in this problem? • What does the word *unit* mean? • What is a unit rate? • How can you change the units you are given so you have only one unit for time?
If students finish early….	• If Elena and Josiah are running together, which of them will limit the distance they can run in an hour? • How far can Josiah and Elena run in 3 hours if they run at Elena's rate? • Why are unit rates useful? List 3 reasons to share with the class, using Josiah and Elena as examples.

Discuss/Analyze

Whole Group Questions

Pick groups to share their work. Select a sequence that will progress students through higher order thinking. Emphasize the definition of unit rate and that there are multiple acceptable answers for the first part of the problem, as the structure of the unit rate is not specified.

A rate is a ratio that compares two quantities that are measured in different units.

- What rates did you find in this problem?
- Which are unit rates and which are not?
- What do you notice about the units of the rates?
- How would you define *rate* in your own words?
- Does anyone have a different definition?

A unit rate is the comparison of two measurements in which the denominator has a value of one unit.

- What does the word *unit* mean?
- What is a unit rate?

- How did you decide whether to divide the distance by the time or the time by the distance?
- What does *seconds per mile* (alternately, *minutes per mile*, *miles per second*, or *miles per minute*) mean?
- Why does dividing the time by the distance (or distance by time) make it a unit rate?
- What kind of units of measurement does the unit rate you found have? Did anyone have different units? Why are both correct? Is one more meaningful than the others in this problem?
- Did anyone use the unit rate to do Part b) of the problem? Explain.
- Why are unit rates useful?

Name ________________________________

Task 2: Running Buddies, Part 2

Josiah and Elena's running group has gained popularity, and a few of their friends want to join them. They are concerned about the compatibility of running times, so they've required everyone to submit their best times. If a runner's time matches either Josiah's or Elena's, they will be permitted to join. The problem is, everyone has measured his or her running times for different distances!

Name	Time	Distance
Josiah	5 minutes, 30 seconds	½ mile
Elena	5 minutes, 15 seconds	½ mile
Simon	22 minutes, 5 seconds	2 miles
Oceana	16 minutes, 30 seconds	1 ½ miles
Delia	4 minutes, 12 seconds	2/5 miles
George	9 minutes	¾ mile

Can you compare the times and decide who should join the running group?

Task 2: Running Buddies, Part 2	**7th Grade**

Josiah and Elena's running group has gained popularity, and a few of their friends want to join them. They are concerned about the compatibility of running times, so they've required everyone to submit their best times. If a runner's time matches either Josiah's or Elena's, they will be permitted to join. The problem is, everyone has measured his or her running times for different distances!

Name	Time	Distance
Josiah	5 minutes, 30 seconds	½ mile
Elena	5 minutes, 15 seconds	½ mile
Simon	22 minutes, 5 seconds	2 miles
Oceana	16 minutes, 30 seconds	1 ½ miles
Delia	4 minutes, 12 seconds	2/5 miles
George	9 minutes	¾ mile

Can you compare the times and decide who should join the running group?

Teacher Notes:

This task picks up where Task 1 of this arc left off, further developing students' skills with ratios and unit rates while adding recognition of proportional relationships. Since the structure of the unit rate is not specified, students need to realize that all comparisons must be done using the same unit rates.

Tennessee State Standards for Mathematical Content	Tennessee State Standards for Mathematical Practice
7.RP.A.1 Compute unit rates associated with ratios of fractions, including ratios of lengths, areas and other quantities measured in like or different units. *For example, if a person walks ½ miles in each ¼ hour, compute the unit rate as the complex fraction ½/¼ miles per hour, equivalently 2 miles per hour.* **7.RP.A.2** Recognize and represent proportional relationships between quantities. **a.** Decide whether two quantities are in a proportional relationship, e.g., by testing for equivalent ratios in a table or graphing on a coordinate plane and observing whether the graph is a straight line through the origin.	1. Make sense of problems and persevere in solving them. 2. Reason abstractly and quantitatively. 3. Construct viable arguments and critique the reasoning of others. 4. Model with mathematics. 5. Use appropriate tools strategically. 6. Attend to precision. 7. Look for and make use of structure. 8. Look for and express regularity in repeated reasoning.

Essential Understandings:

- A rate is a ratio that compares two quantities that are measured in different units.
- A unit rate is the comparison of two measurements in which the denominator has a value of one unit.
- Equivalent ratios can be observed using tables.
- Multiple representations such as words, ratios, equations, tables, or graphs are used to model proportional relationships in the real world and determine alternate equivalent representations.

Explore Phase

Possible Solution Paths

Since Simon and Oceana's times are proportional to Josiah's, and Delia's is proportional to Elena's, Simon, Oceana, and Delia should be allowed to join the running group. However, George's is different from both Josiah's and Elena's, so he shouldn't join. As the problem does not specify how the student should decide which runners have times similar to Josiah and Elena's, there are several methods that may be used.

Calculating Time per ½ Mile

As Josiah's and Elena's times for ½ mile are given, students may wish to find the other runners' times for ½ mile (or another distance) and compare them. Simon's ½ mile time would be calculated as follows:

- Convert the 2 mile time to one unit of measurement.

$$22\min + 5\sec = (22 + \frac{5}{60})\min$$
$$= 22\frac{1}{12}\min$$

- Calculate the amount of time to run ½ mile based on the amount of time it takes to run 2 miles.

$$\frac{22\frac{1}{12}\min}{2mi} = \frac{x\min}{\frac{1}{2}mi}$$
$$2x = 22\frac{1}{12} \times \frac{1}{2}$$
$$2x = \frac{265}{12} \times \frac{1}{2}$$
$$2x = \frac{265}{24}$$
$$x = \frac{\frac{265}{24}}{2} = \frac{265}{24} \div 2$$
$$x = \frac{265}{24} \times \frac{1}{2} = \frac{265}{48}$$
$$x = 5\frac{25}{48}\min$$

Assessing and Advancing Questions

Assessing Questions:

- How did you compare the times of the runners?
- Can you explain exactly how you converted one of the runner's times into something that can be compared with Josiah's and Elena's?
- Why is it that Simon's, Oceana's, and Josiah's times are related, even though they seem different initially?
- What math language describes this situation?

Advancing Questions:

- What was Josiah's ½ mile time? Elena's?
- Can this situation be modeled with ratios?
- Could you use a table to represent this situation?
- What information would have to be in the table?
- How many tables would you use?

- Change the fractional part of a minute to seconds.

$$\frac{25}{48}\text{min} \times \frac{60\,\text{sec}}{1\,\text{min}} = \frac{125}{4}\text{sec} \approx 31\,\text{sec}$$

The times for the other runners can be calculated similarly and are shown in the following table.

Runner	Time for ½ Mile
Simon	5 min, 31 sec
Oceana	5 min, 30 sec
Delia	5 min, 15 sec
George	6 min

This method is similar to using double-line diagrams. If using a double-line diagram, it would be most useful to make the first line 2 miles long and then make separate lines for each runner's time, calculating how long it would take each runner to run two miles by scaling up and down, as follows:

Finding Unit Rates
Students may choose to find the unit rate for each runner, since they already know Josiah's and Elena's from the previous task. Simon's can be calculated as follows:

- Calculate the number of seconds and minutes equivalent to Simon's time of 22 minutes and 5 seconds.

$$\begin{aligned} 22\min + 5\sec &= (22 + \frac{5}{60})\min \\ &\approx (22 + 0.08)\min \\ &= 22.08\min \end{aligned}$$

OR

$$\begin{aligned} 22\min + 5\sec &= (22 \times 60 + 5)\sec \\ &= (1{,}320 + 5)\sec \\ &= 1{,}325\sec \end{aligned}$$

- Divide the number of miles by the minutes or seconds OR divide the number of minutes or seconds by the number of miles.

$$\frac{2mi}{1325\sec} \approx 0.0015 mi/\sec$$

$$\frac{2mi}{22.08\min} \approx 0.09 mi/\min$$

$$\frac{1{,}325\sec}{2mi} = 662\frac{1}{2}\sec/mi$$

$$\frac{22.08\min}{2mi} = 11\frac{1}{25}\min/mi$$

The unit rates for the other runners can be found similarly and are shown in the table below.

NOTE: The students should calculate just one of the unit rates, preferably doing so in the same way that they did Josiah's and Elena's in the previous task.

				Unit Rates			
Name	**Time (sec)**	**Time (min)**	**Mi.**	**mi/sec**	**mi/min**	**sec/mi**	**min/mi**
Josiah	330	5.5	½	≈0.0015	≈0.09	660	11
Elena	315	5 ¼	½	≈0.0016	≈0.095	630	10 ½
Simon	1,325	≈22.08	2	≈0.0015	≈0.09	662½	11 1/25
Oceana	990	16.5	1 ½	≈0.0015	≈0.09	660	11
Delia	252	4.2	2/5	≈0.0016	≈0.10	630	10 ½
George	540	9	¾	≈0.0014	≈0.083	720	12

Assessing Questions:
- How did you compare the times of the runners?
- What is a unit rate?
- Why did you use unit rates?
- How did you calculate the unit rates?
- Could there be more than one unit rate for each runner?

Advancing Questions:
- What was Josiah's unit rate? Elena's?
- Can this situation be modeled with ratios?
- Could you use a table to represent this situation?
- What information would have to be in the table?
- How many tables would you use?

Converting to Miles per Hour	Assessing Questions:
Because it is a familiar method of measuring speed and because they already calculated Josiah's, students may choose to calculate the mile per hour rate for each runner in order to solve the problem. The following is the calculation for Simon:	• How did you compare the times of the runners? • Why does changing the times to miles per hour help solve the problem? • Did you have to use miles per hour, or would other units have worked as well?

Converting to Miles per Hour

Because it is a familiar method of measuring speed and because they already calculated Josiah's, students may choose to calculate the mile per hour rate for each runner in order to solve the problem. The following is the calculation for Simon:

- Change the time to hours, instead of minutes and seconds.

Since an hour is 60 minutes (or 3,600 seconds), the units can be converted by dividing the number of minutes by 60 and the number of seconds by 3,600.

$$22\min + 5\sec = \left(\frac{22}{60} + \frac{5}{3{,}600}\right)hrs$$
$$\approx 0.368hrs$$

- Divide the number of miles run by the number of hours it took.

$$\frac{2mi}{0.368hrs} = \frac{x}{1hr}$$
$$0.368x = 2(1)$$
$$x \approx 5.43mph$$

The mile per hour rate for each runner can be found similarly and is in the table below.

Runner	Time (hours)	Distance (miles)	Rate (mph)
Josiah	NA	NA	≈5.4/5.5
Elena	$\frac{7}{80} \approx 0.088$	$\frac{1}{2}$	≈5.7
Simon	$\frac{53}{144} \approx 0.368$	2	≈5.4
Oceana	$\frac{11}{40} \approx 0.275$	$1\frac{1}{2}$	≈5.4
Delia	$\frac{7}{100} = 0.07$	$\frac{2}{5}$	≈5.7
George	$\frac{3}{20} = 0.15$	$\frac{3}{4}$	5

Assessing Questions:

- How did you compare the times of the runners?
- Why does changing the times to miles per hour help solve the problem?
- Did you have to use miles per hour, or would other units have worked as well?

Advancing Questions:

- What is Josiah's rate in miles per hour?
- Can this situation be modeled with ratios?
- Could you use a table to represent this situation?
- What information would have to be in the table?
- How many tables would you use?

Calculating Times for Different Distances

Students may calculate the amount of time that it would take each runner to run multiples of the distances they reported running. The following tables represent the values they should find.

Assessing Questions:

- How did you compare the times of the runners?
- Can you explain exactly how you converted one of the runner's times into something that can be compared with Josiah's and Elena's?
- Did you use unit rates to compare the times?

Josiah

D	$\frac{1}{2}$	1	$1\frac{1}{2}$	2	$2\frac{1}{2}$	3	$3\frac{1}{2}$	4
T	$5\frac{1}{2}$	11	$16\frac{1}{2}$	22	$27\frac{1}{2}$	33	$38\frac{1}{2}$	44

Elena

D	$\frac{1}{2}$	1	$1\frac{1}{2}$	2	$2\frac{1}{2}$	3	$3\frac{1}{2}$	4
T	$5\frac{1}{4}$	$10\frac{1}{2}$	$15\frac{3}{4}$	21	$26\frac{1}{4}$	$31\frac{1}{2}$	$36\frac{3}{4}$	42

Simon

D	2	4
T	$\boxed{22\frac{1}{12}}$	$\boxed{44\frac{1}{6}}$

Oceana

D	$1\frac{1}{2}$	3	$\boxed{4\frac{1}{2}}$
T	$\boxed{16\frac{1}{2}}$	33	$\boxed{49\frac{1}{2}}$

Delia

D	**T**
$\frac{2}{5}$	$4\frac{1}{5}$
$\frac{4}{5}$	$8\frac{2}{5}$
$\frac{6}{5}$	$12\frac{3}{5}$
$1\frac{3}{5}$	$16\frac{4}{5}$
2	21
$2\frac{2}{5}$	$25\frac{1}{5}$
$2\frac{4}{5}$	$29\frac{2}{5}$
$3\frac{1}{5}$	$33\frac{3}{5}$
$3\frac{3}{5}$	$37\frac{4}{5}$
4	42

George

D	**T**
$\frac{3}{4}$	9
$1\frac{1}{2}$	18
$2\frac{1}{4}$	27
3	36
$3\frac{3}{4}$	45
$4\frac{1}{2}$	54

Why or why not?

Advancing Questions:

- Can this situation be modeled with ratios?
- Could you use a table to represent this situation?
- What information would have to be in the table?
- What increments should you use for time?
- How would you find the values for the table?
- How many tables would you use?

<table>
<tr><th>Possible Student Misconceptions</th><th>Assessing and Advancing Questions</th></tr>
<tr><td>Students think that having different values for each runner means that none of the runners’ speeds are compatible with Josiah’s and Elena’s speeds.</td><td>• If Simon can run 2 miles in 22 minutes, 5 seconds, how long would it take him to run 4 miles?
• Can you calculate the times for each runner to run different distances?
• What do you notice about the distances and times when you compare each runner?</td></tr>
<tr><td>Students do not make the connection between the mathematics they are doing and the vocabulary (i.e. unit rate and proportion).</td><td>• When you calculate the distance a runner can run in one second/minute/hour, what would we call that in mathematics?
• If a runner can run a distance in the same amount of time as another runner, what is the mathematical language that is used to describe that type of relationship?</td></tr>
<tr><th>Entry/Extensions</th><th>Assessing and Advancing Questions</th></tr>
<tr><td>If students can’t get started….</td><td>• Is there information in the previous task that can help you solve this one?
• How can you compare the data you have for Josiah and Elena to the data you have for the other runners?
• What makes it difficult to compare the times of the runners as they are presented?
• How can you adjust the times to be easier to compare?</td></tr>
<tr><td>If students finish early….</td><td>• Can you graph some distances and times for the runners on a coordinate axis? What do you notice is the same about all of the graphs? Are any of the graphs the same? Why? Which ones are different? Could you have predicted which ones would be the same and which would be different?
• Can you think of a way that Josiah and Elena could test the compatibility of the runners without doing math problems?</td></tr>
<tr><th colspan="2">Discuss/Analyze</th></tr>
<tr><th colspan="2">Whole Group Questions</th></tr>
<tr><td colspan="2">Pick groups to share their work. Select a sequence that will progress students through higher order thinking. Since the structure of the unit rate is not specified, students need to realize that all comparisons must be done using the same unit rates.

A rate is a ratio that compares two quantities that are measured in different units.
• Are there rates in this problem?
• What are they?
• What units are on the rates?
• What is a rate?</td></tr>
</table>

A unit rate is the comparison of two measurements in which the denominator has a value of one unit.

- What is a unit rate?
- Did you use unit rates to compare the times? Why or why not?
- How did you calculate the unit rates?
- Could there be more than one unit rate for each runner?
- What units of measurement are your unit rates in?
- Would other units of measurement have worked as well?

Multiple representations such as words, ratios, equations, tables, or graphs are used to model proportional relationships in the real world and determine alternate equivalent representations.

- If a runner's rate is the same as that of another runner, what mathematical language could be used to describe that situation?
- What makes it difficult to compare the times of the runners as they are presented?
- How did you decide if these times are compatible or proportional?
- Did anyone do it differently?
- How could you solve this problem using a diagram?
- How could you solve it using a table?
- Which method of representing the rates seems to be the best? Why?

Name ______________________________

Task 3: What's the Catch?

Phone Co.	Monthly	Cost / # of Messages
Snappy & Co.	$0	$0.25/message
Venti Mobile	$20	$0.15/message
Cheshire Wireless	$0	$0.50/2 messages
Ritzy Cellular	$30	$0.10/message

Your parents have agreed to let you get a cell phone! Finally! The catch is that you have to research the plans available and decide which one would give you the best value.

a) Which plan would fit your needs best if you send between 0 and 100 messages each month?

b) Which plan would fit your needs best if you send over 200 messages each month?

c) Make 3 observations about the plans, using math vocabulary, graphs, or tables.

Task 3: What's the Catch? **7th Grade**

Your parents have agreed to let you get a cell phone! Finally! The catch is that you have to research the plans available and decide which one would give you the best value.

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Venti Mobile	$20	$0.15/message
Cheshire Wireless	$0	$0.50/2 messages
Ritzy Cellular	$30	$0.10/message

a) Which plan would fit your needs best if you send between 0 and 100 messages each month?
b) Which plan would fit your needs best if you send over 200 messages each month?
c) Make 3 observations about the plans, using math vocabulary, graphs, or tables.

Teacher Notes:

This task continues to develop the ratio and proportion concepts from Tasks 1 and 2, adding graphs of proportional relationships. Questioning should guide students to make observations about the slope of the lines of proportional relationships and the constant of proportionality (unit rate). Part c) requires them to make their own observations about the plans, which may require some guidance toward observations that have to do with the differences between proportional relationships and relationships which are not proportional.

Tennessee State Standards for Mathematical Content	Tennessee State Standards for Mathematical Practice
7.RP.A.2 Recognize and represent proportional relationships between quantities. **a.** Decide whether two quantities are in a proportional relationship, e.g., by testing for equivalent ratios in a table or graphing on a coordinate plane and observing whether the graph is a straight line through the origin. **b.** Identify the constant of proportionality (unit rate) in tables, graphs, equations, diagrams, and verbal descriptions of proportional relationships.	1. Make sense of problems and persevere in solving them. 2. Reason abstractly and quantitatively. 3. Construct viable arguments and critique the reasoning of others. 4. Model with mathematics. 5. Use appropriate tools strategically. 6. Attend to precision. 7. Look for and make use of structure. 8. Look for and express regularity in repeated reasoning.

Essential Understandings:

- Equivalent ratios can be observed using tables.
- The graph of two variables that are directly proportional is a line that passes through the origin (0,0).
- Multiple representations such as words, ratios, equations, tables, or graphs are used to model proportional relationships in the real world and determine alternate equivalent representations.
- A proportion is an equation that states that two ratios are equal.

Explore Phase

Possible Solution Paths	Assessing and Advancing Questions
a) I can choose either Snappy or Cheshire because they cost the same and are the least expensive when 100 or fewer messages are sent. Students should explore the cost of each plan for different numbers of messages between 0 and 100 and compare them to choose the best plan. They may choose to do this in one of the following ways.	**Assessing Questions:** • How did you get an overview of the plans to make your decision? • Why did you choose the numbers of messages that you used in your table (or points)? • How did you calculate the monthly cost of Snappy & Co. for each cell of the table (or point of the graph)?

1) *Table*

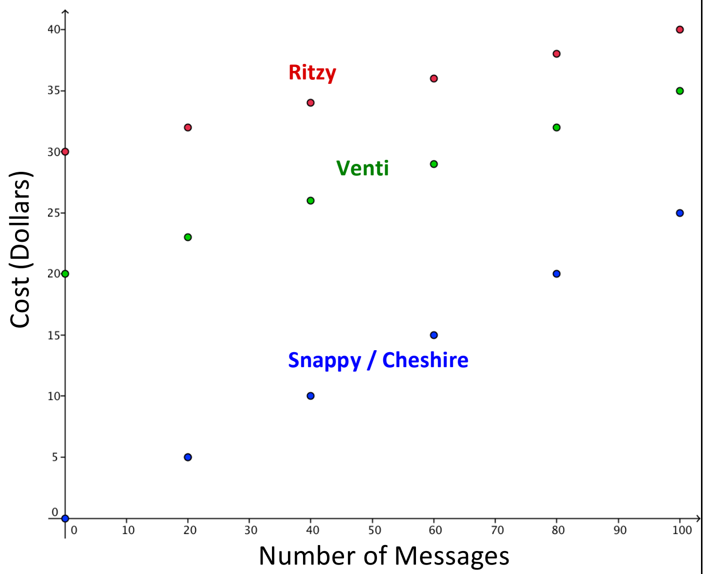
Students may make a table, calculating the monthly cost for different numbers of messages on different plans, as follows:

	0	**20**	**40**	**60**	**80**	**100**
Snappy & Co.	$0.00	$5.00	$10.00	$15.00	$20.00	$25.00
Venti Mobile	$20.00	$23.00	$26.00	$29.00	$32.00	$35.00
Cheshire Wireless	$0.00	$5.00	$10.00	$15.00	$20.00	$25.00
Ritzy Cellular	$30.00	$32.00	$34.00	$36.00	$38.00	$40.00

They should notice that the values for the Snappy and Cheshire companies are the same, and that they are cheaper than both of the other companies when the customer sends 100 or fewer messages.

2) *Graph*

Students may graph the relationships to solve the problem.



Students should notice that the Snappy and Cheshire graphs are the same, and that they are consistently cheaper than the other two plans.

Advancing Questions:

- Can you find the cost of each plan for various numbers of messages?
- Do you notice patterns?
- Could you represent these patterns using a graph?

b) I should choose Ritzy Cellular if I send more than 200 messages each month because it is the least expensive when more than 200 messages are sent.

Students should explore the cost of each plan for different numbers of messages over 200 and compare them to choose the best plan. They may choose to do this in one of the following ways.

Assessing Questions:

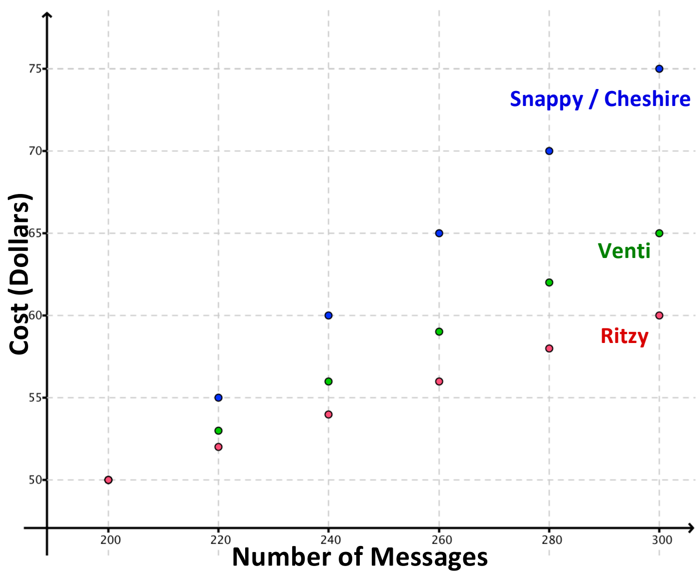
- How did you get an overview of the plans to make your decision?
- Why did you choose the numbers of messages that you used in your table (or points on your graph)?
- How did you calculate the per month cost of Snappy & Co. for each cell in the table (or point on the graph)?

1) *Table*
Students may make a table, calculating the per month cost for different numbers of messages on different plans, as follows:

	200	220	240	260	280	300
Snappy & Co.	$50.00	$55.00	$60.00	$65.00	$70.00	$75.00
Venti Mobile	$50.00	$53.00	$56.00	$59.00	$62.00	$65.00
Cheshire Wireless	$50.00	$55.00	$60.00	$65.00	$70.00	$75.00
Ritzy Cellular	$50.00	$52.00	$54.00	$56.00	$58.00	$60.00

They should notice that Snappy and Cheshire become more expensive than the other two companies when the customer sends 200 or more messages.

2) *Graph*
Students may graph the relationships to solve the problem.



They should notice from the graph that after 200 messages, Snappy & Cheshire become more expensive than the other two.

- What type of language describes the different plans?
- Are some of them proportional? How do you know?

Advancing Questions:

- How could you get an overview of the costs of the plans in order to make your decision?
- For what numbers of messages should you calculate the costs in this part of the problem?

c) Students should make observations related to proportional reasoning, such as equations for the proportions, how to recognize a graph of a proportion, and how to recognize a proportion in a table. *Answers to this part may vary, since it is open-ended.*

Assessing Questions:

- What vocabulary did you use in your observations?
- What does *proportional* mean?
- What are *equivalent ratios*?
- What is a *constant of proportionality*?
- What is a *unit rate*?

The following are some examples:

1) *Vocabulary – Proportionality*
Since both Snappy & Co. and Cheshire Wireless increase by the same amounts per message, they have proportional relationships. On the other hand, Venti Mobile and Ritzy Cellular have flat monthly fees, which make them not proportional. For example, with Venti and Ritzy, the unit rate for 1 message is not the same as the unit rate for 10 messages with Venti and Ritzy.

2) *Graph*
(Referring to graph in part a) Since Snappy and Cheshire go through the origin, they represent proportional relationships, since the rate for each message would be the same regardless of how many messages were sent. Venti and Ritzy, however, are not proportional, since they do not pass through the origin.

3) *Table*
(Referring to either the table in part a), b), or both) From the table, I can tell that Snappy and Cheshire are proportional, since they start at (0, \$0.00) and then increase by the same amount in the number of messages from left to right and in the cost of the plan. Venti and Ritzy increase by the same amount each time, but the flat fee for the month makes them not proportional, since this monthly fee becomes less and less per message as the number of messages increases.

4) *Vocabulary – Proportionality with Calculations*
Snappy and Cheshire are proportional, because when I set up ratios using values representing the number of messages sent and the corresponding monthly cost, they are proportional, as below:

$$\frac{20}{\$5.00} = \frac{60}{\$15.00}$$

By equivalent fractions: $\frac{60 \div 3}{\$15.00 \div 3} = \frac{20}{\$5.00}$

By cross-multiplication:
$$20 \cdot 15 = 60 \cdot 5$$
$$300 = 300$$

Venti and Ritzy are not proportional by the same reasoning.

- Did you make any statements about proportionality? How did you support them?
- Is there more than one way to show that a relationship is proportional?
- What types of relationships have constants of proportionality? Unit rates?

Advancing Questions:

- Can you use some of the terms we've learned recently to describe the relationships in this problem? For instance, can you use *proportional*, *equivalent ratio*, and *unit rate* to make observations?
- Which relationships are proportional? How do you know?
- Can you refer to earlier parts of the problem to either support your observation or come up with another observation?

Venti $\frac{20}{\$23.00} = \frac{60}{\$15.00}$ $20 \cdot 15 = 60 \cdot 23$ $300 \neq 1{,}380$ $\frac{20}{\$32.00} = \frac{60}{\$36.00}$ $20 \cdot 36 = 60 \cdot 32$ $720 \neq 1{,}920$ Ritzy 5) *Vocabulary – Constant of Proportionality* Since both Snappy and Cheshire are proportional, we can find the unit rates or constants of proportionality for both of these companies by dividing the cost of the plan by the number of messages sent at any point on the graph or table. The constant of proportionality for both of them is \$0.25 per message. Since Venti and Ritzy are not proportional, they do not have constants of proportionality or unit rates, since dividing the cost by the number of messages yields a different value at every point on the graph or table.	
Possible Student Misconceptions	**Assessing and Advancing Questions**
Students use too few values to determine which plan should be used. *For example, in part a), they may only calculate the cost of each plan for 100 messages.*	• If a plan is the cheapest for a certain number of messages, do you know for sure that it is always the cheapest? • How can you be more certain that the plan you've chosen will always be the cheapest from 0 to 100 messages?
Students think that the constant rate of the plans means that they are all proportional.	• Can you tell me in your own words what *proportional* means? • How can you test to see if a relationship is proportional? • Can you test each of these and then say which are proportional and which are not? • Are all linear relationships proportional?
Entry/Extensions	**Assessing and Advancing Questions**
If students can't get started….	• How would you decide how much each plan would cost for a certain number of messages? • Can you pick different numbers of messages and calculate the cost to compare them? • What method could you use to organize the data you're generating?
If students finish early….	• Can you use the equations you found for the cost of the plans to create a graph that shows the differences in proportional relationships and relationships that are not proportional? • What part of the graph represents the unit rate? • What is the significance of the *y*-intercept of a line, when we're talking about proportions?

Discuss/Analyze
Whole Group Questions

Pick groups to share their work. Select a sequence that will progress students through higher order thinking. Highlight any student answers that made use of a graph. Emphasize that in order for the relationship to be proportional, the graph must pass through the origin.

Multiple representations such as words, ratios, equations, tables, or graphs are used to model proportional relationships in the real world and determine alternate equivalent representations.

- How did you get an overview of the plans to make your decision?
- Why did you choose the numbers of messages that you used in your table (or points)?
- How did you calculate the per month cost of Snappy & Co. for each cell in the table (or point on the graph)?
- Can you tell me in your own words what *proportional* means?
- Do you notice patterns in the values for each plan?
- Are some of them proportional? How do you know?
- Did you make any statements about proportionality in your observations? How did you support them?
- Is there more than one way to show that a relationship is proportional?

A proportion is an equation that states that two ratios are equal.

- Did anyone use equations to represent the relationships in this problem?
- How can you tell if two ratios are proportional using an equation?

The graph of two variables that are directly proportional is a line that passes through the origin (0,0).

- Could you represent the patterns you observed in the tables using a graph?
- What is a *constant of proportionality*?
- What is a *unit rate*?
- What types of relationships have constants of proportionality? Unit rates?
- Are all linear relationships proportional?
- Could you find an equation for the cost of the plans based on the graphs?
- What part of the graph represents the unit rate?
- What is the significance of the *y*-intercept of a line, when we're talking about proportions?

Name ______________________________

Task 4: The Veggie Dispute

The teacher asked each member of your class to measure the growth of a vegetable in the school garden every day and record its changes over the course of a week. Your classmates have differing opinions about whether the growth of the vegetables is proportional or not, and they've come to you to settle the disagreement.

Veggie	Day 1	Day 2	Day 3	Day 4	Day 5
Tomato	0.5 in	1.0 in	1.5 in	2.0 in	2.5 in
Carrot	3.9 cm	4.5 cm	5.1 cm	5.7 cm	6.3 cm
Onion	4.1 cm	5.2 cm	6.4 cm	7 cm	8.5 cm
Zucchini	2.25 cm	4.5 cm	6.75 cm	9 cm	11.25 cm

Review the data given and determine whether the relationships are proportional or not. Justify your answers using tables, graphs, or equations.

Task 4: The Veggie Dispute — **7th Grade**

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Zucchini	2.25 cm	4.5 cm	6.75 cm	9 cm	11.25 cm

Teacher Notes:

This task solidifies the understanding students have gained in Tasks 1 through 3 by asking them to use all of the methods of testing proportions in the same problem.

Tennessee State Standards for Mathematical Content	Tennessee State Standards for Mathematical Practice
7.RP.A.1 Compute unit rates associated with ratios of fractions, including ratios of lengths, areas and other quantities measured in like or different units. *For example, if a person walks ½ miles in each ¼ hour, compute the unit rate as the complex fraction ½/¼ miles per hour, equivalently 2 miles per hour.* **7.RP.A.2** Recognize and represent proportional relationships between quantities. **a.** Decide whether two quantities are in a proportional relationship, e.g., by testing for equivalent ratios in a table or graphing on a coordinate plane and observing whether the graph is a straight line through the origin. **b.** Identify the constant of proportionality (unit rate) in tables, graphs, equations, diagrams, and verbal descriptions of proportional relationships.	1. Make sense of problems and persevere in solving them. 2. Reason abstractly and quantitatively. 3. Construct viable arguments and critique the reasoning of others. 4. Model with mathematics. 5. Use appropriate tools strategically. 6. Attend to precision. 7. Look for and make use of structure. 8. Look for and express regularity in repeated reasoning.

Essential Understandings:

- A rate is a ratio that compares two quantities that are measured in different units.
- A unit rate is the comparison of two measurements in which the denominator has a value of one unit.
- Equivalent ratios can be observed using tables.
- The graph of two variables that are directly proportional is a line that passes through the origin (0,0).
- Multiple representations such as words, ratios, equations, tables, or graphs are used to model proportional relationships in the real world and determine alternate equivalent representations.
- A proportion is an equation that states that two ratios are equal.

Explore Phase	
Possible Solution Paths	**Assessing and Advancing Questions**

Possible Solution Paths

Only the growth of the tomato and zucchini are proportional relationships, since they are the only ones with equivalent ratios starting at (0,0).

Students may choose to provide justification in one of the following ways:

1) *Growth Rate Calculation*
Students may try to find the growth rate for each vegetable. They may do this either by dividing the length of the vegetable by the number of days it has been being observed or by checking to see if the change in length from one day to the next is always the same. If using the second method, all of the following calculations would need to be done:

Tomato	
Day Range	*Growth*
Day 1 to 2	1.0 *in* – 0.5 *in* = 0.5 *in*
Day 2 to 3	1.5 *in* – 1.0 *in* = 0.5 *in*
Day 3 to 4	2.0 *in* – 1.5 *in* = 0.5 *in*
Day 4 to 5	2.5 *in* – 2.0 *in* = 0.5 *in*

Carrot	
Day Range	*Growth*
Day 1 to 2	4.5 – 3.9 = 0.6 cm
Day 2 to 3	5.1 – 4.5 = 0.6 cm
Day 3 to 4	5.7 – 5.1 = 0.6 cm
Day 4 to 5	6.3 – 5.7 = 0.6 cm

Onion	
Day Range	*Growth*
Day 1 to 2	5.2 – 4.1 = 1.2 cm
Day 2 to 3	6.4 – 5.2 = 1.2 cm
Day 3 to 4	7.0 – 6.4 = 0.6 cm
Day 4 to 5	8.5 – 7.0 = 1.5 cm

Zucchini	
Day Range	*Growth*
Day 1 to 2	4.5 – 2.25 = 2.25 cm
Day 2 to 3	6.76 – 4.5 = 2.25 cm
Day 3 to 4	9.0 – 6.75 = 2.25 cm
Day 4 to 5	11.25 – 9.0 = 2.25 cm

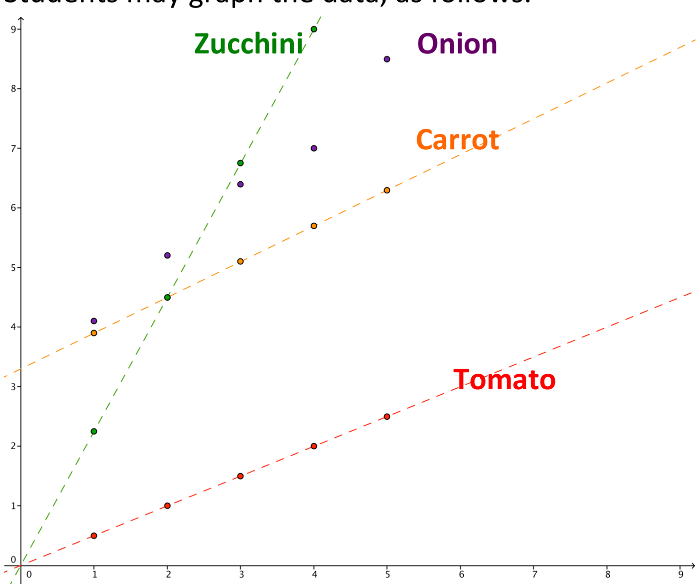
Assessing and Advancing Questions

Assessing Questions:

- How did you decide if the relationships were proportional or not?
- What must be true of a relationship in order for it to be proportional?
- Is it enough to show that the relationship has a constant rate of change for the values given?
- What must we assume in this problem to say that any of the growth relationships are proportional?
- How did you find the unit rates for the growth of each vegetable?

Advancing Questions:

- What must be true of a relationship in order for it to be proportional?
- How can you tell if the growth rate is constant?
- Is that enough information?
- Would another representation of the data help you decide which relationships are proportional?

If observing the difference between each measurement to see if the growth is a proportional relationship, then students should recognize that they must not only consider the values given but also what the value is at Day 0.	
2) *Unit Rate Vocabulary/Rationale* Students may analyze the values in the table. They should note that, if the measurements for the tomato and the zucchini are assumed to have been zero before the measurement start date, then these relationships increase the by the same amount each time and are proportional. The measurements for the carrot, although increasing by the same amount each time, cannot be reasonably assumed to have been zero before day 1, and thus cannot be represented using a proportion. The carrot's measurement relationship is the most likely to be misunderstood, since the measurements increase by the same amount from day-to-day but this relationship would not hold if we went backward from Day 1.	**Assessing Questions:** • How did you decide if the relationships were proportional or not? • What must be true of a relationship in order for it to be proportional? • Is it enough to say that the relationship has a constant rate of change for the values given? • What must we assume in this problem to say that any of the growth relationships are proportional? **Advancing Questions:** • What must be true of a relationship in order for it to be proportional? • How can you tell if the growth rate is constant? • Is that enough information? • Would another representation of the data help you decide which relationships are proportional?
3) *Graph / Equation* Students may graph the data, as follows.  They should observe that the data points of the tomato, carrot, and zucchini are linear, but the data points of the carrot do not fall on a line that goes through the origin. Also, the measurements of the onion are not linear at all.	**Assessing Questions:** • How did you decide if the relationships were proportional or not? • What must be true of a relationship in order for it to be proportional? • Is it enough to say that the relationship has a constant rate of change for the values given? • What must we assume in this problem to say that any of the growth relationships are proportional? • What types of graphs illustrate proportional relationships? • What types of equations model proportional relationships? **Advancing Questions:** • What must be true of a relationship in order for it to be proportional? • How can you tell if the growth rate is constant? • Is that enough information? • Would another representation of the data help you decide which relationships are proportional?

<table>
<tr><td>Students may take this reasoning further and write equations for each of the linear relationships. They should be as follows:
<table><tr><th>Veggie</th><th>Equation</th></tr><tr><td>Tomato</td><td>$y = 0.5x$</td></tr><tr><td>Carrot</td><td>$y = 0.6x + 3.3$</td></tr><tr><td>Onion</td><td>N/A</td></tr><tr><td>Zucchini</td><td>$y = 2.25x$</td></tr></table>Those that are linear with a y-intercept of 0 can be identified as proportional.</td><td></td></tr>
<tr><th>Possible Student Misconceptions</th><th>Assessing and Advancing Questions</th></tr>
<tr><td>Students think that a relationship is automatically proportional if it has a constant rate.</td><td>• What is the rate of change for this relationship?
• Does that rate of change stay the same when you consider the height of the plant at Day 0?
• What height must the plant have at Day 0 in order for its growth to be a proportional relationship?</td></tr>
<tr><td>Students do not realize that proportions can be represented many different ways (tables, graphs, equations, etc.).</td><td>• In math, how can values from a table be represented?
• Could we use a graph to display these values?
• Can you write an equation to match the graph?
• What do you notice about these representations? How can you tell if the relationships are proportional?</td></tr>
<tr><th>Entry/Extensions</th><th>Assessing and Advancing Questions</th></tr>
<tr><td>If students can't get started….</td><td>• What is true about a proportional relationship?
• How can you tell if the unit rate is the same from day to day?
• Is that enough information?
• Would another representation of the data help you decide which relationships are proportional?</td></tr>
<tr><td>If students finish early….</td><td>• If you did not create a graph to display the data, can you do so now? What do you notice about the graphs?
• If you did not write equations to model the linear relationships, can you do so now? What is true of the equations that model the proportional relationships? Where can the constant of proportionality be found in the equations?</td></tr>
</table>

Discuss/Analyze
Whole Group Questions
Pick groups to share their work. Select a sequence that will progress students through higher order thinking. Highlight any student answers that made use of a graph. Emphasize that in order for the relationship to be proportional, the graph must pass through the origin. **A unit rate is the comparison of two measurements in which the denominator has a value of one unit.** • How did you find the unit rates for the growth of each vegetable? • How can you tell if the growth rate is constant? • Is that enough information? • How can you tell if the unit rate is the same from day to day? **The graph of two variables that are directly proportional is a line that passes through the origin (0,0). Multiple representations such as words, ratios, equations, tables, or graphs are used to model proportional relationships in the real world and determine alternate equivalent representations.** • What must be true of a relationship in order for it to be proportional? • What must we assume in this problem to say that any of the growth relationships are proportional? • Is it enough to show that the relationship has a constant rate of change for the values given? • What types of graphs illustrate proportional relationships? • What types of equations model proportional relationships? • What types of relationships can seem proportional but aren't? • What types of relationships are obviously not proportional? Are some criteria true for these relationships, while others are false? • Does that rate of change stay the same when you consider the height of the plant at Day 0? • What height must the plant have at Day 0 in order for its growth to be a proportional relationship? • What do you notice about the graphs? • What is the constant of proportionality for the proportional relationships? How does that relate to the graphs? • If you did not write equations to model the linear relationships, can you do so now? What do you notice about the equations? What is true of the equations that model the proportional relationships? Where can the constant of proportionality be found in the equations?

Name ______________________________

Task 5: Charlotte's Chocolates

Charlotte makes truffles for special occasions in her community. Her recipe uses 1 cup of cream and 2 cups of dark chocolate to make 20 truffles. Charlotte's granny is famous for her chocolatier skills, and she has offered to share her coveted chocolate truffle recipe with Charlotte! Granny's recipe uses 2 cups of cream to 5 cups of dark chocolate and makes 50 truffles.

a) Using math symbols, words, tables, or diagrams, represent each of the ratios in these recipes in at least 2 ways.

b) Graph the ratios and provide equations to match the graphs. Are the ratios the proportional? How can you tell? Are the recipes the same?

c) Charlotte needs to whip up a batch of truffles and has 10 cups of chocolate and 4 cups of cream on hand. Which recipe should she use to make the maximum number of truffles?

d) Which representation of the recipes seems the most useful to you for truffle-making? Why?

Task 5: Charlotte's Chocolates	**7th Grade**
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Teacher Notes:	
This task develops students' skills with ratios and proportions, extending the understanding and representations presented in Tasks 1-4 of this arc to include writing equations and creating graphs as well as understanding the significance of the points (0,0) and $(1,r)$. A full understanding of the meaning of points on the graph of a proportional relationship will likely only be gained by active participation in the whole group discussion at the end of the task.	
Tennessee State Standards for Mathematical Content	**Tennessee State Standards for Mathematical Practice**
7.RP.A.1 Compute unit rates associated with ratios of fractions, including ratios of lengths, areas and other quantities measured in like or different units. *For example, if a person walks ½ miles in each ¼ hour, compute the unit rate as the complex fraction ½/¼ miles per hour, equivalently 2 miles per hour.* **7.RP.A.2** Recognize and represent proportional relationships between quantities. **a.** Decide whether two quantities are in a proportional relationship, e.g., by testing for equivalent ratios in a table or graphing on a coordinate plane and observing whether the graph is a straight line through the origin. **b.** Identify the constant of proportionality (unit rate) in tables, graphs, equations, diagrams, and verbal descriptions of proportional relationships. **c.** Represent proportional relationships by equations. *For example, if total cost t is proportional to the number n of items purchased at a constant price p, the relationship between the total cost and the number of items can be expressed as* $t = pn$. **d.** Explain what a point (x, y) on the graph of a proportional relationship means in terms of the situation, with special attention to the points $(0,0)$ and $(1,r)$, where *r* is the unit rate.	1. Make sense of problems and persevere in solving them. 2. Reason abstractly and quantitatively 3. Construct viable arguments and critique the reasoning of others. 4. Model with mathematics. 5. Use appropriate tools strategically. 6. Attend to precision. 7. Look for and make use of structure. 8. Look for and express regularity in repeated reasoning.

Essential Understandings:

- A rate is a ratio that compares two quantities that are measured in different units.
- A unit rate is the comparison of two measurements in which the denominator has a value of one unit.
- Equivalent ratios can be observed using tables.
- The graph of two variables that are directly proportional is a line that passes through the origin (0,0).
- Multiple representations such as words, ratios, equations, tables, or graphs are used to model proportional relationships in the real world and determine alternate equivalent representations.
- A proportion is an equation that states that two ratios are equal.
- Proportional relationships are used to define equations and determine a constant of proportionality.
- The *y*-coordinate of the point on the graph of a proportional relationship where $x = 1$ is the unit rate.
- The ratio of the *y*-coordinate to the *x*-coordinate for any point is equivalent to the constant of proportionality, *k*, when analyzing a graph of two variables that are directly proportional.
- Ratios, unit rates, and proportions are used to solve for unknown quantities.

Explore Phase

Possible Solution Paths

a) (*Must provide 2 of the following representations.)*

Math Symbols

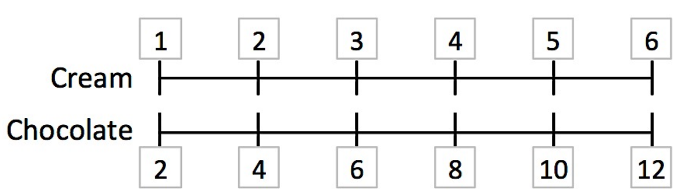
Students may represent the recipes using ratio notation, as follows:

Charlotte's	1:2	$\frac{1}{2}$	1 to 2
Granny's	2:5	$\frac{2}{5}$	2 to 5

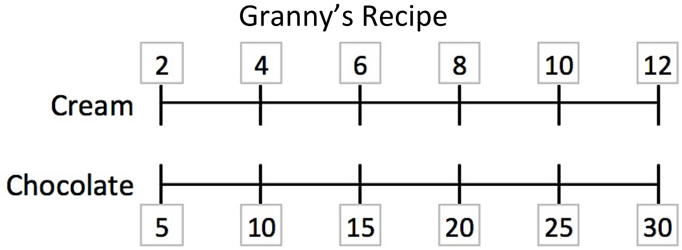
Diagram

Students may use diagrams, such as the double number line diagrams below.

Charlotte's Recipe



Granny's Recipe



Assessing and Advancing Questions

Assessing Questions:

- How did you choose to represent the recipes?
- What math concept are these examples of?
- How do you know that the two recipes are not the same?

Advancing Questions:

- When you hear the language "1 cup of cream and 2 cups of dark chocolate" what math concept are you reminded of?
- What are some different ways we represent ratios?
- Can you use these to represent each of the recipes in 3 different way and answer the question?

Table

Students may use tables to represent the ratios.

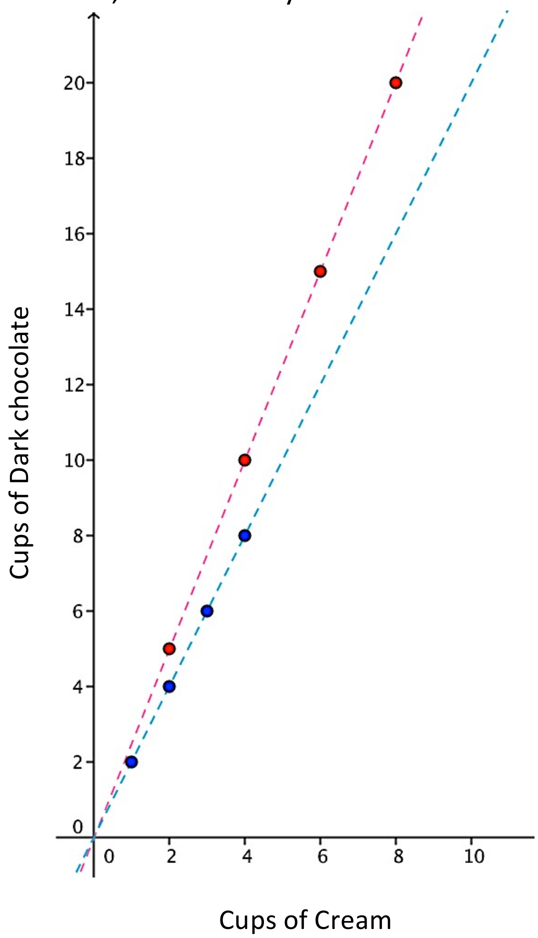
Charlotte's Recipe

Cream	1	2	3	4
Chocolate	2	4	6	8

Granny's Recipe

Cream	2	4	6	8
Chocolate	5	10	15	20

b) Students may graph the ratios as below where the dots represent increments of batches. They will note that the different quantities of the recipes are proportional, but that they are not the same recipes.



Equations

Students may extend the graph and tables above to equations that represent the recipes, such as the following:

Charlotte's Recipe: $y = 2x$

Granny's Recipe: $y = 2.5x$

Assessing Questions:

- Can you describe the features of the graph to me?
- Based on your graph, how do you know the recipes are both proportional?
- Based on your graph, how do you know the recipes are not the same?
- If you only have one cup of cream, how much chocolate would you need to make half of a batch of Granny's truffles?

Advancing Questions:

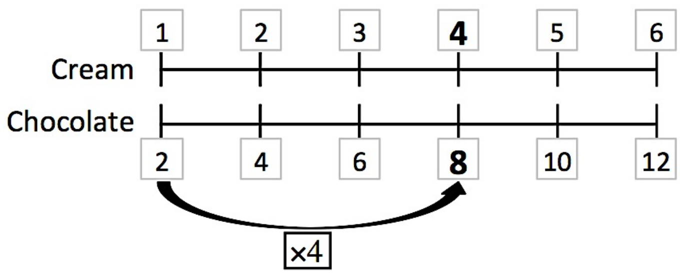
- What two items are we relating?
- Can you draw a coordinate plane and label the *x*-axis with one of your variables and the *y*-axis with the other?
- In Charlotte's recipe how much cream and dark chocolate is required for one batch? Can you plot that point on your graph? Can you plot other points?
- In Granny's recipe how much cream and dark chocolate is required for one batch? Can you plot that point on your graph? Can you plot other points?

c) Charlotte should use Granny's recipe, which makes 100 truffles with the given ingredients.

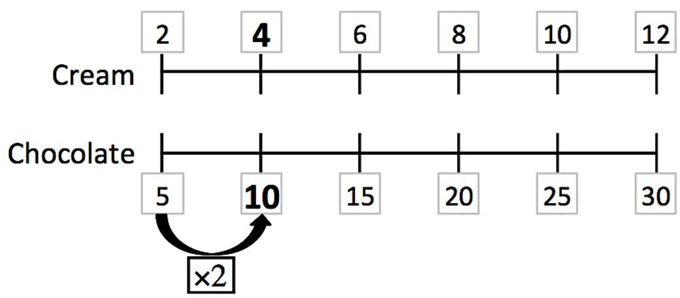
Diagram

Students may use the diagrams they created in Part a), as below.

Charlotte's Recipe



Granny's Recipe



From these diagrams, they should notice that the cream is the limiting ingredient in Charlotte's recipe, but doubling Granny's recipe uses all of the available ingredients. Thus, Charlotte can either quadruple her own recipe, making 20 truffles x 4 = 80 truffles, or she can double Granny's, making 50 truffles x 2 = 100 truffles.

Table

Students may use their tables from part a) to answer this part of the question in the same manner as with a diagram.

Charlotte's Recipe

Cream	1	2	3	**4**
Chocolate	2	4	6	**8**

Granny's Recipe

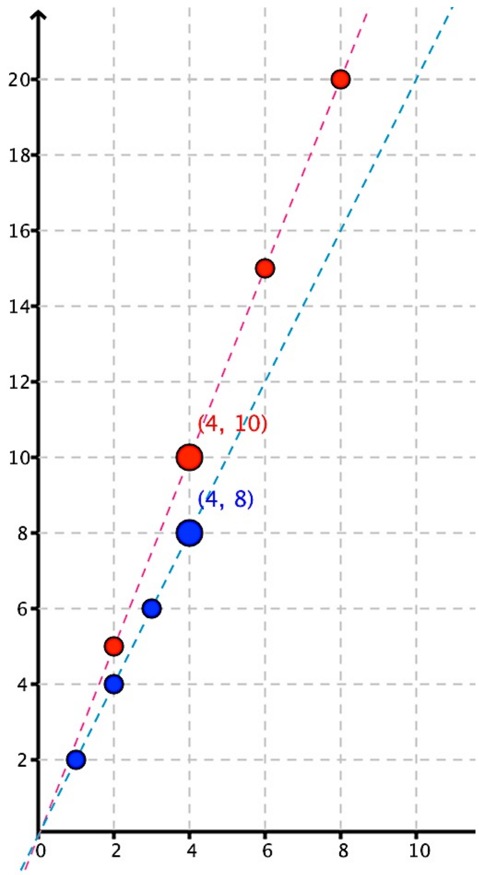
Cream	2	**4**	6	8
Chocolate	5	**10**	15	20

Assessing Questions:

- How did you know that Charlotte's recipe was quadrupled and Granny's was doubled?
- How did you use that information to figure out how many truffles she could make using each recipe?
- How many truffles could she make using each recipe?

Advancing Questions:

- Can you use one of the representations from Part a) to help you figure out the number of cups of each ingredient she can use for each recipe?
- Can she use all of the ingredients if she follows her own recipe?
- Should you choose the version of the recipe that uses 4 cups of cream or 10 cups of chocolate?
- How many times greater is the version of her recipe she needs to use than the original recipe?
- How many times greater is the version of Granny's recipe that she needs to use than Granny's original recipe?
- Can you use the number of times greater each recipe is to calculate how many truffles she will be able to make with each recipe?

Graph Students may analyze the graphs they made for part b) to answer this question, as below. They must notice that they must choose the points on the graph that use no more than the ingredients that are available, i.e. (4,8) and (4,10). After that, they may calculate the number of truffles that can be made similarly to in the *Diagram* solution.  *Equation* Since the cream limits the number of truffles that can be made using Charlotte's recipe, the equations found in Part b) give the following: <table><tr><th>Charlotte's Recipe</th><th>Granny's Recipe</th></tr><tr><td>$y = 2x$ $y = 2 \times 4$ $y = 8$ Recipe is quadrupled.</td><td>$y = 2.5x$ $y = 2.5 \times 4$ $y = 10$ Recipe is doubled.</td></tr></table> The question can then be answered as in *Diagram* solution.	

<table>
<tr>
<td>
d) Answers will vary.

Some examples of acceptable answers are given below.

Math Symbols
Students may prefer representing the recipes with ratio notation, since from this representation you can easily multiply each part of the ratio by the same number to find the number of cups of cream to cups of chocolate needed. You may point out that if they use that representation, it may be more difficult to calculate the number of cups of chocolate needed given cups of cream or vice versa.

Diagram
Students may prefer representing the recipes with diagrams, since from this representation you can easily see the cups of cream to cups of chocolate needed. You may point out that if they use that representation, it may be more difficult to find the number of cups of one ingredient needed given a rational (but not integer) number of cups of the other.

Table
Similar to above.

Graph
Students may prefer the graph representation, since the line connecting the equivalent ratios can help you find an approximation for the cups of one ingredient needed given any amount of the other ingredient. However, you may wish to point out that these measurements would only be approximations, if they are between actual points on the line.

Equation
Students may prefer the equation representations of the proportions, since they can input any number of cups of cream and find the number of cups of chocolate needed and vice versa. This is the probably the best answer, but any may be acceptable given enough explanation of their reasoning.
</td>
<td>
Assessing Questions:
• Why did you choose that representation as the most useful?
• What happens if you try to find the number of cups of chocolate needed for $2\frac{1}{3}$ cups of cream?
• Is there a representation that makes this easier to calculate for fraction values?

Advancing Questions:
• What representation do you think might be the best?
• Can you try finding the number of cups of chocolate needed for a certain number of cups of cream using the representation you chose?
• If the number of cups of cream that you have is not a whole number, does that change the representation you like best?
</td>
</tr>
</table>

<table>
<tr><th>Possible Student Misconceptions</th><th>Assessing and Advancing Questions</th></tr>
<tr><td>Students may not connect the recipes given to the ratio concept.</td><td>• Can you read Charlotte's recipe to me?
• What does that phrase ("1 cup of cream and 2 cups of dark chocolate") remind you of?
• Can you write that using math symbols?</td></tr>
<tr><td>Students may write the equations incorrectly, perhaps thinking that since the cups of chocolate in Charlotte's recipe are increasing by 2 each time and that equation is $y = 2x$ and the cups of chocolate in Granny's recipe are increasing by 5 each time that the equation for Granny's recipe would be $y = 5x$.</td><td>• If you have not already, can you make graphs that represent each of the recipes?
• What determines the equation of a line?
• What is the *y*-intercept of each of these lines?
• What is the slope?</td></tr>
<tr><th>Entry/Extensions</th><th>Assessing and Advancing Questions</th></tr>
<tr><td>If students can't get started….</td><td>• When you hear the language "1 cup of cream and 2 cups of dark chocolate" what math concept are you reminded of?
• What are some different ways we represent ratios?</td></tr>
<tr><td>If students finish early….</td><td>• If you did not already, can you create graphs and write equations for each of the recipes?
• What is the significance of the graphs of proportions going through the point (0,0)? Should all proportions have the point (0,0) in common?
• What is the value when the number of cups of cream is 1?
• What does the point $(1, y)$ mean on the graph of any proportion?</td></tr>
<tr><th colspan="2">Discuss/Analyze</th></tr>
<tr><th colspan="2">Whole Group Questions</th></tr>
<tr><td colspan="2">Pick groups to share their work. Select a sequence that will progress students through higher order thinking. Highlight any student answers that made use of a graph. Emphasize that in order for the relationship to be proportional, the graph must pass through the origin and that the point (1, *r*) gives the unit rate, *r*.

Equivalent ratios can be observed using tables.
• Did anyone use a table to represent the recipes?
• How did you create your tables? Did anyone do it differently?
• How did that help you answer the question in Part b)?

Multiple representations such as words, ratios, equations, tables, or graphs are used to model proportional relationships in the real world and determine alternate equivalent representations.
• What representation did you think was the best for truffle-making? Why?
• Did anyone choose a different representation? Why?
• Can you try finding the number of cups of chocolate needed for a certain number of cups of cream using the representation you chose?
• What happens if you try to find the number of cups of chocolate needed for $2\frac{1}{3}$ cups of cream?</td></tr>
</table>

- Is there a representation that makes this easier to calculate for fraction values?
- Can we all agree on one that is the best?

The graph of two variables that are directly proportional is a line that passes through the origin (0,0). The *y*-coordinate of the point on the graph of a proportional relationship where $x = 1$ is the unit rate.

- How did you create your graphs? Did anyone do it differently?
- What types of graphs are these?
- Do you think the graphs of proportional relationships will always be lines? Why or why not?
- What does the *y*-intercept of these graphs tell you?
- Do you think the *y*-intercept of a proportional relationship should always be the same? Why or why not?
- What does the value of *y* when *x* is 1 mean on these graphs?
- What term that we have learned can be used for the *y*-value that corresponds to $x = 1$?
- Everyone, choose one point on one of the graphs. Can you explain to me in words what that point means in this context?

Proportional relationships are used to define equations and determine a constant of proportionality.

- How do these equations relate to the graphs we were discussing before?
- What does the multiplier on the *x* mean? What term do we use to describe this when we are talking about lines? What term do we use when we are talking about proportions?
- Is the slope of *any* line a constant of proportionality? Why or why not?

Name ________________________________

Task 6: Mental Math – Mall Edition!

Your favorite store is having a 25% off *everything* sale! You try on everything that catches your eye, love it all, and then realize that 25% off is still far from free.

Item	Unit Price
Graphic Tee	$3.99
Blue Jeans	$43.99
Shoes	$31.99
Watch	$19.99
Cologne	$23.99
Jacket	$47.99
Socks	$7.99

a) If you buy something 25% off, what percentage of the original price do you pay? Can you represent both of these percentages in several equivalent ways?

b) If your mom offers to buy you $200 worth of clothes, which items from the table to the right can you buy? Come up with a plan based on the 25% discount and sales tax of 10%. Note that you may buy more than one of each should spend as close to $200 as possible. Describe how you would do the math mentally, in order to avoid getting caught doing math at the mall.

Task 6: Mental Math – Mall Edition! **7th Grade**

Your favorite store is having a 25% off *everything* sale! You try on everything that catches your eye, love it all, and then realize that 25% off is still far from free.

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Teacher Notes:

In this task, students will extend ratio reasoning to taking percentages. Rather than using decimal representation of percentages, students will view them as a ratio with one of the values being 100. This should make sense to them based on previous understandings of percentages; however, they may not be accustomed to taking percentages this way, especially with a reduced fraction representation. They should be strongly encouraged to see the value of taking percentages this way in real-world situations, as using a calculator or pencil and paper is not always practical. An important component of this task will be the whole group discussion in which students will share other situations where percentages can be more easily calculated using the ratio concept. Teachers may want to direct students to round the dollar amounts for the calculations since they are supposed to be writing what they would do mentally as if they were at the store.

Tennessee State Standards for Mathematical Content	Tennessee State Standards for Mathematical Practice
7.RP.A.3 Use proportional relationships to solve multistep ratio and percent problems. *Examples: simple interest, tax, markups and markdowns, gratuities and commissions, fees, percent increase and decrease, percent error.*	1. Make sense of problems and persevere in solving them. 2. Reason abstractly and quantitatively. 3. Construct viable arguments and critique the reasoning of others. 4. Model with mathematics. 5. Use appropriate tools strategically. 6. Attend to precision. 7. Look for and make use of structure. 8. Look for and express regularity in repeated reasoning.

Essential Understandings:

- Ratios, unit rates, and proportions are used to solve for unknown quantities.
- Multiple representations such as words, ratios, equations, tables, or graphs are used to model proportional relationships in the real world and determine alternate equivalent representations.

Explore Phase	
Possible Solution Paths	**Assessing and Advancing Questions**
a) If something is 25% off then you are paying 75% of the original price. These percentages can be represented by the following: **25%** | **0.25** | $\frac{25}{100}=\frac{1}{4}$ **75%** | **0.75** | $\frac{75}{100}=\frac{3}{4}$	**Assessing Questions:** • Can you explain how you came up with the answer 75%? • Why does "25% off" mean you're paying 75%? • Why are the decimal/fraction representations of the percentages equivalent? • What does the fraction representation remind you of? • Is a percentage a ratio? **Advancing Questions:** • Does 25% off mean you are paying 25% of the original price? • What is 25% of a number, say 16? • What amount is left over after you take away that 25%? • What percentage of the original number, 16, is that? • If you calculate 25% of an amount, what percentage is left over?
b) Plans will vary. Example below. Students could find 10% of a $200 purchase to give an estimate of how much money they should set aside to pay taxes at the end (i.e. how much under $200 their pre-tax purchase should be): $$200\times\frac{10}{100}=200\times\frac{1}{10}=20$$ Students should then come up with a way to prioritize which items they will buy. They may want to start with more expensive items and round to whole dollar amounts to make calculations easier. Another way to cut some of the calculating is the calculate 75% of the price, rather than taking 25% and then subtracting that from the original price. Additionally, turning 75% into the ratio $\frac{75}{100}$ and then reducing it to the equivalent ratio $\frac{3}{4}$ before multiplying makes the calculations much easier, as illustrated below.	**Assessing Questions:** • How did you calculate the tax? • How did you find the cost of each item after the discount? • How did you represent the percentages? • Which seems easier, using the decimal representation of a percentage or the fraction representation? Why? **Advancing Questions:** • When doing mental math, what concept is useful when you're dealing with decimal numbers or numbers with a lot of different digits? • What values can you round in this problem to make it easier to do the calculations? • Referring to Part a), can you calculate the percentage discount and percentage paid for one or two items? • Which representation of the percentages seems to make the calculations easiest? • Should you calculate the discount or the amount paid for each item?

Item	Price	Discount Price	Running Total
Jacket	$48	$\frac{\$48}{1}\times\frac{3}{4}=\frac{12}{1}\times\frac{3}{1}=\$36$	$36
Jeans	$2(\$44)$ $=\$88$	$\frac{\$88}{1}\times\frac{3}{4}=\frac{22}{1}\times\frac{3}{1}=\$66$	$102
Shoes	$32	$\frac{\$32}{1}\times\frac{3}{4}=\frac{8}{1}\times\frac{3}{1}=\$24$	$126
Cologne	$24	$\frac{\$24}{1}\times\frac{3}{4}=\frac{6}{1}\times\frac{3}{1}=\$18$	$144
Watch	$20	$\frac{\$20}{1}\times\frac{3}{4}=\frac{5}{1}\times\frac{3}{1}=\$15$	$159
Tees	$4 ea.	$\frac{\$4}{1}\times\frac{3}{4}=\frac{1}{1}\times\frac{3}{1}=\$3$	$162
Socks	$3(\$8)$ $=\$24$	$\frac{\$24}{1}\times\frac{3}{4}=\frac{6}{1}\times\frac{3}{1}=\$18$	$180

Once students get closer to the target number of $180, they will have to make decisions about which things to buy, which is why only one t-shirt and 3 pairs of socks were added. Students should be able to explain that since they rounded prices and percentages up, the amount will definitely be under $200 when tax is added.

Possible Student Misconceptions	**Assessing and Advancing Questions**
Students think that they must use the decimal form of a percentage to calculate percentages.	• What decimal number is equivalent to 25%? • How is 0.25 read? • What is that number as a fraction? • What simpler fraction is equivalent to that one? • If you are calculating 25% of $24 by hand, which representation of the percentage makes the most sense to use?
Students do not realize that the percent discount and the percent paid adds to 100% (i.e. the discount on an item and the sale price add to the original price).	• What is the amount of a 25% discount on a $24 item? • What is the sale price? • What percentage of the original price is the sale price? • What is the sum of the discount percentage and the sale price percentage? • Do you think that the discount percentage and the sale price percentage always add to 100%? Why or why not?

Entry/Extensions	Assessing and Advancing Questions
If students can’t get started….	• What does percent mean? • If a percentage is the amount something is out of 100, how can we represent that as a fraction? Decimal? • What percentage is the discount? • What is percentage is left once the discount percentage is removed?
If students finish early….	• What are some other situations where you may need to take percentages? • Collaborate with your group to come up with 4 different situations and some sample percentages, representations and calculations to share with your classmates. • If the discount is 30%, what shopping plan would you use with Mom’s $200?

Discuss/Analyze

Whole Group Questions

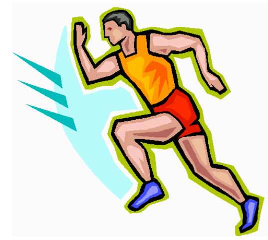
Pick groups to share their work. Select a sequence that will progress students through higher order thinking. Highlight any student answers that made use of a graph. Emphasize that rather than using decimal representations of percentages, students will view them as ratios with one of the values being 100.

Ratios, unit rates, and proportions are used to solve for unknown quantities.
Multiple representations such as words, ratios, equations, tables, or graphs, are used to model proportional relationships in the real world and determine alternate equivalent representations.

- What percentages did we deal with in this problem?
- How were they related?
- Can someone demonstrate how a percentage would be taken of one of the item costs using the decimal representation of the percentage?
- Can someone demonstrate how the percentage would be taken using the reduced ratio/fraction representation?
- Which representation was the most useful in calculating the discounts/sale prices?
- In other problems, do you typically use decimals or ratios to calculate percentages?
- In what other types of situations do we commonly need to calculate percentages?
- Does the ratio method help in these situations?

Name ______________________________

Task 7: Empire State Building Run-Up



The Empire State Building in New York City has 86 flights of stairs, covering a distance of 1,050 feet. Every year, 100 lucky people get to race to the top in the annual Empire State Building Run-Up.

The fastest recorded time was 9 minutes, 33 seconds by Australian runner Paul Crake in 2003. What is Crake's average speed in meters per second?

Tennessee Department of Education: Lesson Guide 7

Task 7: Empire State Building Run-Up — **7th Grade**

The Empire State Building in New York City has 86 flights of stairs, covering a distance of 1,050 feet. Every year, 100 lucky people get to race to the top in the annual Empire State Building Run-Up.

The fastest recorded time was 9 minutes, 33 seconds by Australian runner Paul Crake in 2003. What is Crake's average speed in meters per second?

Teacher Notes:

In this task, students will use what they have learned about ratios and proportions to convert units (by using unit multipliers), which they may recall from Task 2 of this arc.
Note that the meters to feet conversion factor is not given in the problem. You may either have students look this up, if they have internet access, or give it to them. 1 meter is approximately 3.3 feet.

Tennessee State Standards for Mathematical Content	Tennessee State Standards for Mathematical Practice
7.RP.A.3 Use proportional relationships to solve multistep ratio and percent problems. *Examples: simple interest, tax, markups and markdowns, gratuities and commissions, fees, percent increase and decrease, percent error.*	1. Make sense of problems and persevere in solving them. 2. Reason abstractly and quantitatively. 3. Construct viable arguments and critique the reasoning of others. 4. Model with mathematics. 5. Use appropriate tools strategically. 6. Attend to precision. 7. Look for and make use of structure. 8. Look for and express regularity in repeated reasoning.

Essential Understandings:

- Ratios, unit rates, and proportions are used to solve for unknown quantities.
- Multiple representations such as words, ratios, equations, tables, or graphs are used to model proportional relationships in the real world and determine alternate equivalent representations.

Explore Phase

Possible Solution Paths

Crake's average speed was 0.56 m/s.

Students may choose to calculate this speed with or without ratios but should be encouraged to use ratios. Below is an example a solution path without using ratios.

Ratios
To change 9 minutes, 33 seconds into seconds only:

$$9\,\text{min} \times \frac{60\,\text{sec}}{1\,\text{min}} = 540\,\text{sec}$$

$$9\,\text{min}, 33\,\text{sec} = 540\,\text{sec} + 33\,\text{sec} = 573\,\text{sec}$$

Assessing and Advancing Questions

Assessing Questions:

- What conversion factors did you have to use to solve this problem?
- How did you use them?
- How did using ratios guide the operations you did (i.e. multiplying or dividing)?

Advancing Questions:

- What unit of measure is needed in your final answer? Do these units give you any clues about how to solve this problem?
- What conversion factor could you use to change minutes to seconds?
- Can this be written as a ratio? Is this the only ratio that can be written for this situation?

To change the units (ft/sec into m/sec): $$\frac{1050ft}{573\sec} \times \frac{1m}{3.3ft} \approx 0.56m/s$$	• What distance did Crake run? • What units do you need for the distance? • How can you convert the distance to meters? • What do you need to do with the two values you have to get units of meters per second?
Calculations To change 9 minutes, 33 seconds into seconds only: $9\min \times 60 = 540\sec$, $540 + 33 = 573\sec$ To change feet into meters, students much use the conversion factor $1m \approx 3.3ft$, or a similar conversion factor. $1050 \div 3.3 \approx 318.18m$ Finally, to get the units of meters per second, the number of meters must be divided by the number of seconds it took to run that distance: $$\frac{318.18m}{573\sec} \approx 0.56m/s$$	**Assessing Questions:** • What conversion factors did you have to use to solve this problem? • How did you use them? • How did you decide whether to multiply or divide? • How do you know that you did the correct operation? • How did the units you needed in your final answer help you solve the problem? **Advancing Questions:** • What unit of measure is needed in your final answer? Do these units give you any clues about how to solve this problem? • What makes the time given difficult to work with? • How can you convert the time to one unit of measurement? • What distance did Crake run? • What units do you need for the distance? • How can you convert the distance to meters? • What do you need to do with the two values you have to get units of meters per second?
Possible Student Misconceptions	**Assessing and Advancing Questions**
Students do not understand why unit conversion factors can be written as ratios.	• How many feet are in one yard? • Can you say this with ratio language? • Can you represent it with a table or diagram, like we do with other ratios?
Students do not understand why it is beneficial to represent conversion factors as ratios.	• How do you convert seconds to minutes? Minutes to seconds? • How do you determine when to multiply and when to divide? • Can you represent the conversion factor 1 minute = 60 seconds with a ratio? • If you use that to convert, how do you know whether to multiply or divide?

Entry/Extensions	Assessing and Advancing Questions
If students can't get started….	• Do the units you need in your final answer give you any clues about how to solve this problem? • What conversion factor could you use to change minutes to seconds? • Can this be written as a ratio? Is this the only ratio that can be written for this situation?
If students finish early….	• How can the conversion factors be expressed using ratio language? • What equation could be used to represent the number of seconds, *y*, in *x* minutes? • Can you convert Crake's speed to miles per hour using three ratios and only one step? • What ratios would you need to change the time to hours? • What ratio would you need to change meters to miles? • How would you need to arrange these to get the units to work out correctly?

Discuss/Analyze

Whole Group Questions

Pick groups to share their work. Select a sequence that will progress students through higher order thinking. Highlight any student answers that made use of a graph. Emphasize the use of ratios for converting mixed units of measure.

Ratios, unit rates, and proportions are used to solve for unknown quantities.

- What ratios did you use in this problem?
- How did writing the conversion factors as ratios help us solve the problem?

Multiple representations such as words, ratios, equations, tables, or graphs are used to model proportional relationships in the real world and determine alternate equivalent representations.

- How can the conversion factors be expressed using ratio language?
- What equation could be used to represent the number of seconds, *y*, in *x* minutes?

Name ______________________________

Task 8: Working Backward

For each of the following mathematical representations, come up with a real-life situation that it could represent. Express your real-life situation in multiple ways, using mathematical symbols and words. Why would it be useful to represent this situation using math symbols?

a) $\frac{25}{100}$

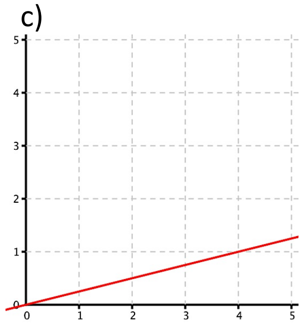
d) $\frac{1}{4}$ mile, 154.25 seconds

b) $c = 30n$

e)

m	0	4	7	8	11
s	0	2	$\frac{7}{2}$	4	$\frac{11}{2}$

c)



Task 8: Working Backward — 7th Grade

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a) $\frac{25}{100}$

b) $c = 30n$

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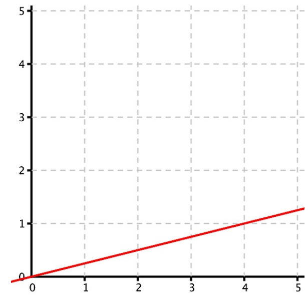

d) $\frac{1}{4}$ mile, 154.25 seconds

e)

m	0	4	7	8	11
s	0	2	$\frac{7}{2}$	4	$\frac{11}{2}$

Teacher Notes:

This task, the final one in this arc, summarizes what the student has learned in the previous 7 tasks. Students will solidify their ability to create multiple representations of proportional relationships, describe ratios and proportions with appropriate language, and calculate or observe unit rates and constants of proportionality given different representations. This task is open-ended, lending itself to a collaborative, group effort, rather than individual, although it can be completed either way. To limit the amount of time the teacher spends critiquing each situation and representation, groups can be larger or different parts can be assigned to different groups and then presented to the entire class for discussion.

Tennessee State Standards for Mathematical Content

7.RP.A.1 Compute unit rates associated with ratios of fractions, including ratios of lengths, areas and other quantities measured in like or different units. *For example, if a person walks ½ miles in each ¼ hour, compute the unit rate as the complex fraction ½/¼ miles per hour, equivalently 2 miles per hour.*

7.RP.A.2 Recognize and represent proportional relationships between quantities.

a. Decide whether two quantities are in a proportional relationship, e.g., by testing for equivalent ratios in a table or graphing on a coordinate plane and observing whether the graph is a straight line through the origin.

b. Identify the constant of proportionality (unit rate) in tables, graphs, equations, diagrams, and verbal descriptions of proportional relationships.

c. Represent proportional relationships by equations. *For example, if total cost t is proportional to the number n of items purchased at a constant price p, the relationship between the total cost and the number of items can be expressed as* $t = pn$.

d. Explain what a point (x, y) on the graph of a proportional relationship means in terms of the

Tennessee State Standards for Mathematical Practice

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
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situation, with special attention to the points $(0,0)$ and $(1,r)$, where *r* is the unit rate.

7.RP.A.3 Use proportional relationships to solve multistep ratio and percent problems. *Examples: simple interest, tax, markups and markdowns, gratuities and commissions, fees, percent increase and decrease, percent error.*

Essential Understandings:

- A rate is a ratio that compares two quantities that are measured in different units.
- A unit rate is the comparison of two measurements in which the denominator has a value of one unit.
- Equivalent ratios can be observed using tables.
- The graph of two variables that are directly proportional is a line that passes through the origin (0,0).
- Multiple representations such as words, ratios, equations, tables, or graphs are used to model proportional relationships in the real world and determine alternate equivalent representations.
- A proportion is an equation that states that two ratios are equal.
- Proportional relationships are used to define equations and determine a constant of proportionality.
- The *y*-coordinate of the point on the graph of a proportional relationship where $x=1$ is the unit rate.
- The ratio of the *y*-coordinate to the *x*-coordinate for any point is equivalent to the constant of proportionality, *k*, when analyzing a graph of two variables that are directly proportional.
- Ratios, unit rates, and proportions are used to solve for unknown quantities.

Explore Phase

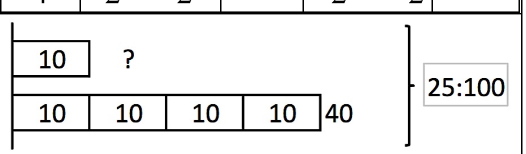
Possible Solution Paths

a) Answers will vary. Should include a real-world situation represented multiply ways and an explanation of why it would be useful to represent the situation that way. Examples below.

Situation

A 25% discount could be represented with the ratio $\frac{25}{100}$.

Other Representations

<table>
<tr><td rowspan="2">Table</td><td>1</td><td>10</td><td>20</td><td>30</td><td>40</td></tr>
<tr><td>$\frac{1}{4}$</td><td>$\frac{5}{2}=2\frac{1}{2}$</td><td>5</td><td>$\frac{15}{2}=7\frac{1}{2}$</td><td>10</td></tr>
<tr><td>Diagram</td><td colspan="5">
</td></tr>
<tr><td>Equation</td><td colspan="5">$y=\frac{25}{100}x$ or $y=\frac{1}{4}x$</td></tr>
</table>

Assessing and Advancing Questions

Assessing Questions:

- How is this ratio related to the situation that you chose?
- What other ways did you represent it?
- Are there other ways that ratios can be represented?
- Why did you choose the representations you chose?
- Did you observe anything else about the proportional relationships formed from this ratio, such as the constant of proportionality?

Advancing Questions:

- In what sort of real-life situations do we use ratios that are out of 100?
- Can you turn that into a description with words?
- In what ways can we represent ratios?
- Which ones of those do you think would be most useful in the situation you've written?
- Why is it useful to write this situation in any of these specific ratio forms?

Usefulness
Representing a 25% discount as a ratio is useful because it can be reduced, $\frac{25}{100} = \frac{1}{4}$, making it easier to calculate the discount amount. If represented with an equation, we can see that this relationship has a *constant of proportionality* $\frac{25}{100} = \frac{1}{4}$.

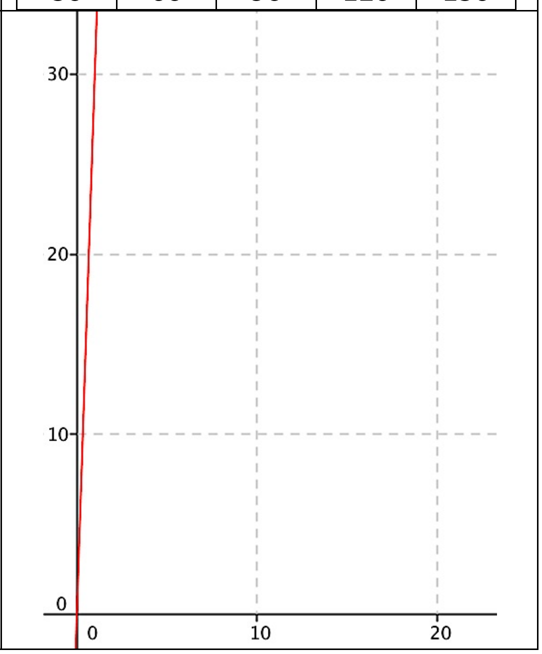
b) Answers will vary. Should include a real-world situation represented multiply ways and an explanation of why it would be useful to represent the situation that way. Examples below.

Situation
The ratio of the number of children on a field trip to the number of buses taken on the trip is 30:1.

Other Representations

Table	1	2	3	4	5
	30	60	90	120	150

Graph



Usefulness
Using an equation to represent this proportional relationship is useful, because if you know how many buses are being used, you can calculate the maximum number of children on the bus. In the table, some of these values are calculated, which could be useful to determine the number of buses you need if you know the number of children that needed to be transported. The graph of the proportion shows that if no children were going, no

Assessing Questions:
- Why did you choose the situation that you chose for the equation given?
- What math language applies to an equation like this?
- What does the number multiplied by the *n* mean?
- In what other ways did you represent this proportional relationship?
- Is it possible to observe the constant of proportionality or unit rate in these other representations?
- What does the point $(0,0)$ represent?
- Why is the point $(1,30)$ significant?
- What would a point with an *x*-coordinate of 0.5 mean?

Advancing Questions:
- Can you think of a ratio that corresponds to this equation?
- Can you think of a situation in which the ratio 1 to 30 would be used?
- In what ways can we represent ratios?
- Which ones of those do you think would be most useful in the situation you've written?
- Why is it useful to write this situation in any of these specific ratio forms?

buses would need to be used (i.e. point $(0,0)$ is on the line. The graph also makes the unit rate apparent, since when the number of buses being used is 1 (*x*-coordinate), the number of children that can be transported is 30 (*y*-coordinate). The graph could be misleading, however, since the points between whole numbers of buses actually need to be rounded up as fractional parts of buses doesn't make sense.	
c) Answers will vary. Should include a real-world situation represented multiply ways and an explanation of why it would be useful to represent the situation that way. Examples below. *Situation* The recipe calls for 1 cup of milk for every 4 cups of flour. *Other Representations* The equation $y = \frac{1}{4}x$, where *x* is the number of cups of milk and *y* is the number of cups of flour, corresponds to the graph given, since the constant of proportionality (or *unit rate*) is $\frac{1}{4}$. *Usefulness* It would be useful to represent the recipe in this way so that someone could easily adapt it to make different quantities.	**Assessing Questions:** • Why did you choose the situation that you chose for the equation given? • What math language applies to a graph/situation like this? • How did you figure out what the equation would be? • In what other ways did you represent this proportional relationship? • Is it possible to observe the constant of proportionality or unit rate in these other representations? • What does the point $(0,0)$ represent? • Why is the point $(1,\frac{1}{4})$ significant? • Do the points between the integer points make sense in your situation? **Advancing Questions:** • Can you think of a ratio that corresponds to this graph? • What is the constant of proportionality for this graph? • Can you think of a situation in which the ratio 4 to 1 would be used? • In what ways can we represent ratios? • Which ones of those do you think would be most useful in the situation you've written? • Why is it useful to write this situation in any of these specific ratio forms?

d) Answers will vary. Should include a real-world situation represented multiply ways and an explanation of why it would be useful to represent the situation that way. Examples below.

Situation

Mario runs an average speed of $\frac{1}{4}$ mile every 154.25 seconds.

Other Representations

Table					
	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{3}{4}$	1	$1\frac{1}{4}$
	308.5	462.75	617	771.25	925.5

Equation	
	If *x* is the number of miles run and *y* is the amount of time it took to run them, $y = 617x$ (units are in seconds) $y = 10.28x$, (units changed to minutes)

Usefulness

Useful for Mario to find the amount of time it would take him to run a certain number of miles. He may wish to change the given ratio to a unit rate in order to see how many seconds or minutes it would take him to run 1 mile, as follows:

In seconds:

$$\frac{154.25\,\text{sec}}{\frac{1}{4}mi} = 154.25\,\text{sec} \times \frac{4}{1}mi = 617\,\text{sec}/\,mi$$

In minutes:

$$154.25\,\text{sec} \times \frac{1\,\text{min}}{60\,\text{sec}} \approx 2.57\,\text{min}$$

$$\frac{2.57\,\text{min}}{\frac{1}{4}mi} = 2.57\,\text{min} \times \frac{4}{1}mi = 10.28\,\text{min}/\,mi$$

Assessing Questions:

- Why did you choose the situation that you chose for the information given?
- What math language applies to a situation like this?
- How did you convert this information to a table/equation/etc.?
- Is it possible to observe the constant of proportionality or unit rate in these representations?

Advancing Questions:

- Can you think of a ratio that corresponds to this information?
- Can you think of a situation in which the ratio $\frac{1}{4}$ mile to 154.25 seconds would be used?
- In what ways can we represent ratios?
- Which ones of those do you think would be most useful in the situation you've written?
- Why is it useful to write this situation in any of these specific ratio forms?

e) Answers will vary. Should include a real-world situation represented multiply ways and an explanation of why it would be useful to represent the situation that way.

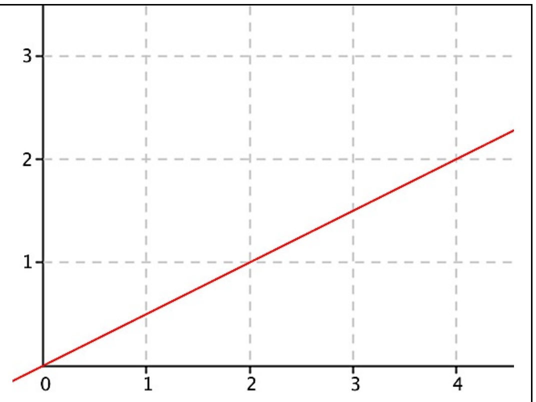
Situation

The scale of a map is $\frac{1}{2}$ inch equals 1 mile.

Assessing Questions:

- Why did you choose the situation that you chose for the table given?
- What math language applies to a situation like this?
- How did you convert this information to a graph/equation/etc.?
- Is it possible to observe the constant of proportionality or unit rate in these

Other Representations

Graph	
Equation	$s = \frac{1}{2}m$

Usefulness
Since the table given does not increase the number of miles by 1 each time, it is easier to observe the unit rate or constant of proportionality from the graph or equation. Using the table, one can only see the number of inches necessary for 0, 4, 7, 8, or 11 miles, but using the graph or table, one could calculate the number of inches needed to represent any number of miles.

representations?

Advancing Questions:

- Can you think of a ratio that corresponds to this table?
- What ratio is represented by this table?
- Can you think of a situation in which the ratio 4 to 2 would be used?
- In what ways can we represent ratios?
- Which ones of those do you think would be most useful in the situation you've written?
- Why is it useful to write this situation in any of these specific ratio forms?

Possible Student Misconceptions	**Assessing and Advancing Questions**
Students do not connect the information given to the ratio/proportion concept.	• What types of mathematical ideas are represented with fraction notation? • What mathematical idea is represented by a line through the point $(0,0)$? • What type of equation represents a proportional relationship? • If we put the word "to" between two values, what type of relationship is represented? • In a table, if the value for one variable is always a multiple of the other, what type of relationship is represented?
Students do not make the connection between the mathematics they are doing and the vocabulary (i.e. *unit rate* and *proportion*).	• What is the *unit rate*? • How do you recognize it in a table/graph/equation/diagram/etc.? • What point must always be present when a proportional relationship is graphed? • What does a point mean on any given proportional relationship?
Students have difficulty adapting different representations of proportional relationships (tables, graphs, equations, etc.).	• Can you find equivalent ratios for the given ratio/equation/graph/table? • In math, what are other ways that we can represent pairs of values?

Entry/Extensions	Assessing and Advancing Questions
If students can't get started….	• In what sort of real-life situations do we use ratios that are out of 100? • Can you think of a ratio that corresponds to this equation? • Can you think of a ratio that corresponds to this graph? • Can you think of a ratio that corresponds to this information? • Can you think of a ratio that corresponds to this table? • Can you turn that into a description with words? • In what ways can we represent ratios? • Which ones of those do you think would be most useful in the situation you've written? • Why is it useful to write this situation in any of these specific ratio forms?
If students finish early….	• Can you create graphs for each of the proportional relationships in parts a) through e) (if you haven't done so already)? • What are the unit rates of each of these? • Can you graph a point that illustrates the unit rate? • What point do all of these graphs have in common? • What does a point on each of the graphs mean, given the situation you wrote that matches the relationship?

Discuss/Analyze

Whole Group Questions

Pick groups to share their work. Select a sequence that will progress students through higher order thinking. Highlight any student answers that made use of a graph. Emphasize that ratios have a variety of useful applications.

Multiple representations such as words, ratios, equations, tables, or graphs are used to model proportional relationships in the real world and determine alternate equivalent representations.

- What are some ways that you represented ratios and proportional relationships in your answers?
- Which representations are your favorite? Why?
- Which seem to be the most useful?
- Are there situations where certain representations are not useful?

A unit rate is the comparison of two measurements in which the denominator has a value of one unit.

- What is a unit rate?
- How can you find a unit rate given a table of equivalent ratios?
- How can you find it given an equation/graph/diagram?
- Why is the unit rate useful?

The graph of two variables that are directly proportional is a line that passes through the origin (0,0).

- Who created graphs for the relationships given?
- What do you notice about the graphs for these proportions?
- What points do they go through?
- Which points are significant?

A proportion is an equation that states that two ratios are equal.

- What equations are in this task?
- What is similar about the equations?
- What is different?
- What types of values satisfy an equation of a proportional relationship?
- What term describes the relationship between these values?

Proportional relationships are used to define equations and determine a constant of proportionality.

- What is the constant of proportionality?
- How can you find the constant of proportionality given a table of equivalent ratios?
- How can you find it given an equation/graph/diagram?
- Why is the constant of proportionality useful?