Grade 6: Reasoning with Ratio and Rates

A Set of Related Tasks and Lesson Guides

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Introduction

Tasks and Lesson Guides

ARC OVERVIEW

In this set of related tasks, 6th Grade students will practice ratio skills by developing and solidifying knowledge, language, notation, representations, and general skills related to using ratios to solve problems.

The Arc Preview table on page 5 provides all of the task questions contained in this arc. The tasks are aligned to standards 6.RP.A.1, 6.RP.A.2 and 6.RP.A.3.

- Task 1 develops students' ability to use ratio knowledge and language to describe relationships, as well as guiding them toward an understanding of the similarities and differences of *a*:*b* versus *b*:*a*.
- Task 2 continues developing skills with ratios, adding the concept of equivalent ratios.
- Task 3 develops student understanding of rates, unit rates, and the ratio language used to describe these relationships.
- Task 4 solidifies the concepts developed in Tasks 1 through 3.
- Task 5 continues to develop student understanding of unit rates and representations of ratios, introducing graphs of equivalent ratios as well.
- Task 6 develops ratio understanding by exploring percentages as ratios.
- Task 7 develops understanding of unit conversions by representing conversion factors as ratios.
- Task 8 solidifies student understanding of all of the concepts outlined in Tasks 1 through 7.

Before starting this task arc, students should be familiar with tables, equations, and graphs of linear relationships. They should also have an introduction to diagrams that can be used to represent ratios, such as tape diagrams and double number line diagrams. Students may also need a review of percentages for Tasks 6 and 8 and a review of converting units for tasks 7 and 8.

Note that the some of the Essential Understandings listed in each task were modified from those contained in Pearson's EnVision Math series. Others were taken from NCTM's Developing Essential Understanding series. Tennessee State Mathematics Standards were retrieved from http://www.tn.gov/education/standards/math.shtml.

By the end of these eight tasks, students will be able to answer the following overarching questions:

- What is a ratio?
- How are ratios written?
- What makes ratios and fractions different?
- Are the ratios *a*:*b* and *b*:*a* the same? Why or why not?
- What is a unit rate and how can it be used?
- Can the two values in a ratio have different units? Explain.
- How can we use pictures to solve ratio problems?
- What are equivalent ratios? Can you give me some examples? How do we find them?
- What types of pictures and organization can we use to help us visualize equivalent ratios?
- What types of ratios can represent percentages?
- What types of ratios can be used for converting units?

The assessing questions, advancing questions, and whole group questions provided in this guide will ensure that students are working in ways aligned to the Standards for Mathematical Practice. Although the students will not be aware that this is occurring, the teacher can guide the process so that each MP (Mathematical Practice) is covered through good explanations, understanding of context, and clarification of reasoning behind solutions.

Arc Preview

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<td>

Task 1: Bountiful Blueberries

The farmer's market is selling 4 pints of blueberries for $10.00.

a) Can you write two different ratios for this situation?

b) Can you represent each ratio from part a) in multiple ways?

c) Explain why each of the sentences is correct:

- Tia said, "That means we can write the ratio 10:4, or $2.50 per pint of blueberries."
- Wendy said, "I thought we had to write the ratio the other way, 4:10, or 0.4 pints of blueberries per dollar."

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Goals for Task 1:

- Use ratio knowledge and language to describe ratio relationships
- Recognize that a given situation may be represented by more than one ratio

Standards for Task 1:

6.RP.A.1 Understand the concept of a ratio and use ratio language to describe a ratio relationship between two quantities. *For example, "The ratio of wings to beaks in the bird house at the zoo was 2:1, because for every 2 wings there was 1 beak." "For every vote candidate A received, candidate C received nearly three votes."*

6.RP.A.2 Understand the concept of a unit rate *a/b* associated with a ratio *a:b* with $b \neq 0$, and use rate language in the context of a ratio relationship. *For example, "This recipe has a ratio of 3 cups of flour to 4 cups of sugar, so there is 3/4 cup of flour for each cup of sugar." "We paid $75 for 15 hamburgers, which is a rate of $5 per hamburger." (Expectations for unit rates in this grade are limited to non-complex fractions.)*

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Task 2: When Will I Ever Use This?

For each relationship below, write a real-world scenario, describing each with ratio language, symbols, and alternate representations. Then comment on why it is helpful to use a ratio in that situation.

a) $\frac{2}{4}$

b) 25 to 100

c) 7:9

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Goals for Task 2:

- Use ratio knowledge and language to describe ratio relationships
- Find ratios equivalent to a given ratio

Represent equivalent ratios with tables, diagrams, equations, and/or graphs

Standards for Task 2:

6.RP.A.1 Understand the concept of a ratio and use ratio language to describe a ratio relationship between two quantities. *For example, "The ratio of wings to beaks in the bird house at the zoo was 2:1, because for every 2 wings there was 1 beak." "For every vote candidate A received, candidate C received nearly three votes."*

6.RP.A.3 Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations.

a. Make tables of equivalent ratios relating quantities with whole-number measurements, find missing values in the tables, and plot the pairs of values on the coordinate plane. Use tables to compare ratios.

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</table>

Task 3: Perfect Peppers	Goals for Task 3:
The farmer's market has too many peppers. This morning a new sign was posted that says, "5 pounds of perfect peppers for $2." a) How many pounds of peppers can you buy for $1? How much does 1 pound of peppers cost? Describe how these values can be used. b) Evelyn needs 4 pounds of peppers to make fajitas. How much money will she will need? How do you know? c) Marcus has $5 to buy peppers. How many pounds of peppers he can buy? How do you know?	• Use ratio knowledge/language to describe ratio relationships and solve real world problems. • Recognize that a given situation may be represented by more than one ratio • Use unit rates and rate language in a ratio context • Find ratios equivalent to a given ratio **Standards for Task 3:** **6.RP.A.1** Understand the concept of a ratio and use ratio language to describe a ratio relationship between two quantities. *For example, "The ratio of wings to beaks in the bird house at the zoo was 2:1, because fore very 2 wings there was 1 beak." "For every vote candidate A received, candidate C received nearly three votes."* **6.RP.A.2** Understand the concept of a unit rate *a/b* associated with a ratio *a:b* with $b \neq 0$, and use rate language in the context of a ratio relationship. *For example, "This recipe has a ratio of 3 cups of flour to 4 cups of sugar, so there is 3/4 cup of flour for each cup of sugar." "We paid $75 for 15 hamburgers, which is a rate of $5 per hamburger." (Expectations for unit rates in this grade are limited to non-complex fractions.)*

Task 4: Nail It Down!

Solidifying Task

You and your dad are buying the supplies needed to build a doghouse. He sends you to the nail and screw aisle with instructions to buy 80 8-penny nails and 120 16-penny nails. You find nails packaged as in the table below.

Count	Type	Price
20	8-penny	\$1.00
10	16-penny	\$1.00
40	Assorted (16 8-penny; 24 16-penny)	\$3.00

a) How much does it cost to purchase the nails you need with the 8-penny and 16-penny nails packaged separately? How much does it cost to purchase them in an assorted package?

b) You decide that you will build and sell doghouses in small, medium, and large, with the size of your original doghouse being the smallest. Every time you increase the size, the number of nails needed is 3/2 what was needed for the size before. How many nails do you need for the medium and large doghouses?

c) The hardware store will let you buy replacement nails by breaking up packages, charging the same rate as in the table. What is the cost of one 16-penny nail? What is the cost of one 8-penny nail? Does it matter which package you use to calculate these costs?

Goals for Task 4:

- Use ratio knowledge and language to describe ratio relationships
- Recognize that a given situation may be represented by more than one ratio
- Use unit rates and rate language in a ratio context
- Find ratios equivalent to a given ratio
- Use knowledge of ratios and unit rates to solve real world problems
- Represent equivalent ratios with tables, diagrams, equations, and/or graphs

Standards for Task 4:

6.RP.A.1 Understand the concept of a ratio and use ratio language to describe a ratio relationship between two quantities. *For example, "The ratio of wings to beaks in the bird house at the zoo was 2:1, because for every 2 wings there was 1 beak." "For every vote candidate A received, candidate C received nearly three votes."*

6.RP.A.2 Understand the concept of a unit rate *a/b* associated with a ratio *a:b* with *b ≠ 0*, and use rate language in the context of a ratio relationship. *For example, "This recipe has a ratio of 3 cups of flour to 4 cups of sugar, so there is 3/4 cup of flour for each cup of sugar." "We paid \$75 for 15 hamburgers, which is a rate of \$5 per hamburger." (Expectations for unit rates in this grade are limited to non-complex fractions.)*

6.RP.A.3 Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations.

a. Make tables of equivalent ratios relating quantities with whole-number measurements, find missing values in the tables, and plot the pairs of values on the coordinate plane. Use tables to compare ratios.

Task 5: Tricky Trail Mix	**Goals for Task 5:**
Jonathon's dad is making trail mix for a camping trip. The ratio of cereal to raisins needed for the recipe is 5 to 3. a) What are some ratios that could be used to represent this situation? b) Jonathon's dad bought 15 cups of cereal for the trail mix. What is the unit rate of cups of raisins per cup of cereal? How many cups of raisins does he need? c) If the number of people attending doubles, how many cups of raisins and cereal will Jonathon's dad need to make trail mix for this larger group? Illustrate your reasoning using a mathematical picture or model.	• Use ratio knowledge and language to describe ratio relationships • Recognize that a given situation may be represented by more than one ratio • Find ratios equivalent to a given ratio • Use knowledge of ratios and unit rates to solve real world problems • Represent equivalent ratios with tables, diagrams, equations, and/or graphs. **Standards for Task 5: This should fit above.** **6.RP.A.1** Understand the concept of a ratio and use ratio language to describe a ratio relationship between two quantities. *For example, "The ratio of wings to beaks in the bird house at the zoo was 2:1, because for every 2 wings there was 1 beak." "For every vote candidate A received, candidate C received nearly three votes."* **6.RP.A.2** Understand the concept of a unit rate *a/b* associated with a ratio *a:b* with *b ≠ 0*, and use rate language in the context of a ratio relationship. *For example, "This recipe has a ratio of 3 cups of flour to 4 cups of sugar, so there is 3/4 cup of flour for each cup of sugar." "We paid $75 for 15 hamburgers, which is a rate of $5 per hamburger." (Expectations for unit rates in this grade are limited to non-complex fractions.)* **6.RP.A.3** Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations. **b.** Solve unit rate problems including those involving unit pricing and constant speed. *For example, if it took 7 hours to mow 4 lawns, then at that rate, how many lawns could be mowed in 35 hours? At what rate were lawns being mowed?*

Task 6: Garden Variety

Hiromi has a garden full of delicious vegetables. Yellow vegetables make up 25% of the garden. Green vegetables make up 50% of the garden. The rest of the garden is made up of 6 red vegetables.

a) What percentage of the vegetables is red? How do you know?

b) What is the total number of vegetables in the garden? Explain your reasoning.

c) If the total number of vegetables were increased to 48, how many would be green? How do you know?

Goals for Task 6:

- Use ratio knowledge and language to describe ratio relationships
- Recognize that a given situation may be represented by more than one ratio
- Find ratios equivalent to a given ratio
- Use knowledge of ratios and unit rates to solve real world problems
- Represent equivalent ratios with tables, diagrams, equations, and/or graphs
- Use a percentage expressed as a ratio and to find the part given the whole and vice versa

Standards for Task 6:

6.RP.A.1 Understand the concept of a ratio and use ratio language to describe a ratio relationship between two quantities. *For example, "The ratio of wings to beaks in the bird house at the zoo was 2:1, because for every 2 wings there was 1 beak." "For every vote candidate A received, candidate C received nearly three votes."*

6.RP.A.3 Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations.
c. Find a percent of a quantity as a rate per 100 (e.g., 30% of a quantity means 30/100 times the quantity); solve problems involving finding the whole, given a part and the percent.

Task 7: The Donut Dilemma

Every Saturday, you and your friends jog to the donut shop. Your friend Rodrigo just got a new bike and wants to bike along with the running group.

a) You and your friends jog at a rate of 6 minutes per mile, but Rodrigo measures his bike speed in miles per hour. How could you tell Rodrigo what speed he will need to ride to stay with the group?

b) Rodrigo says he wants to ride 12 mph and thinks that the group could keep up with him. Do you agree? Explain your reasoning.

Goals for Task 7:

- Use ratio knowledge and language to describe ratio relationships
- Recognize that a given situation may be represented by more than one ratio
- Use knowledge of ratios and unit rates to solve real world problems
- Represent equivalent ratios with tables, diagrams, equations, and/or graphs
- Use a conversion factor expressed as a ratio to convert measurements to different units

Standards for Task 7:

6.RP.A.1 Understand the concept of a ratio and use ratio language to describe a ratio relationship between two quantities. *For example, "The ratio of wings to beaks in the bird house at the zoo was 2:1, because for every 2 wings there was 1 beak." "For every vote candidate A received, candidate C received nearly three votes."*

6.RP.A.2 Understand the concept of a unit rate *a/b* associated with a ratio *a:b* with *b ≠ 0*, and use rate language in the context of a ratio relationship.
For example, "This recipe has a ratio of 3 cups of flour to 4 cups of sugar, so there is 3/4 cup of flour for each cup of sugar." "We paid $75 for 15 hamburgers, which is a rate of $5 per hamburger." (Expectations for unit rates in this grade are limited to non-complex fractions.)

6.RP.A.3 Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations.
d. Use ratio reasoning to convert measurement units; manipulate and transform units appropriately when multiplying or dividing quantities.

Task 8: Be the Mathe-Magician! *Solidifying Task* You and your classmates have been asked to solve each of the problems below but can't agree on a solution path. Knowing that you're a math whiz, your group has asked you to decide if they are both correct and provide a diagram or equation to illustrate. a) *If a dress that originally cost $24 is 25% off, how much does it cost?* Rosemary says we can turn 25% into the ratio $\frac{1}{4}$ and multiply it by the original cost of the dress, but Jack says 25% should be changed to 0.25 and multiplied. b) *Convert 3 yards to centimeters.* Maria says that, since 1 yard = 36 inches and 1 inch is about 2.5 centimeters, the problem can be solved by multiplying 3 times 36 times 2.5, but Rosemary says that the problem should be solved using ratios. c) *A cleaner needs to be diluted 1 part cleaner to 10 parts water. How much water should be used for 8 cups of cleaner?* Yungwei says that the equation $y = 10x$ illustrates this situation, but Maria says the equation should be $y = \frac{1}{10}x$.	**Goals for Task 8:** • Describe ratio relationships • Recognize that a given situation may be represented by more than one ratio • Find ratios equivalent to a given ratio • Represent equivalent ratios with tables, diagrams, equations, and/or graphs • Use a percentage expressed as a ratio and to find the part given the whole and vice versa • Use a conversion factor expressed as a ratio to convert measurements to different units **Standards for Task 8:** **6.RP.A.1** Understand the concept of a ratio and use ratio language to describe a ratio relationship between two quantities. *For example, "The ratio of wings to beaks in the bird house at the zoo was 2:1, because for every 2 wings there was 1 beak." "For every vote candidate A received, candidate C received nearly three votes."* **6.RP.A.2** Understand the concept of a unit rate *a/b* associated with a ratio *a:b* with *b ≠ 0*, and use rate language in the context of a ratio relationship. *For example, "This recipe has a ratio of 3 cups of flour to 4 cups of sugar, so there is 3/4 cup of flour for each cup of sugar." "We paid $75 for 15 hamburgers, which is a rate of $5 per hamburger." (Expectations for unit rates in this grade are limited to non-complex fractions.)* **6.RP.A.3** Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations. **a.** Make tables of equivalent ratios relating quantities with whole-number measurements, find missing values in the tables, and plot the pairs of values on the coordinate plane. Use tables to compare ratios. **b.** Solve unit rate problems including those involving unit pricing and constant speed. *For example, if it took 7 hours to mow 4 lawns, then at that rate, how many lawns could be mowed in 35 hours? At what rate were lawns being mowed?* **c.** Find a percent of a quantity as a rate per 100 (e.g., 30% of a quantity means 30/100 times the quantity); solve problems involving finding the whole, given a part and the percent. **d.** Use ratio reasoning to convert measurement units; manipulate and transform units appropriately when multiplying or dividing quantities.

Tasks' Standards Alignment

Task	6.RP.A.1	6.RP.A.2	6.RP.A.3.a	6.RP.A.3.b	6.RP.A.3.c	6.RP.A.3.d	MP 1	MP 2	MP 3	MP 4	MP 5	MP 6	MP 7	MP 8
Task 1 **Bountiful Blueberries**	✔	✔					✔	✔	✔	✔	✔	✔	✔	
Task 2 **When Will I Ever Use This?**	✔		✔				✔		✔	✔	✔	✔	✔	✔
Task 3 **Perfect Peppers**	✔	✔					✔	✔	✔	✔	✔	✔	✔	
Task 4 **Nail It Down!** *Solidifying Understanding*	✔	✔	✔				✔	✔			✔	✔	✔	✔
Task 5 **Tricky Trail Mix**	✔	✔		✔			✔	✔	✔	✔	✔	✔	✔	✔
Task 6 **Garden Variety**	✔				✔		✔	✔	✔	✔	✔	✔	✔	✔
Task 7 **The Donut Dilemma**	✔	✔				✔	✔				✔	✔	✔	✔
Task 8 **Be the Mathe-Magician** *Solidifying Understanding*	✔	✔	✔	✔	✔	✔	✔		✔	✔	✔	✔	✔	✔

The Standards for Mathematical Practice

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Name ________________________________

Task 1: Bountiful Blueberries

The farmer's market is selling 4 pints of blueberries for $10.00.

a) Can you write 2 different ratios for this situation?

b) Can you represent each ratio from part a) in multiple ways?

c) Explain why each of the sentences is correct.

- Tia said, "That means we can write the ratio 10:4, or $2.50 per pint of blueberries."
- Wendy said, "I thought we had to write the ratio the other way, 4:10, or 0.4 pints of blueberries per dollar."

Task 1: Bountiful Blueberries — **6th Grade**

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b) Can you represent each ratio from part a) in multiple ways?

c) Explain why each of the sentences is correct.

- Tia said, "That means we can write the ratio 10:4, or $2.50 per pint of blueberries."
- Wendy said, "I thought we had to write the ratio the other way, 4:10, or 0.4 pints of blueberries per dollar."

Teacher Notes:

Students may need to be reminded that a ratio is a comparison of two quantities, and therefore, the order is not important, ie, 4:10 or 10:4 could both be correct ratios for this task.

Tennessee State Standards for Mathematical Content	**Tennessee State Standards for Mathematical Practice**
6.RP.A.1 Understand the concept of a ratio and use ratio language to describe a ratio relationship between two quantities. *For example, "The ratio of wings to beaks in the bird house at the zoo was 2:1, because for every 2 wings there was 1 beak." "For every vote candidate A received, candidate C received nearly three votes."* **6.RP.A.2** Understand the concept of a unit rate *a/b* associated with a ratio *a:b* with *b* ≠ 0, and use rate language in the context of a ratio relationship. *For example, "This recipe has a ratio of 3 cups of flour to 4 cups of sugar, so there is 3/4 cup of flour for each cup of sugar." "We paid $75 for 15 hamburgers, which is a rate of $5 per hamburger." (Expectations for unit rates in this grade are limited to non-complex fractions.)*	1. Make sense of problems and persevere in solving them. 2. Reason abstractly and quantitatively. 3. Construct viable arguments and critique the reasoning of others. 4. Model with mathematics. 5. Use appropriate tools strategically. 6. Attend to precision. 7. Look for and make use of structure. 8. Look for and express regularity in repeated reasoning.

Essential Understandings:

- A ratio is a special relationship between two quantities where for every *x* units of one quantity there are *y* units of another quantity.
- Understand that ratios can be represented using ":", fraction notation, and the word "to".
- A given situation may be represented by more than one ratio.
- A rate is a ratio that compares two quantities that are measured in different units.
- Tape diagrams and double number line diagrams can show ratio relationships and be used to reason about solutions to problems.

<table>
<tr><th colspan="2">Explore Phase</th></tr>
<tr><th>Possible Solution Paths</th><th>Assessing and Advancing Questions</th></tr>
<tr><td>

a) Students could write ratios with the precise numbers given in the situation such as 4:10 (4 to 10; $\frac{4}{10}$) or 10:4 (10 to 4; $\frac{10}{4}$).

They could also simplify the numbers given to write the ratios 2:5 (2 to 5; $\frac{2}{5}$) or 5:2 (5 to 2; $\frac{5}{2}$).

It is possible for students to simplify the numbers to the unit rate and write the ratio as 2.50:1 or 0.40:1.

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Assessing Questions:

- Can you explain how your ratios relate to the situation?
- Does the order of the numbers change the meanings? Explain.
- Could you have written a different ratio for this situation?

Advancing Questions:

- What is a ratio?
- What symbols and notation can you use to represent ratios?
- What are the quantities mentioned in this situation?
- What is the relationship between these quantities?

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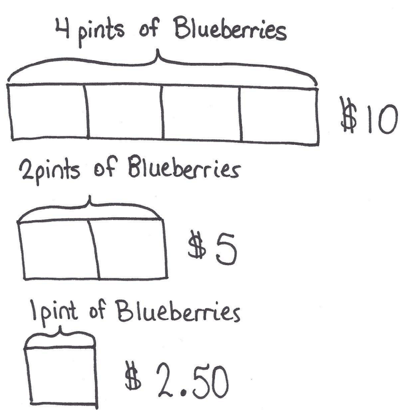
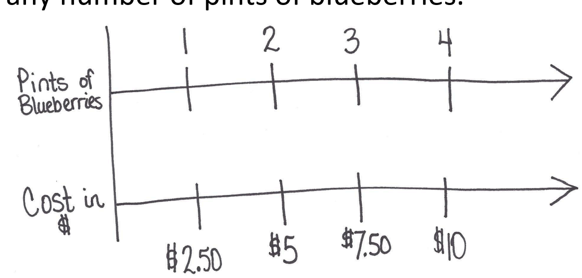
b) Students may represent any of the ratios in part a) using a tape diagram, a double line diagram, a ratio table, or a coordinate plane.

Tape Diagram:
Students may create one tape diagram, illustrating the ratio itself, or they make create several, illustrating different equivalent ratios.



Double Line Diagram:
This type of diagram can be used to find the cost for any number of pints of blueberries.



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Assessing Questions:

- How do your representations fit the ratios from part a)?
- Are there any other ways to represent ratios?
- Do these different representations have different meanings or the same?
- How do your representations help you understand the ratio?

Advancing Questions:

- How could you draw a picture to explain the ratios from part a)?
- Can you draw these ratios using a tape diagram? Double line diagram? Ratio table? Coordinate plane?

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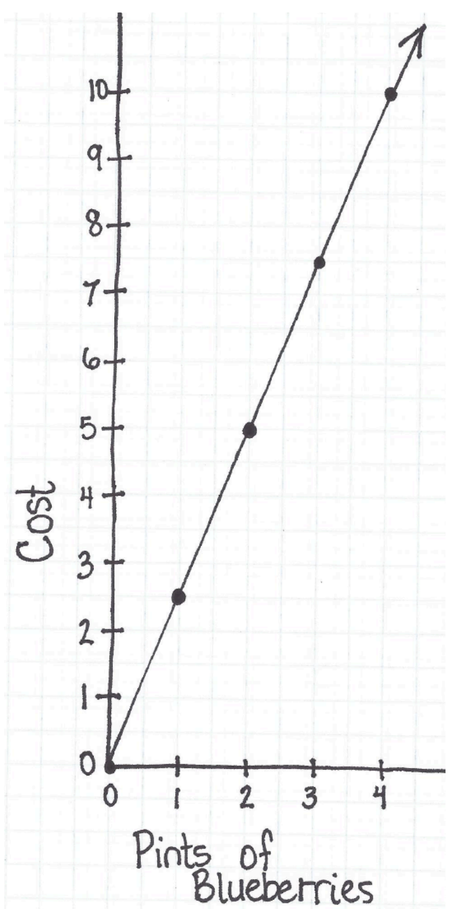
Ratio Table:
A ratio table shows that as the number of pints of blueberries increases by 1, the cost increases by $2.50.

Pints of Blueberries	Cost
4	$10
3	$7.50
2	$5
1	$2.50

Coordinate Plane:
The coordinate plane shows the linear relationship between the number of pints of blueberries to the cost.



<table>
<tr><td>c) Tia is correct in saying that 10:4 represents this situation. Ten dollars will buy 4 pints of blueberries. By using division, 10 ÷ 4 = 2.5, one pint of blueberries costs $2.50.

Wendy is correct in saying that 4:10 represents the situation. Four pints of blueberries cost $10. By using division, 4 ÷ 10 = 0.4, one dollar will buy 0.4 of a pint of blueberries.</td><td>Assessing Questions:
• How do you know that Tia is correct?
• How do you know that Wendy is correct?
• How did Tia find the cost per pint?
• How did Wendy find the number of pints per dollar?
Advancing Questions:
• What is a ratio?
• What symbols and notation can you use to represent ratios?
• What operation is implied within a ratio?
• What are the quantities mentioned in this situation?
• What is the relationship between these quantities?</td></tr>
<tr><td>Possible Student Misconceptions</td><td>Assessing and Advancing Questions</td></tr>
<tr><td>Students think that in ratio story problems, the quantity written first should always be the first quantity in the ratio a:b. It is important for students to recognize that a given situation may be represented by more than one ratio.</td><td>• Margaret's mom gave her a 20-dollar bill to spend on blueberries. How many pints of blueberries can she buy?
• Hunter is making a blueberry dessert that uses 1 pint of blueberries. How much money will he need to purchase the amount of blueberries in his dessert?</td></tr>
<tr><td>Students think that a:b is the same as b:a.</td><td>• If Mark is 60 years old and his son is 30 years old, what is the ratio of ages of the father and the son? (60:30)
• What does that ratio mean in words? (The father is twice as old as the son.)
• What is the ratio of the ages of the son and the father? (30:60)
• What does that ratio mean in words? (The son is half as old as the father.)
• Do the two ratios mean the same thing? Explain.</td></tr>
<tr><td>Entry/Extensions</td><td>Assessing and Advancing Questions</td></tr>
<tr><td>If students can't get started….</td><td>• What is a ratio?
• What symbols and notation can you use to represent ratios?
• What are the quantities mentioned in this situation?
• What is the relationship between these quantities?
• What operation is embedded within a ratio?</td></tr>
<tr><td>If students finish early….</td><td>• Can you calculate how much 12 pints of blueberries would cost? 16? 20?
• Explain your reasoning using multiple representations.</td></tr>
</table>

Discuss/Analyze

Whole Group Questions

Different representations should be selected and sequenced from basic to complex for students to share in this whole group setting. This will allow students to strengthen their understanding as well as give the teacher an opportunity to address misconceptions.

A ratio is a special relationship between two quantities where for every *x* units of one quantity there are *y* units of another quantity.

- What is a ratio?
- How is a ratio used to compare two quantities or values?
- How and where are ratios and rates used in the real world?
- How can I model and represent rates and ratios?

Understand that ratios can be represented using ":", fraction notation, and the word "to".

- What symbols and notation can you use to represent ratios?
- What is another way to write the ratio 2:3?
- What is the connection between a ratio and a fraction?
- What are similarities and differences between fractions, division, and ratios?

A given situation may be represented by more than one ratio.

- What ratios can be written for the situation in this problem?
- Did anyone write a different ratio?
- How can one situation be represented by different ratios?
- Do all of these ratios have the same meaning?

A rate is a ratio that compares two quantities that are measured in different units.

- How is a ratio or rate used to compare two quantities or values?
- Why is it important to know how to solve for unit rates?

Tape diagrams and double number line diagrams can show ratio relationships and be used to reason about solutions to problems.

- What representations did you use for the ratios you wrote for this situation?
- Are there other possible representations?

Name ______________________________

Task 2: When Will I Ever Use This?

For each relationship below, write a real-world scenario, describing each with ratio language, symbols, and alternate representations. Then comment on why it is helpful to use a ratio in that situation.

a) $\frac{2}{4}$

b) 25 to 100

c) 7:9

Task 2: When Will I Ever Use This? — **6th Grade**

For each relationship below, write a real-world scenario, describing each with ratio language, symbols, and alternate representations. Then comment on why it is helpful to use a ratio in that situation.

a) $\frac{2}{4}$ b) 25 to 100 c) 7:9

Teacher Notes:

This task will develop the students' skills with ratio language, symbols, and representations. Its open-ended nature will lead to significant variation in student answers. Any answer should be considered appropriate if it includes the following for each given ratio: (1) a suitable real-world application, (2) an accurate description using ratio language, (3) an alternate symbolic representation, (4) a diagram or table, and (5) an explanation of the usefulness of a ratio in this situation. Using limited guidance and encouraging students to work individually before collaborating in small groups will achieve greater variation, which will enrich the whole group discussion.

Tennessee State Standards for Mathematical Content	Tennessee State Standards for Mathematical Practice
6.RP.A.1 Understand the concept of a ratio and use ratio language to describe a ratio relationship between two quantities. *For example, "The ratio of wings to beaks in the bird house at the zoo was 2:1, because for every 2 wings there was 1 beak." "For every vote candidate A received, candidate C received nearly three votes."* **6.RP.A.3** Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations. **a.** Make tables of equivalent ratios relating quantities with whole-number measurements, find missing values in the tables, and plot the pairs of values on the coordinate plane. Use tables to compare ratios.	1. Make sense of problems and persevere in solving them. 2. Reason abstractly and quantitatively. 3. Construct viable arguments and critique the reasoning of others. 4. Model with mathematics. 5. Use appropriate tools strategically. 6. Attend to precision. 7. Look for and make use of structure. 8. Look for and express regularity in repeated reasoning.

Essential Understandings:

- A ratio is a special relationship between two quantities where for every *x* units of one quantity there are *y* units of another quantity.
- Understand that ratios can be represented using ":", fraction notation, and the word "to".
- A given situation may be represented by more than one ratio.
- Tape diagrams and double number line diagrams can show ratio relationships and be used to reason about solutions to problems.
- Equivalent ratios can be represented with tables, diagrams, equations, and/or graphs.
- Ratios and unit rates are used to solve for unknown quantities.

Explore Phase

Possible Solution Paths

a) Answers will vary. Must provide a situation, table/diagram, and comment on the usefulness of the ratio. The ratio may be represented as $\frac{2}{4}$, 2:4, or *2 to 4*.

Some example situations in which this ratio may be used are the following:

- In a recipe, you need 2 cups of oatmeal to every 4 cups of milk.
- In a classroom, there are 2 girls to every 4 students.
- In a fruit basket, there are 2 grapefruit to every 4 plums.

The first example situation will be used in the following table and diagrams. In the first situation, a student should recognize that the ratio would be helpful in determining how to make more or less than the original recipe. They may choose one of the following representations to explore the ratio's usefulness:

Table:

÷2

Oatmeal	1	**2**	3	4
Milk	2	**4**	6	8

×2

The table shows that dividing the recipe in half will change the number of cups of oatmeal needed to 1 and the cups of milk needed to 2, and also provides the cups of each needed for doubling the recipe. By following the pattern, students can see that the ratio is equivalent to 3 cups of oatmeal to 6 cups of milk.

Assessing and Advancing Questions

Assessing Questions:

- Can you explain how this situation fits with the ratio given?
- Why would a ratio help in this real-life scenario?
- How did you create the table/diagram for this ratio?
- How does the table/diagram help explain the ratio?
- Could you have written a different ratio for this situation?

Advancing Questions:

- What symbols and notation can you use to represent ratios?
- Can you find a ratio equivalent to $\frac{2}{4}$?
- Are there other ways to represent ratios with pictures?
- How could you use one of the symbol or picture representations to explain how this ratio is useful?

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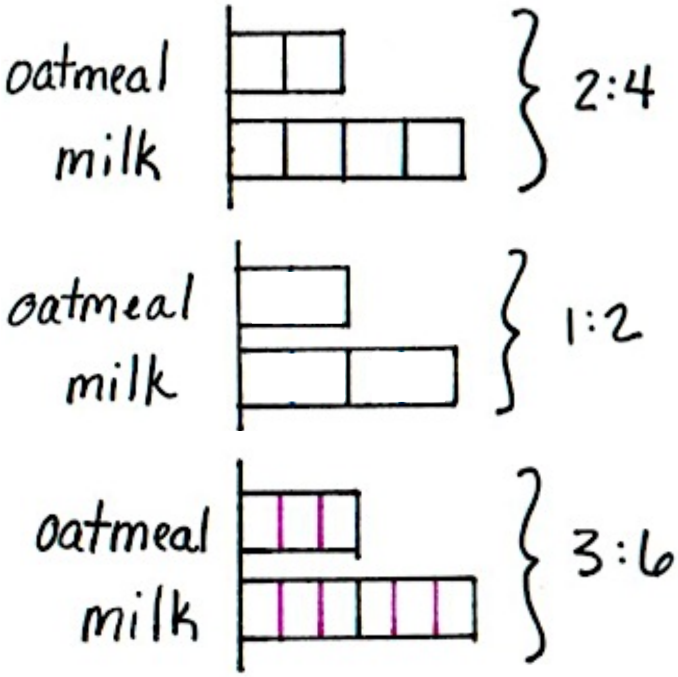
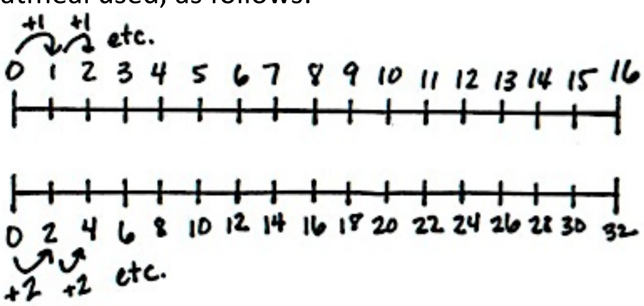
Tape Diagram:
Students may create one tape diagram, illustrating the ratio itself, or they may create several, illustrating different equivalent ratios, as follows:



Double Number Line Diagram:
This type of diagram can be used to find the number of cups of milk needed for any number of cups of oatmeal used, as follows:



The diagram above would be made after the student recognizes that the ratio 2:4 is equivalent to 1:2. Students may alternatively choose to make the top number line in increments of 2 and the bottom in increments of 4, as the given ratio suggests.
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<td>

b) Answers will vary. Must provide a ×4 situation, table/diagram, and comment on the usefulness of the ratio. The ratio may be represented as $\frac{25}{100}$, 25:100, or *25 to 100*.
</td>
<td>

Assessing Questions:

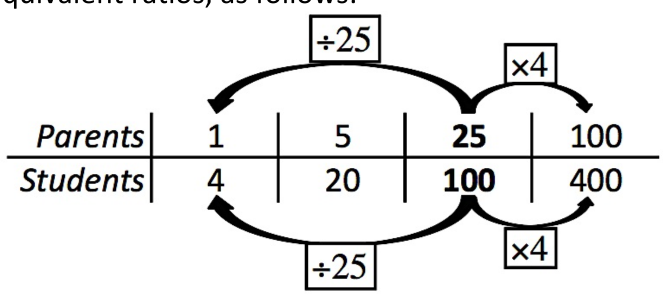
- Can you explain how this situation fits with the ratio given?
- Why would a ratio help in this real-life scenario?
- How did you create the table/diagram for this ratio?
</td>
</tr>
</table>

Some example situations in which this ratio may be used are the following:

- On the field trip, there were 25 parents to every 100 students.
- Of the songs played on the radio station yesterday, 25 out of 100 were top 40 hits.
- Twenty-five of the 100 math problems I worked to study for my test contained fractions.

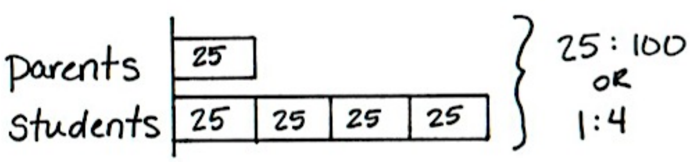
The first situation will be used in the following example table and diagrams. Students should explain that the ratio given could be used to evaluate the exact number of students and parents that may have been on the trip or to find the reduced parent to student ratio to see if it meets school policy for field trips. Variations of the table and diagrams below could be used to explain the usefulness.

Table:
Students may choose to create a table with equivalent ratios, as follows:



This table illustrates finding equivalent ratios by scaling up and down (multiplying and dividing).

Tape Diagram:
Students may prefer to create a tape diagram, such as the one below.



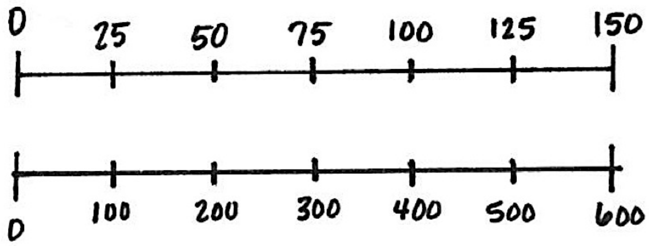
This diagram makes it easy to see that the ratio can be reduced to 1 to 4. Students may also point out that replacing 25 with a different number will give you different values that fit the 25 to 100 ratio. For instance, if 30 were put in each box, the ratio would be 30 to 120, which reduces to 1 to 4 as well.

- How does the table/diagram help explain the ratio?
- Could you have written a different ratio for this situation?

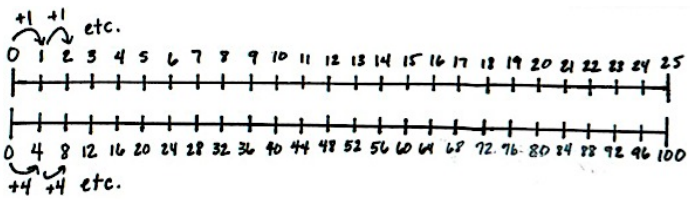
Advancing Questions:

- What symbols and notation can you use to represent ratios?
- Can you find a ratio equivalent to *25 to 100*?
- Are there other ways to represent ratios with pictures?
- How could you use one of the symbol or picture representations to explain how this ratio is useful?

Double Number Line Diagram:
Some students may prefer the double number line, as shown below.



The line representing the number of parents should increase by 25 each time, and the number line representing the number of students should increase by 100. Students may also realize that the ratio reduces to 1 to 4 and create the following number lines:



c) Answers will vary. Must provide a situation, table/diagram, and comment on the usefulness of the ratio. The ratio may be represented as $\frac{7}{9}$, 7:9, or *7 to 9*.

Some example situations in which this ratio may be used are the following:

- In the park, there are 7 skateboarders for every 9 rollerbladers.
- In my school, there are seven 5th Grade students to every nine 6th Grade students.
- In the race, Demetrio ran 7 yards to every 9 yards Rebecca ran.

The first situation will be used in the following example table and diagrams. Students should recognize that the ratio would be helpful in determining the total number of skateboarders and rollerbladers, given the number of one or the other.

Assessing Questions:

- Can you explain how this situation fits with the ratio given?
- Why would a ratio help in this real-life scenario?
- How did you create the table/diagram for this ratio?
- How does the table/diagram help explain the ratio?
- Could you have written a different ratio for this situation?

Advancing Questions:

- What symbols and notation can you use to represent ratios?
- Can you find a ratio equivalent to 7:9?
- Are there other ways to represent ratios with pictures?
- How could you use one of the symbol or picture representations to explain how this ratio is useful?

Table:
The table below illustrates how the ratio can be scaled up.

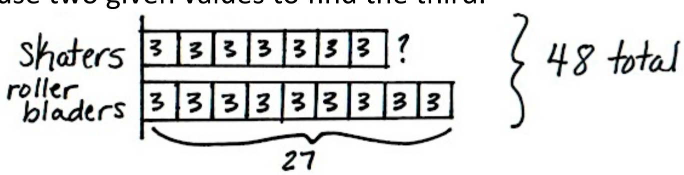
×2

Skateboarders	7	14	21	etc.
Rollerbladers	9	18	27	etc.

×2

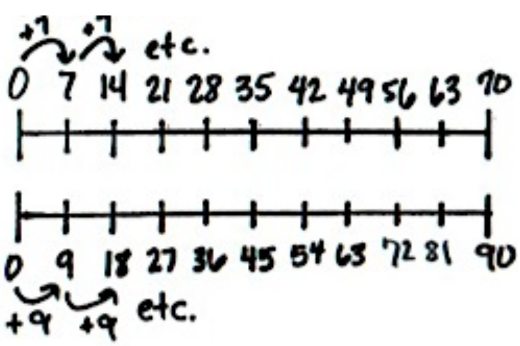
Scaling down cannot be done in this example because the fraction 7/9 is in reduced form. They could give use values from the table to find the number of rollerbladers, given the number of skateboarders, and vice versa.

Tape Diagram:
The diagram below illustrates how students could use two given values to find the third.



In the example above, they are given a total and the number of rollerbladers. They can then divide the number of rollerbladers into 9 equal parts, which helps them to find the number of skateboarders.

Double Number Line Diagram:
The double number line diagram gives the same information as the table in a different format and has the same usefulness.



Possible Student Misconceptions	**Assessing and Advancing Questions**
Students may think that ratios have to have the same units for each value of the ratio (i.e. cups to cups, feet to feet, etc.).	• Can you think of a common real-life ratio? (*Guide toward something with units like mph, minutes per mile, words per minute, etc.*) • What does that ratio literally mean? • What are the units of the first value? The second?

Students may not understand the process of scaling a ratio up and down to find equivalent ratios.	• If the ratio $\frac{2}{4}$ were a fraction, could you find equivalent fractions, reduced and unreduced? • If there are 2 boys to 4 girls in a classroom, how many girls do you think there are to 8 boys? • How is this similar to the equivalent fractions we found before?
Students may think that the fraction notation for a ratio is the same as a fraction itself, which may cause confusion about how ratios are used.	• Can you give me an example of a ratio that is not an example of a part/whole relationship? • Can you give me an example of a fraction that is not an example of a part/whole relationship? • What does the numerator of a fraction mean? The denominator? • What does the top value of one of the ratios you've written in fraction form mean? The bottom value?
Entry/Extensions	**Assessing and Advancing Questions**
If students can't get started….	• What is a ratio? • What are some situations where people compare one thing to another using numbers? • Can you adapt that to fit one of the given ratios?
If students finish early….	• If your ratio situation compares one value to another, can you change it or write a new one to compare a part to a whole? • In part b), are you reminded of any special ratios that come up often in real life? • If you did not write a table for any of the ratios, can you do so? What type of math representation does this table remind you of? • Can you create a table and graph for each ratio? What do you observe about these graphs?

Discuss/Analyze

Whole Group Questions

Different representations should be selected and sequenced from basic to complex for students to share in this whole group setting. This will allow students to strengthen their understanding as well as give the teacher an opportunity to address misconceptions.

A ratio is a special relationship between two quantities where for every *x* units of one quantity there are *y* units of another quantity.

- What is a ratio?
- Do ratios have to compare parts to wholes?
- What is a situation for the ratio in part a)/b)/c)? Does everyone agree that the situation fits that ratio? Why or why not?
- How could the ratios be useful in real life?

Equivalent ratios can be represented with tables, diagrams, equations, and/or graphs.

- What are equivalent ratios? Can you give me some examples? How do we find them?

- What types of pictures and organization can we use to help us visualize equivalent ratios?
- What type of diagrams did you create for the ratio in part a)/b)/c)? Did anyone create a different diagram? Are both correct? Why or why not?
- Are either of the diagram types similar to the table? How?
- Did you notice that one type of diagram was more useful than another in making observations about the ratio?

Understand that ratios can be represented using ":", fraction notation, and the word "to".

- How can ratios be represented using math symbols?
- Do you like some of the notation forms better than others? Why?
- Which notation looks similar to something else we use a lot in math?
- Are ratios and fractions the same?
- What makes them seem the same? What makes them different?

Name ________________________________

Task 3: Perfect Peppers

The farmer's market has too many peppers. This morning a new sign was posted that says, "5 pounds of perfect peppers for $2."

a) How many pounds of peppers can you buy for $1? How much does 1 pound of peppers cost? Describe how these values can be used.

b) Evelyn needs 4 pounds of peppers to make fajitas. How much money will she will need? How do you know?

c) Marcus has $5 to buy peppers. How many pounds of peppers he can buy? How do you know?

Task 3: Perfect Peppers	**6th Grade**
The farmer's market has too many peppers. This morning a new sign was posted that says, "5 pounds of perfect peppers for \$2." a) How many pounds of peppers can you buy for \$1? How much does 1 pound of peppers cost? Describe how these values can be used. b) Evelyn needs 4 pounds of peppers to make fajitas. How much money will she will need? How do you know? c) Marcus has \$5 to buy peppers. How many pounds of peppers he can buy? How do you know?	
Teacher Notes:	
Teachers should emphasize the meaning of the two different ratios, 5 pounds to \$2 and \$2 to 5 pounds, as well as when each form is useful.	
Tennessee State Standards for Mathematical Content	**Tennessee State Standards for Mathematical Practice**
6.RP.A.1 Understand the concept of a ratio and use ratio language to describe a ratio relationship between two quantities. *For example, "The ratio of wings to beaks in the bird house at the zoo was 2:1, because for every 2 wings there was 1 beak." "For every vote candidate A received, candidate C received nearly three votes."* **6.RP.A.2** Understand the concept of a unit rate *a/b* associated with a ratio *a:b* with *b* ≠ 0, and use rate language in the context of a ratio relationship. *For example, "This recipe has a ratio of 3 cups of flour to 4 cups of sugar, so there is 3/4 cup of flour for each cup of sugar." "We paid \$75 for 15 hamburgers, which is a rate of \$5 per hamburger." (Expectations for unit rates in this grade are limited to non-complex fractions.)*	1. Make sense of problems and persevere in solving them. 2. Reason abstractly and quantitatively. 3. Construct viable arguments and critique the reasoning of others. 4. Model with mathematics. 5. Use appropriate tools strategically. 6. Attend to precision. 7. Look for and make use of structure. 8. Look for and express regularity in repeated reasoning.
Essential Understandings:	
• A ratio is a special relationship between two quantities where for every *x* units of one quantity there are *y* units of another quantity. • A given situation may be represented by more than one ratio. • A rate is a ratio that compares two quantities that are measured in different units. • A unit rate is a rate that compares a quantity to one unit of another quantity.	
Explore Phase	
Possible Solution Paths	**Assessing and Advancing Questions**
a) You can buy 2.5 lbs. of peppers for \$1. This value can be found by calculating 5 lbs. of peppers ÷ \$2 = 2.5 lbs. of peppers/dollar. Finding this value is useful when you have a specific amount of money to spend and want to know how many pounds of peppers that you can buy. For example, if you have \$6 to buy peppers, you would	**Assessing Questions:** • How do you know how many peppers can you purchase for \$1? • How do you know how much one pound of peppers will cost? • When might you need to know the price per pound of an item? • When might you need to know the quantity of an

<table>
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calculate $6 x 2.5 lbs. of peppers/dollar, which would tell you that you could buy 15 lbs. of peppers.

One pound of peppers will cost $0.40.

This value can be found by calculating
$2 ÷ 5 lbs. of peppers = $0.40/lb. of peppers.
Finding this value is useful when you have a specific amount of peppers that you would like to buy and want to know how much that amount will cost. For example, if you have a recipe that requires 3 lbs. of peppers, you would calculate 3 x $0.40/lb. of peppers, which would tell you that you would need $1.20 to buy the amount of pepper that you need for your recipe.
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item per dollar?

Which ratio notation gives you a clue about finding these rates?

Advancing Questions:

What do you know from reading the problem?
Can you write a ratio using the values in the problem?
How can this ratio help you?
Which operation might help you solve this problem?

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b) Using the value $0.40/lb. of peppers found in part a), the total amount of money needed can be found by
4 x $0.40/lb. = $1.60

Accept reasonable student representations to show their reasoning, including the following:

Ratio Table:
<table>
<tr><th>Pounds of Peppers</th><th>Cost</th></tr>
<tr><td>1</td><td>0.40</td></tr>
<tr><td>2</td><td>0.80</td></tr>
<tr><td>3</td><td>1.20</td></tr>
<tr><td>4</td><td>1.60</td></tr>
<tr><td>5</td><td>2.00</td></tr>
</table>
</td>
<td>
Assessing Questions:

How do you know the cost of 4 pounds of peppers that Evelyn needs to make fajitas?
How does knowing how much one-pound of peppers cost help you to calculate how much 4 pounds of peppers cost?
How could a ratio table help to explain your reasoning?
What other representations could you use to show how you found your answer?

Advancing Questions:

What is this question asking you to find?
How much does 1 pound of peppers cost?
What operation can help you find the answer?
Could a table help to find the answer?
Are there other representations that could help you to find the answer?

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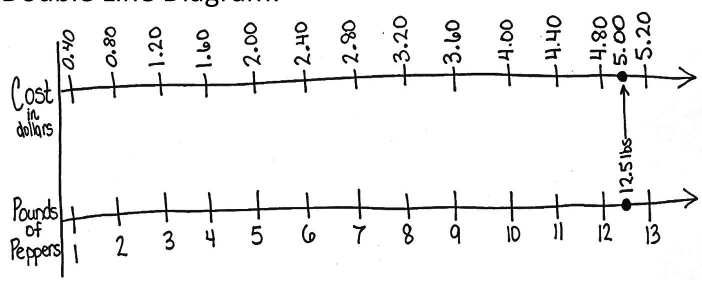
</table>

c) Using the value 2.5 lbs. of peppers/dollar found in part a), the total amount of peppers Marcus can buy is found by $5 x 2.5 lbs. = 12.5 lbs of peppers. Accept reasonable student representations to show their reasoning. Double Line Diagram:  	**Assessing Questions:** • How do you know the quantity of peppers that Marcus can purchase with $5? • How does knowing the quantity of peppers that $1 can purchase help you to solve this problem? • How could a double line diagram help to explain your reasoning? • What other representations could you use to show how you found your answer? **Advancing Questions:** • What is this question asking you to find? • What quantity of peppers can you purchase for $1? • What operation can help you find the answer? • Could a double line diagram help to find the answer? • Are there other representations that could help you to find the answer?
Possible Student Misconceptions	**Assessing and Advancing Questions**
Students think that in ratio story problems, the quantity written first should always be the first quantity in the ratio *a:b*. It is important for students to recognize that a given situation may be represented by more than one ratio.	• If I tell you that corn costs $2 for 4 pounds, how much can you buy with $1? Can you write two ratios from this scenario? • Katherine's mom gave her a 20-dollar bill to spend on peppers. How many pounds of peppers can she buy? • Julie is making a pepper pizza that uses 0.5 pounds of peppers. How much money will she need to purchase for the amount of peppers for her pizza?
Students are confused about the fact that while a ratio can be written two ways, the way you write it *does* depend on how it is described. For example, a comparison of 2 wins to 3 losses is written as 2:3, and not 3:2 (which would be a comparison of losses to wins). It is helpful if students begin labeling the quantities of the things they are comparing both in writing and orally.	• If you have 5 apples and 3 oranges what ratio compares apples to oranges? • If you have 7 strawberries and 4 bananas what ratio compares bananas to strawberries? • Would the meaning be the same if the values in the ratio were reversed?
Students think that the equivalent ratios must be multiples of the values given in original ratio. For example, in ratio 4:1.60, students may think the only equivalent ratios would be 8:3.20, 12:4.80, etc.	• If 3 pounds of strawberries cost $6.00, how much would 4 pounds of strawberries cost? • If 3 pounds of strawberries cost $6.00, how many pounds could you buy with $3.00?
Entry/Extensions	**Assessing and Advancing Questions**
If students can't get started….	• How many pounds of peppers can you buy for $2? • Can you write a ratio using the values in the problem? • How can this ratio help you?

	• Which operation might help you solve this problem? • Is there a table or diagram that might help you to solve this problem?
If students finish early….	• Can you create a graph, table, double number line, or a tape diagram that shows how many peppers you can buy with $2, $10, and $20? • Research the price of your favorite food. Create a tape diagram that shows how much for 1-5 of your favorite food. • Compare your favorite food representations with a friend. Do any of your values match up?

Discuss/Analyze

Whole Group Questions

Different representations should be selected and sequenced from basic to complex for students to share in this whole group setting. This will allow students to strengthen their understanding as well as give the teacher an opportunity to address misconceptions.

A ratio is a special relationship between two quantities where for every *x* units of one quantity there are *y* units of another quantity.

- What representations did you use to explain your reasoning?
- How do these representations show the relationship between quantities?
- How do these representations help you solve problems for an unknown amount of one of the quantities?

A given situation may be represented by more than one ratio.

- What ratio shows pounds of peppers per dollar?
- What ratio shows cost per pound of pepper?
- Why is each of these ratios useful?

A rate is a ratio that compares two quantities that are measured in different units.

- What is a rate?
- What is a unit rate?
- Why is it important to know how to use rates?
- How are rates used in everyday life?

A unit rate is a rate that compares a quantity to one unit of another quantity.

- What is the unit rate of pounds of peppers to dollars?
- What is the unit rate of dollars per pounds of peppers?

Name ______________________________

Task 4: Nail It Down!

You and your dad are buying the supplies needed to build a doghouse. He sends you to the nail and screw aisle with instructions to buy 80 8-penny nails and 120 16-penny nails. You find nails packaged as in the table.

Count	Type	Price
20	8-penny	$1.00
10	16-penny	$1.00
40	Assorted (16 8-penny; 24 16-penny)	$3.00

a) How much does it cost to purchase the nails you need with the 8-penny and 16-penny nails packaged separately? How much does it cost to purchase them in an assorted package?

b) You decide that you will build and sell doghouses in small, medium, and large, with the size of your original doghouse being the smallest. Every time you increase the size, the number of nails needed is 3/2 what was needed for the size before. How many nails do you need for the medium and large doghouses?

c) The hardware store will let you buy replacement nails by breaking up packages, charging the same rate as in the table. What is the cost of one 16-penny nail? What is the cost of one 8-penny nail? Does it matter which package you use to calculate these costs?

Task 4: Nail It Down!	**6th Grade**

You and your dad are buying the supplies needed to build a doghouse. He sends you to the nail and screw aisle with instructions to buy 80 8-penny nails and 120 16-penny nails. You find nails packaged as in the table to the right.

Count	Type	Price
20	8-penny	$1.00
10	16-penny	$1.00
40	Assorted (16 8-penny; 24 16-penny)	$3.00

a) How much does it cost to purchase the nails you need with the 8-penny and 16-penny nails packaged separately? How much does it cost to purchase them in an assorted package?

b) You decide that you will build and sell doghouses in small, medium, and large, with the size of your original doghouse being the smallest. Every time you increase the size, the number of nails needed is 3/2 what was needed for the size before. How many nails do you need for the medium and large doghouses?

c) The hardware store will let you buy replacement nails by breaking up packages, charging the same rate as in the table, with each nail in the assorted package having equal cost. What is the cost of one 16-penny nail? What is the cost of one 8-penny nail? Does it matter which package you use to calculate these costs?

Teacher Notes:

This task solidifies students' understanding of ratios, including ratio language and symbols, unit rates, equivalent ratios, ratio diagrams and representations, and solving problems using ratios. As the problem is rather long and involved, working in groups from the beginning may be a more efficient way to solve the problem. Groups should be arranged so that at least one student in each group has a level of comfort with ratio operations that is likely to lead the group to choose a solution path that fully utilizes their ratio knowledge. Groups who finish early should be given one of the three extension tasks and asked to share this with the group during the final discussion time, in order to lead the class toward a more thorough understanding of the usefulness of ratios, unit rates that are set up differently, and the similarities of graphs of equivalent ratios. Note that in part c), students must assume that all the nails in the assorted pack are the same price.

Tennessee State Standards for Mathematical Content	**Tennessee State Standards for Mathematical Practice**
6.RP.A.1 Understand the concept of a ratio and use ratio language to describe a ratio relationship between two quantities. *For example, "The ratio of wings to beaks in the bird house at the zoo was 2:1, because for every 2 wings there was 1 beak." "For every vote candidate A received, candidate C received nearly three votes."* **6.RP.A.2** Understand the concept of a unit rate *a/b* associated with a ratio *a:b* with *b ≠ 0*, and use rate language in the context of a ratio relationship. *For example, "This recipe has a ratio of 3 cups of flour to 4 cups of sugar, so there is 3/4 cup of flour for each cup of sugar." "We paid $75 for 15 hamburgers, which is a rate of $5 per hamburger." (Expectations*	1. Make sense of problems and persevere in solving them. 2. Reason abstractly and quantitatively. 3. Construct viable arguments and critique the reasoning of others. 4. Model with mathematics. 5. Use appropriate tools strategically. 6. Attend to precision. 7. Look for and make use of structure. 8. Look for and express regularity in repeated reasoning.

for unit rates in this grade are limited to non-complex fractions.) **6.RP.A.3** Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations. **a.** Make tables of equivalent ratios relating quantities with whole-number measurements, find missing values in the tables, and plot the pairs of values on the coordinate plane. Use tables to compare ratios.	

Essential Understandings:

- A ratio is a special relationship between two quantities where for every *x* units of one quantity there are *y* units of another quantity.
- Understand that ratios can be represented using ":", fraction notation, and the word "to".
- A given situation may be represented by more than one ratio.
- A rate is a ratio that compares two quantities that are measured in different units.
- A unit rate is a rate that compares a quantity to one unit of another quantity.
- Tape diagrams and double number line diagrams can show ratio relationships and be used to reason about solutions to problems.
- Equivalent ratios can be represented with tables, diagrams, equations, and/or graphs.

Explore Phase

Possible Solution Paths	Assessing and Advancing Questions
a) Basic Arithmetic: **If bought packaged separately, the nails will cost $16, but if bought in an assorted package, they will only cost $15. I should buy them in assorted packages.** Students should recognize that multiplying the cost of the packages of nails by the number of nails needed, they can find the total cost. In order to find the number of packages needed, they must divide the number of nails per package into the total number of each kind needed. Sample calculations follow. *Separately* Packages of 8-penny needed: $80 \div 20 = 4$ Cost of 8-penny nails: $4 \times \$1 = \$4$ Packages of 16-penny needed: $120 \div 10 = 12$ Cost of 16-penny nails: $12 \times \$1 = \$12$ Total cost: $\$4 + \$12 = \$16$	**Assessing Questions:** • How did you find the cost of the nails when purchased separately? • How did you find the cost when purchased together? • What are some benefits of using ratios, instead of doing calculations without them? **Advancing Questions:** • What numbers are given in the problem? • What are you asked to find? • In order to answer the question, what do you need to know? • How could you find the number of packages of nails needed, when they are bought separately (in assorted packs)? • How can we use the cost of one pack to find the cost of several?

Assorted
Packages of assorted needed (8-penny): $80 \div 16 = 5$
Packages of assorted needed (16-penny):
$120 \div 24 = 5$
Note that it is not sufficient for the student to show that 5 packages would provide enough of *one* type of nail, as it may not provide enough of the other.

Cost of nails: $5 \times \$3 = \$15$

a) Using Ratios:

Separately
Students may use a ratio to represent the cost of a package of nails. For instance, the cost of 8-penny nails could be represented by the ratio $\frac{20}{1}$, 20:1, or 20 to 1. (Note that the reverse, 1:20, would also be correct.) The cost of 16-penny nails could be represented by the ratio $\frac{10}{1}$, 10:1, or 10 to 1. Using these ratios, students may create any of the following diagrams to solve the problem:

<u>Table</u>

×2

16-penny	**10**	20	30	40	…	120
Cost	**1**	2	3	4	…	12

×2

×2

8-penny	**20**	40	60	80
Cost	**1**	2	3	4

×2

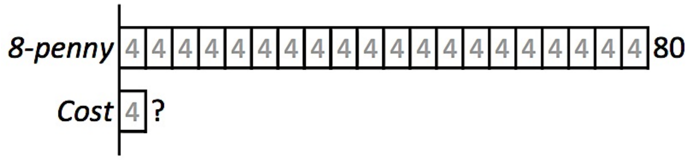
<u>Tape Diagram</u>
Students should draw 20 boxes to represent the number of nails in each bag of 8-penny nails. The cost should have one box, because each package costs $1. This represents the ratio 20:1. After they've represented the ratio correctly, they should write "80" to the side of the boxes that represent the number of nails, since that is the total number of 8-

Assessing Questions:
- How did you find the cost of the nails when purchased separately?
- How did you find the cost when purchased together?
- Why were ratios helpful in this problem?
- Could you have used ratios differently to solve the problem?

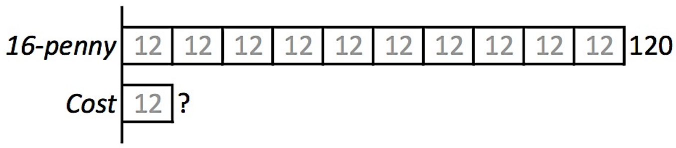
Advancing Questions:
- What numbers are given in the problem?
- How are these values related?
- Can you represent that using a ratio?
- What are you asked to find?
- In order to answer the question, what do you need to know?
- How could you find the number of packages of nails needed, when they are bought separately (in assorted packs)?
- How can we use the cost of one pack to find the cost of several?

×2

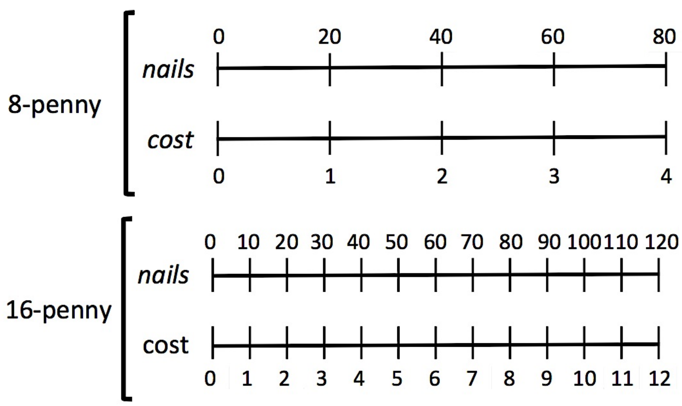
penny nails they need. Using this picture, they can calculate that each box must represent a 4 if the 20 boxes are to sum to 80. Thus, the unknown quantity, Cost, must be $4.



The cost of 16-penny nails can be found similarly using the diagram below.



Double Number Line Diagram
Students should simply line up number lines that are divided into increments matching the ratio they have chosen and see what cost corresponds to the number of nails they need.



Assorted Packages:
The student may choose to represent the number of 8-penny to 16-penny nails in a package as a ratio, in order to determine how many assorted packages they must purchase. This ratio would be $\frac{16}{24}$, 16:24, or 16 to 24 (similarly, $\frac{24}{16}$, 24:16, or 24 to 16). By creating any of the diagrams shown below, students should recognize that they would need to purchase 5 of the assorted bags of nails in order to have 80 8-penny nails and 120 16-penny nails. Since each assorted bag cost $3, the total cost would be $15.

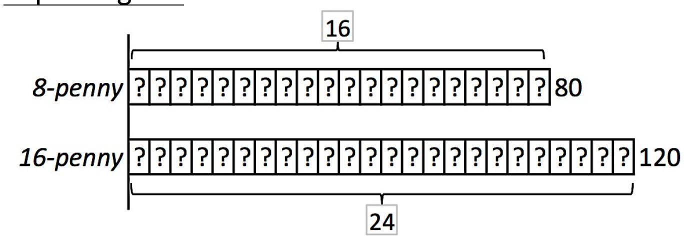
Table

×2

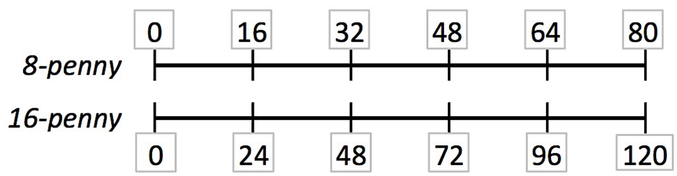
8-penny	16	32	48	64	80
16-penny	24	48	72	96	120

×2

Tape Diagram



Double Number Line Diagram



b) Basic Arithmetic:

For the medium-sized doghouses, I would need 120 8-penny nails and 180 16-penny nails. For the large doghouses, I would need 180 8-penny nails and 270 16-penny nails.

They may do the following calculations to arrive at the number of nails needed:

Medium Doghouses

8-penny:	*16-penny:*
$80 \times 1\frac{1}{2} = \frac{80}{1} \times \frac{3}{2} = \frac{40}{1} \times \frac{3}{1} = 120$	$120 \times 1\frac{1}{2} = \frac{120}{1} \times \frac{3}{2} = \frac{60}{1} \times \frac{3}{1} = 180$

Large Doghouses

8-penny:	*16-penny:*
$120 \times 1\frac{1}{2} = \frac{120}{1} \times \frac{3}{2} = \frac{60}{1} \times \frac{3}{1} = 180$	$180 \times 1\frac{1}{2} = \frac{180}{1} \times \frac{3}{2} = \frac{90}{1} \times \frac{3}{1} = 270$

Assessing Questions:

- What does the problem mean when it says, "the number of nails needed is 3/2 what was needed for the size before"?
- How did you represent that mathematically?
- How did you know that you should multiply?

Advancing Questions:

- What does the problem mean when it says, "the number of nails needed is 3/2 what was needed for the size before"?
- How many nails are needed for the smallest doghouse?
- How would you find 1/2 of those numbers?
- How would you find 3/2 of those numbers?

Alternatively, students may find 1/2 of each quantity of nails needed and then multiply by 3.

b) Using Ratios:

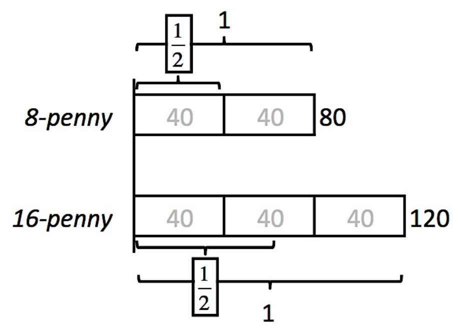
Students may use the ratio $\frac{8-penny}{16-penny}=\frac{80}{120}$ to find the number of nails needed. They must first recognize that the scale factor for each size larger is $\frac{3}{2}$. With the diagrams, it is helpful for the student to recognize that $\frac{3}{2}$ is $\frac{1}{2}\times 3$. Viewing the scale factor this way makes it easier to find the answer from the diagrams below:

Medium

Table

8-penny	40	**80**	120
16-penny	60	**120**	180
Scale Factor	1/2	**1**	3/2

Tape Diagram



8-penny: $\frac{1}{2}\times 80=40$

$40\times 3=120$

16-penny: $\frac{1}{2}\times 120=60$

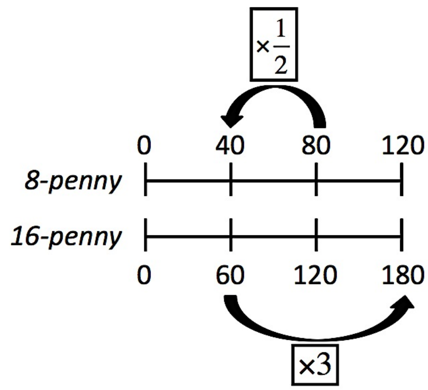
$60\times 3=180$

Assessing Questions:

- How did you find the answer to this part of the problem?
- What made a ratio seem like a good way to solve it?
- How did you know what scale factor you needed to use?
- How did you know how to use the scale factor to get the answers?

Advancing Questions:

- What values are you given for this part of the problem?
- How are these values related?
- Can you represent these relationships using ratios?
- Are there other representations that could make it more clear what is happening here?

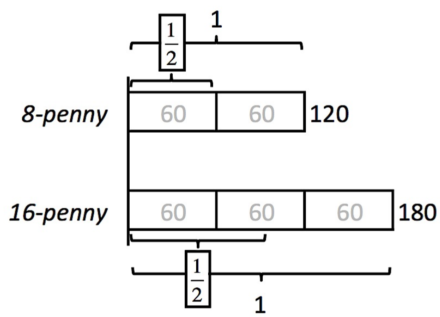
Double Number Line	

Large

Table	*see table below*

8-penny	60	**120**	180
16-penny	90	**180**	270
Scale Factor	1/2	**1**	3/2

Tape Diagram



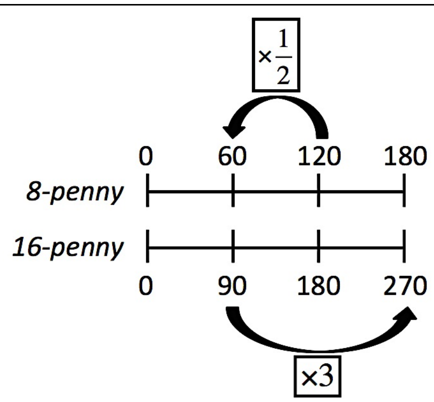
8-penny: $\frac{1}{2}\times 120 = 60$

$60\times 3 = 180$

16-penny: $\frac{1}{2}\times 180 = 90$

$90\times 3 = 270$

Double Number Line



c) If purchased from the separate packages, 8-penny nails would be \$0.05 each and 16-penny nails would be \$0.10 each. If purchased from the assorted packages, 8-penny and 16-penny nails	**Assessing Questions:** • How did you solve this part of the problem? • What is it called when we find the amount of one quantity per 1 of another quantity?

<table>
<tr>
<td>
would cost the same amount, $0.08 each.

The 8-penny nails are cheaper if purchased from the separate packages, but the 16-penny nails are cheaper if purchased from an assorted package.

Students should understand that they are calculating a unit rate to answer this part of the question. They should also notice that calculating the unit rate in nails to dollars is not very helpful, since they would be finding the number of nails they could purchase with $1, rather than the cost of each nail, which is what the question asks. After setting up the ratios $\left(\frac{dollars}{nails}\right)$, the students should divide the numerator by the denominator to reach a decimal answer and then round to the nearest cent (hundredth), or they can work the whole problem in cents, once they've figured out how many cents per nail.

Separate
<table>
<tr><td>8-penny</td><td>$\frac{\$1}{20 nails} = \$0.05 / nail$</td></tr>
<tr><td>16-penny</td><td>$\frac{\$1}{10 nails} = \$0.10 / nail$</td></tr>
</table>

Assorted
<table>
<tr><td>8-penny & 16-penny</td><td>$\frac{\$3}{40 nails} = \$0.075 / nail \approx \$0.08 / nail$</td></tr>
</table>
</td>
<td>
• How is this unit rate useful?
• Could you have calculated different unit rates using the same information?
• Would those unit rates be useful in this scenario?

Advancing Questions:
• What are the different relationships between cost and number of nails in this problem?
• If you organize those on your paper, can you see a way to write the relationships so that they could be changed to "dollars per 1 nail" or "cents per 1 nail"?
• Now that you have them organized, can you change that to a unit rate?
• How can you use those unit rates to find the cheapest way to purchase extra nails?
</td>
</tr>
<tr>
<th>Possible Student Misconceptions</th>
<th>Assessing and Advancing Questions</th>
</tr>
<tr>
<td>Students may think ratios have to have the same units for each value of the ratio (i.e. cups to cups, feet to feet, etc.).</td>
<td>
• Can you think of a common real-life ratio? (Guide toward something with units like mph, minutes per mile, words per minute, etc.)
• What does that ratio literally mean?
• What are the units of the first value? The second?
</td>
</tr>
<tr>
<td>Students may not understand the process of scaling a ratio up and down to find equivalent ratios.</td>
<td>
• If the ratio $\frac{2}{4}$ were a fraction, could you find equivalent fractions?
• If there are 2 boys to 4 girls in a classroom, how many girls do you think there are to 8 boys?
• How is this similar to the equivalent fractions we found before?
• How is "scaling a ratio up and down" similar to creating equivalent fractions?
</td>
</tr>
<tr>
<td>Students may not understand how to change a rate</td>
<td>• What ratios represent the rates of dollars to</td>
</tr>
</table>

into a unit rate.	nails? • Choosing one of these for now, can you write it simply as a fraction? • How could you change it to an equivalent fraction having one in the denominator? • What operation did you perform on the denominator? In order to have an equivalent fraction, what did you do to the numerator?
Entry/Extensions	**Assessing and Advancing Questions**
If students can't get started….	• What values are you given for this part of the problem? • How are these values related? • Can you represent these relationships using ratios? • Are there other representations that could make what is happening here more clear? • How could you find the number of packages of nails needed, when they are bought separately (in assorted packs)?
If students finish early….	• Could you have calculated different unit rates than the ones in Part c) using the same information? How would those unit rates be useful? • Can you write 3 benefits of using ratios, instead of doing calculations without them? • Using the tables you created in each part of the problem, could you create graphs? What do you notice that these graphs have in common? Can you make 2 general statements about graphs of equivalent ratios based on this activity?

Discuss/Analyze

Whole Group Questions

Different representations should be selected and sequenced from basic to complex for students to share in this whole group setting. This will allow students to strengthen their understanding as well as give the teacher an opportunity to address misconceptions.

A ratio is a special relationship between two quantities where for every *x* units of one quantity there are *y* units of another quantity.

- Fill in the blanks: " For the small doghouse, there are __________ 8-penny nails for every ________ 16-penny nails." How many combinations of answers are possible? Is this a ratio? Are there other ways to write it?

Understand that ratios can be represented using ":", fraction notation, and the word "to".
A given situation may be represented by more than one ratio.

- Did anyone solve parts a) and b) using ratios?
- What ratios did you use? Did anyone use different ratios?
- How did you represent your ratios? Did anyone represent them differently?

- Do you like some of the notation forms better than others? Why?
- Which notation looks similar to something else we use a lot in math?

Tape diagrams and double number line diagrams can show ratio relationships and be used to reason about solutions to problems.
Equivalent ratios can be represented with tables, diagrams, equations, and/or graphs.

- What are equivalent ratios? Can you give me some examples? How do we find them?
- What types of pictures and organization can we use to help us visualize equivalent ratios?
- What type of diagrams did you create for the ratios in parts a) and b)? Did anyone create different diagrams? Are both correct? Why or why not?
- Did you notice that one type of diagram was more useful than another in making observations about the ratio?
- What do we often do with tables that we create?
- Could you graph the values in the tables you created in this problem?
- If you graphed the table values, what did you notice about the graphs of equivalent ratios?

A rate is a ratio that compares two quantities that are measured in different units.
A unit rate is a rate that compares a quantity to one unit of another quantity.

- What rates are given in the problem for the cost of 8-penny and 16-penny nails?
- What rate is given for 8-penny and 16-penny together?
- If you want to buy 1 nail, what specific type of rate would be helpful?
- What is a unit rate?
- Does it matter that the units on the quantities in the rate are different (i.e. dollars and nails)? Why or why not?
- Are there multiple ways to find the unit rates?
- How would you use a unit rate that gave the cost per nail?
- How would you use a unit rate that gave the number of nails per dollar?

Name ______________________________

Task 5: Tricky Trail Mix

Jonathon's dad is making trail mix for a camping trip. The ratio of cereal to raisins needed for the recipe is 5 to 3.

a) What are some ratios that could be used to represent this situation?

b) Jonathon's dad bought 15 cups of cereal for the trail mix. What is the unit rate of cups of raisins per cup of cereal? How many cups of raisins does he need?

c) How many cups of raisins and cereal will Jonathon's dad need to double the recipe? Illustrate your reasoning using a mathematical picture or model.

Task 5: Tricky Trail Mix	**6th Grade**

Jonathon's dad is making trail mix for a camping trip. The ratio of cereal to raisins needed for the recipe is 5 to 3.

a) Write and describe two ratios that could be used to represent this situation.

b) Jonathon's dad bought 15 cups of cereal for the trail mix. What is the unit rate of cups of raisins per cup of cereal? How many cups of raisins does he need?

c) How many cups of raisins and cereal will Jonathon's dad need to double the recipe? Illustrate your reasoning using a mathematical picture or model.

Teacher Notes:

This task will develop the student's skills with ratio language and representations, as well as unit rates. These representations may include tables, diagrams, equations, and/or graphs. Using limited guidance and encouraging students to work individually before collaborating in small groups will achieve greater variation in responses, which will enrich the whole group discussion. Draw attention to the possible "part to whole" ratios in part a) if no student provides one during whole group discussion.

Tennessee State Standards for Mathematical Content	**Tennessee State Standards for Mathematical Practice**
6.RP.A.1 Understand the concept of a ratio and use ratio language to describe a ratio relationship between two quantities. *For example, "The ratio of wings to beaks in the bird house at the zoo was 2:1, because for every 2 wings there was 1 beak." "For every vote candidate A received, candidate C received nearly three votes."* **6.RP.A.2** Understand the concept of a unit rate *a/b* associated with a ratio *a:b* with *b ≠ 0*, and use rate language in the context of a ratio relationship. *For example, "This recipe has a ratio of 3 cups of flour to 4 cups of sugar, so there is 3/4 cup of flour for each cup of sugar." "We paid $75 for 15 hamburgers, which is a rate of $5 per hamburger." (Expectations for unit rates in this grade are limited to non-complex fractions.)* **6.RP.A.3** Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations. **b.** Solve unit rate problems including those involving unit pricing and constant speed. *For example, if it took 7 hours to mow 4 lawns, then at that rate, how many lawns could be mowed in 35 hours? At what*	1. Make sense of problems and persevere in solving them. 2. Reason abstractly and quantitatively. 3. Construct viable arguments and critique the reasoning of others. 4. Model with mathematics. 5. Use appropriate tools strategically. 6. Attend to precision. 7. Look for and make use of structure. 8. Look for and express regularity in repeated reasoning.

rate were lawns being mowed?	

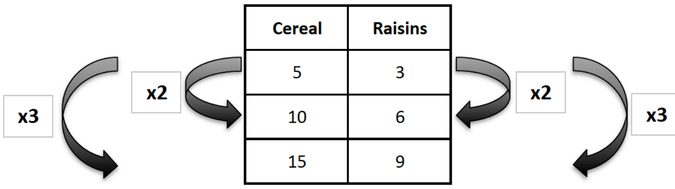
Essential Understandings:

- A ratio is a special relationship between two quantities where for every *x* units of one quantity there are *y* units of another quantity.
- Equivalent ratios can be represented with tables, diagrams, equations, and/or graphs.
- Understand that ratios can be represented using ":", fraction notation, and the word "to".
- A given situation may be represented by more than one ratio.
- Tape diagrams and double number line diagrams can show ratio relationships and be used to reason about solutions to problems.
- A rate is a ratio that compares two quantities that may be measured in different units.
- Ratios and unit rates are used to solve for unknown quantities.

Explore Phase

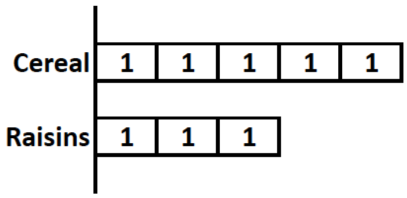
Possible Solution Paths	**Assessing and Advancing Questions**
a) Ratios that could be used to represent this situation include the following: • Part to part: Cups of raisins to cups of cereal; 3:5 or $\frac{3}{5}$ • Part to part: Cups of cereal to cups of raisins; 5:3 or $\frac{5}{3}$ • Part to whole: Cups of cereal to cups of trail mix; 5:8 or $\frac{5}{8}$ • Part to whole: Cups of raisins to cups of trail mix; 3:8 or $\frac{3}{8}$ • Whole to part: Cups of trail mix to cups of raisins; 8:3 or $\frac{8}{3}$ • Whole to part: Cups of trail mix to cups of cereal; 8:5 or $\frac{8}{5}$	**Assessing Questions:** • Why would a ratio help in this real-life scenario? • Can you explain how these ratios fits with the situation given? • What symbols and notation can you use to represent ratios? • Do these different ratios have different meanings or the same? **Advancing Questions:** • What quantities are being compared in this situation? • What are ways in which ratios can be written? • Are there other ways to write ratios?
b) The unit rate of cups of raisins to cups of cereal is $\frac{3}{5}$ cup of raisins needed for every cup of cereal. Jonathon's dad needs 9 cups of raisins to make trail mix with 15 cups of cereal. *Using Unit Rate:* Since the raisin to cereal ratio is 3:5, then the ratio of raisins in 1 cup of cereal is $\frac{3}{5}$: 1. Therefore the unit rate is $\frac{3}{5}$. Since Jonathon's dad has 15 cups of cereal, he needs $\frac{3}{5}$ x 15 = 9 cups of raisins. *Using a Ratio Table:* Students may choose to create a table that	**Assessing Questions:** • How do you know that Jonathon's dad needs 9 cups of raisins to make trail mix with 15 cups of cereal? • Why would a ratio help in this real-life scenario? • How did you create the table/diagram for this ratio? • How does the table/diagram help explain the ratio? • Could you have written a different ratio for this situation? • What symbols and notation can you use to represent ratios? • Do these different representations have different meanings or the same?

illustrates finding equivalent ratios by scaling up and down (multiplying and dividing), as follows:



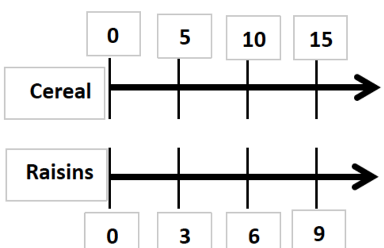
Cereal	Raisins
5	3
10	6
15	9

Using a Tape Diagram:
Students may prefer to create a tape diagram, such as the one below.



This diagram makes it easy to see that the ratio is 5 cups of cereal to 3 cups of raisins. Students should recognize that replacing 1 with a different number would give you different values that fit the 5 to 3 ratio. For instance, if 3 were put in each box, the ratio would be 15 to 9, which simplifies to 5 to 3 as well.

Using a Double Number Line Diagram:
Some students may prefer the double number line, as shown below.

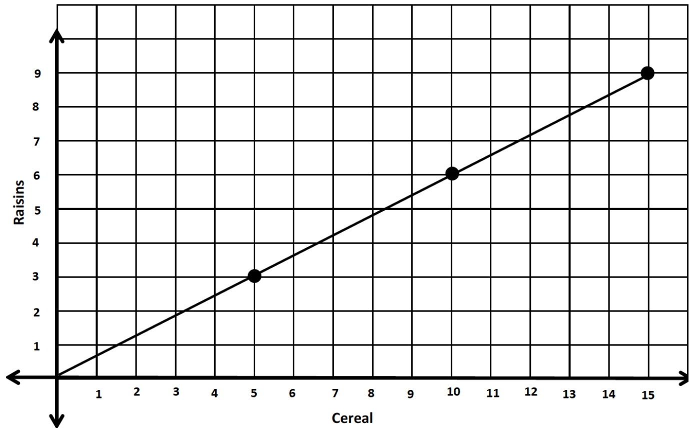


The line representing the number of cups of cereal should increase by 5 each time, and the number line representing the cups of raisins should increase by 3.

Using a Coordinate Plane:
Some students may prefer to use a coordinate plane to show the linear relationship between the cups of cereal and cups of raisins.

Advancing Questions:

- What are the quantities being compared in this situation?
- Could you use scaling up or down with the original ratio from part a)?
- Can you find a ratio equivalent to 3:5?
- Are there other ways to represent ratios with pictures?
- How could you use one of the symbol or picture representations to explain how this ratio is useful?



c) If the number of people attending the hike doubles, Jonathon's dad will need 18 cups of raisins and 30 cups of cereal to make trail mix for this larger group.

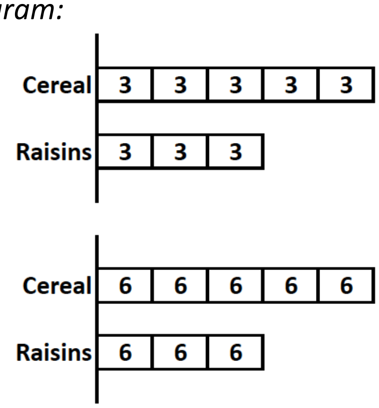
Ratio Table:

Cereal	Raisins
5	3
10	6
15	9
20	12
25	15
30	18

x2 x2

The ratio table shows the original number of cups needed for each ingredient in the trail mix. Each quantity doubled is shown in the bottom row as the amount of each ingredient needed to feed a group double the size of the original.

Tape Diagram:



The top diagram shows the number of cups needed for each ingredient for the original group of hikers. The bottom diagram shows the number of cups needed for each ingredient for double the original

Assessing Questions:

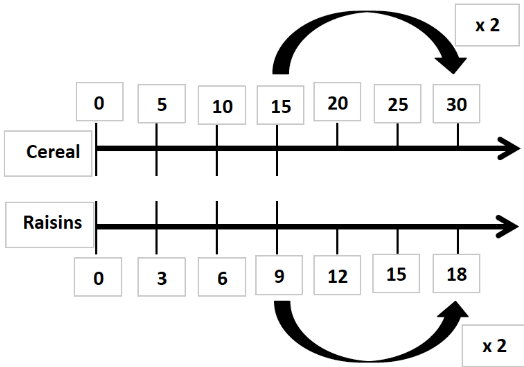
- How do you know that Jonathon's dad will need 18 cups of raisins and 30 cups of cereal?
- How are these quantities related to your original ratio?
- How did you create the table/diagram for this ratio?
- How does the table/diagram help explain the ratio?
- Are there other tables/diagrams that could be used for this ratio?
- What symbols and notation can you use to write these quantities as a ratio?
- Do these different representations have the same meaning?

Advancing Questions:

- What are the 2 quantities being compared in this situation?
- What are the ways in which a ratio can be written?
- What are the ways in which a ratio can be represented using a mathematical picture or model?
- How could you use one of the symbol or picture representations to explain how this ratio is useful?

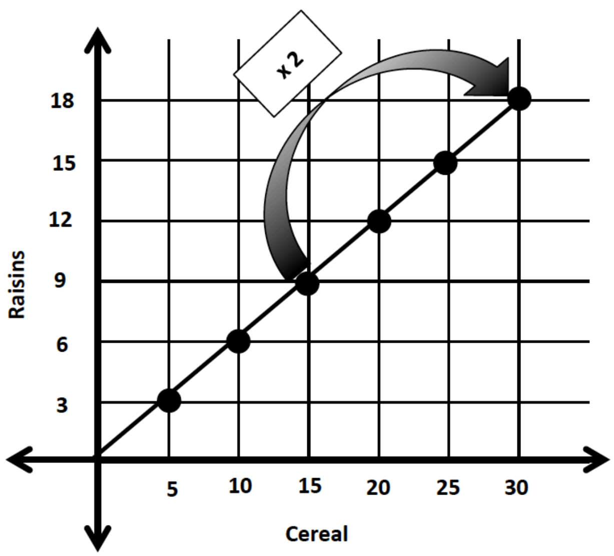
group of hikers.

Double Number Line Diagram:



The double number line shows the amount of cups for each ingredient needed to make trail mix. The arrow leads to the amount needed for group double the size of the original group.

Coordinate Plane:



The scale of the coordinate plane uses the ratio 3 cups of raisins to 5 cups of cereal to show the amount of ingredients needed for the original group. The arrow points to the amount needed for a group double the size of the original group.

Possible Student Misconceptions	**Assessing and Advancing Questions**
Students are not be familiar with multiple forms of representations of ratios.	• How could a table be used to calculate equivalent ratios? • In what real world situation could a table be useful in explaining ratios? • How could a tape diagram be used to calculate

	equivalent ratios? • In what real world situation could a tape diagram be useful in explaining ratios? • How could a double number line be used to calculate equivalent ratios? • In what real world situation could a double number line be useful in explaining ratios? • How could a coordinate plane be used to calculate ratios? • In what real world situation could a coordinate plane be useful in explaining ratios?
Students are not be able to find a unit rate or have difficulty using it once they have found it since it is a not whole number.	• What is a unit rate? • Can you tell how many cups of raisins are needed for every cup of cereal? Vice versa? • How is knowing the unit rate useful?
Entry/Extensions	**Assessing and Advancing Questions**
If students can't get started….	• What are the 2 quantities being compared in this situation? • What are the ways in which a ratio can be written? • What are the ways in which a ratio can be represented using a mathematical picture or model?
If students finish early….	• Are there any other possible ratios that could be used to represent this situation? • Are there any other representations that could be used to model equivalent ratios for this situation? • What are the benefits to each type of representation? • If only half of the original amount of hikers showed up, how many cups of trail mix would be needed? How many cups of cereal? How many cups of raisins?

Discuss/Analyze

Whole Group Questions

Different representations should be selected and sequenced from basic to complex for students to share in this whole group setting. This will allow students to strengthen their understanding as well as give the teacher an opportunity to address misconceptions.

A ratio is a special relationship between two quantities where for every *x* units of one quantity there are *y* units of another quantity.

- What is a ratio?
- What are ways in which a ratio can compare quantities?
- What ratios that could represent the original situation? Does everyone agree that these ratios could fit the situation? Why or why not?
- How could the ratios be useful in real life?

Equivalent ratios can be represented with tables, diagrams, equations, and/or graphs.

- What are equivalent ratios? Can you give me some examples? How do we find them?
- What types of pictures and organization can we use to help us visualize equivalent ratios?
- What type of diagrams did you create for the ratio in part b) and c)? Did anyone create a different diagram? Are both correct? Why or why not?
- Is either of the diagram types similar to the table? How?
- Did you notice that one type of diagram was more useful than another in making observations about the ratio?

Understand that ratios can be represented using ":", fraction notation, and the word "to".

- How can ratios be represented using math symbols?
- Do you like some of the notation forms better than others? Why?
- Which notation looks similar to something else we use a lot in math?
- Are ratios and fractions the same?
- What makes them seem the same? What makes them different?

A given situation may be represented by more than one ratio.

- What ratios could represent the original situation? Does everyone agree that these ratios could fit the situation? Why or why not?
- How is it possible that different ratios can represent the same situation?
- Do the different ratios mean the same thing or something different? Explain.
- In part b) what is the ratio of raisins to trail mix?

Tape diagrams and double number line diagrams can show ratio relationships and be used to reason about solutions to problems.

- What does a tape diagram look like in part b) or part c)? What does this help to describe about the ratio?
- What does a double number line diagram look like in part b) or part c? What does this help to describe about the ratio?
- Are there any other representations that could be used in part b) or part c)?

A rate is a ratio that compares two quantities that may be measured in different units.

- What are the 2 units being compared in this set of problems?
- Can you think of other common units that are compared in everyday life?

Ratios and unit rates are used to solve for unknown quantities.

- How did you use the ratio in part a) to help solve parts b) and c)?
- How can determining the ratio of 2 quantities help to solve for unknown quantities?
- How can determining the unit rate help to solve for unknown quantities?

Name ______________________________

Task 6: Garden Variety

Hiromi has a garden full of delicious vegetables. Yellow vegetables make up 25% of the garden. Green vegetables make up 50% of the garden. The rest of the garden is made up of 6 red vegetables.

a) What percentage of the vegetables is red? How do you know?

b) What is the total number of vegetables in the garden? Explain your reasoning.

c) If the total number of vegetables is increased to 48, how many would be green? How do you know?

Task 6: Garden Variety	**6th Grade**
Hiromi has a garden full of delicious vegetables. Yellow vegetables make up 25% of the garden. Green vegetables make up 50% of the garden. The rest of the garden is made up of 6 red vegetables. a) What percentage of the vegetables is red? How do you know? b) What is the total number of vegetables in the garden? Explain your reasoning. c) If the total number of vegetables is increased to 48, how many would be green? How do you know?	

Teacher Notes:

This task will develop the student's skills with using a percentage expressed as a ratio and finding the part given the whole and vice versa. Using limited guidance and encouraging students to work individually before collaborating in small groups will achieve greater variation, which will enrich the whole group discussion.

Tennessee State Standards for Mathematical Content	Tennessee State Standards for Mathematical Practice
6.RP.A.1 Understand the concept of a ratio and use ratio language to describe a ratio relationship between two quantities. *For example, "The ratio of wings to beaks in the bird house at the zoo was 2:1, because for every 2 wings there was 1 beak." "For every vote candidate A received, candidate C received nearly three votes."* **6.RP.A.3** Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations. **c.** Find a percent of a quantity as a rate per 100 (e.g., 30% of a quantity means 30/100 times the quantity); solve problems involving finding the whole, given a part and the percent.	1. Make sense of problems and persevere in solving them. 2. Reason abstractly and quantitatively. 3. Construct viable arguments and critique the reasoning of others. 4. Model with mathematics. 5. Use appropriate tools strategically. 6. Attend to precision. 7. Look for and make use of structure. 8. Look for and express regularity in repeated reasoning.

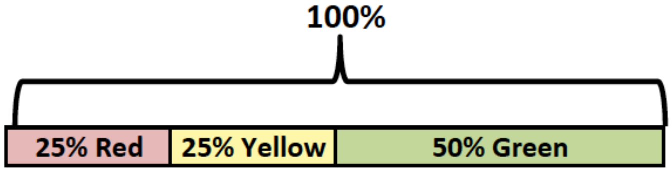
Essential Understandings:

- A ratio is a special relationship between two quantities where for every x units of one quantity there are y units of another quantity.
- Equivalent ratios can be represented with tables, diagrams, equations, and/or graphs.
- Understand that ratios can be represented using ":", fraction notation, and the word "to".
- A given situation may be represented by more than one ratio.
- Tape diagrams and double number line diagrams can show ratio relationships and be used to reason about solutions to problems.
- A percent is a special kind of ratio in which a part is compared to a whole with 100 parts.

Explore Phase

Possible Solution Paths	Assessing and Advancing Questions

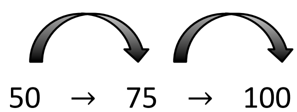
a) The red vegetables take up 25% of the garden.



By beginning with the total of 100%, students can subtract 50% for the green vegetables and 25% for the yellow vegetables, which leaves 25% of the garden for red vegetables.

$$100\% - 50\% - 25\% = 25\%$$

Students could also count up to 100% to find the percentage of red vegetables. Starting with 50% green vegetables then counting by 25s for 25% yellow vegetables and the final 25% for red vegetables.



Assessing Questions:

- How do you know that 25% of the vegetables in the garden are red?
- Why did you choose to use subtraction to solve this problem?
- Is there any other strategy that could have helped you to solve for the percentage of vegetables that are red?

Advancing Questions:

- What are the percentages of vegetables in the garden that you know?
- What percentage would represent all of the vegetables?
- How can you use these percentages to help you find the percentage of vegetables that are red?
- How can a picture help you to find the percentage of red vegetables?

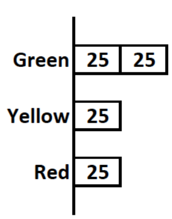
b) There are 24 total vegetables in the garden.

After finding the solution for part a), students could use that knowledge to create the following equation:

$$\left(\frac{25}{100}\right)T = 6$$
$$\left(\frac{1}{4}\right)T = 6$$
$$\left(\frac{1}{4}\right)T \times 4 = 6 \times 4$$
$$T = 24$$

Simplifying $\frac{25}{100}$ to $\frac{1}{4}$ can make diagrams and tables less complicated.

Using a Tape Diagram:



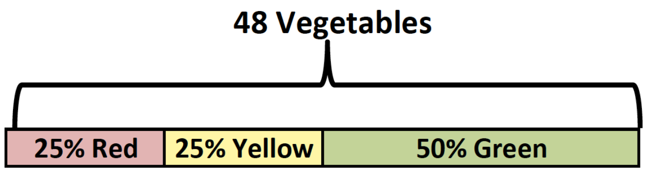
or

Assessing Questions:

- How did you solve for the total number of vegetables in the garden?
- Are there any other strategies to solving this problem?

Advancing Questions:

- How many vegetables are in the garden?
- What is it that you are looking for? A percentage or an amount?
- Could you use an equation to solve this problem?
- Could a tape diagram help you to solve this problem?
- Are there any other representations that might help you to solve this problem?



This diagram should say 24 vegetables.

The red and yellow vegetables have the same quantity. This means that there are 6 red vegetables and 6 yellow vegetables. Together the red and yellow vegetables make up 50% of the garden, which is the same as the green vegetables. Therefore, there is the same number of green vegetables as the combined total of red and yellow vegetables. This means that there are 12 green vegetables. The total numbers of vegetables in the garden is 24 (6 red, 6 yellow, and 12 green).

c) If the total number of vegetables is increased to 48, 24 of the vegetables would be green.

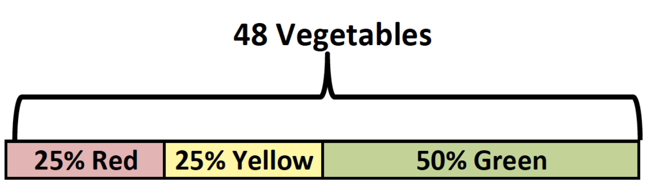
Using an Equation:
The original ratio of green vegetables to the total number of vegetables is $\frac{12}{24}$, which can be simplified to $\frac{1}{2}$.

Students could solve for the number of green vegetables in the garden with a total of 48 vegetables by multiplying

$$\frac{1}{2} \text{ x } 48 = 24.$$

Some students may use division to solve 48 ÷ 2 = 24, by recognizing that multiplying by $\frac{1}{2}$ is the same as dividing by 2.

Using a Tape Diagram:



Using the tape diagram, students can break apart the total of 48 vegetables according to the percentages. Half of the total of 48 is 24 green vegetables.

Assessing Questions:
- How do you know that the number of green vegetables would be 24 in a garden with 48 total vegetables?
- How are these quantities related to your original ratio?
- How did you create the table/diagram for this problem?
- How does the table/diagram help to solve this problem?
- Are there other tables/diagrams that could be used to solve this problem?

Advancing Questions:
- What is the question asking?
- How many vegetables are now in the garden?
- What percentage of the vegetables are green?
- How can use what you know from parts a) and b) to solve this problem?

Possible Student Misconceptions	Assessing and Advancing Questions
Students may have trouble solving an equation with a fraction multiplier on the variable.	• In the equation $\left(\frac{1}{4}\right)T = 6$, how do you solve for T? • How do you divide fractions? • Which operation did you use to solve for T?
Students may have difficulty understanding that the whole can be more or less than 100 in quantity but still be 100%.	• What is the total number of vegetables in the garden? • What is the total percentage of vegetables in the garden?
Entry/Extensions	**Assessing and Advancing Questions**
If students can't get started….	• What are the percentages of vegetables in the garden? • What percentage would represent all of the vegetables? • How can you use these percentages to help you solve this problem? • Could you write these percentages as fractions? • Could using a diagram, equation, or chart help you to solve this problem?
If students finish early….	• If Hiromi's garden tripled in area and the percentage of each vegetable remained the same, how many of each color would be in the garden? • If Hiromi decided to add 6 orange vegetables to the garden, what percentage of the garden would be green? Red? Yellow? Orange?

Discuss/Analyze

Whole Group Questions

Different representations should be selected and sequenced from basic to complex for students to share in this whole group setting. This will allow students to strengthen their understanding as well as give the teacher an opportunity to address misconceptions.

A ratio is a special relationship between two quantities where for every *x* units of one quantity there are *y* units of another quantity.
Understand that ratios can be represented using ":", fraction notation, and the word "to".
A given situation may be represented by more than one ratio.

- What is a ratio?
- What are ways in which a ratio can compare quantities?
- What is a ratio of vegetables in the garden?
- Does anyone have a different ratio for vegetables in the garden?
- Does everyone agree that these ratios could fit the situation? Why or why not?
- How is it possible that different ratios can represent the same situation?
- Do the different ratios mean the same thing or different? Explain.

Tape diagrams and double number line diagrams can show ratio relationships and be used to reason about solutions to problems.
Equivalent ratios can be represented with tables, diagrams, equations, and/or graphs.

- What does a tape diagram look like for this problem?
- What does this help to describe about the ratio?
- Are there any other representations that could be used in this problem?
- Are there any benefits to using a particular representation over another?

A percent is a special kind of ratio in which a part is compared to a whole with 100 parts.

- What is the percentage that represents a whole?
- Is a percent a part to part ratio or a part to whole ratio?
- What is the percentage of red vegetables in the garden? How do you know?

Name ______________________________

Task 7: The Donut Dilemma

Every Saturday, you and your friends jog to the donut shop. Your friend Rodrigo just got a new bike and wants to bike along with the running group.

a) You and your friends jog at a rate of 6 minutes per mile, but Rodrigo measures his bike speed in miles per hour. How could you tell Rodrigo what speed he will need to ride to stay with the group?

b) Rodrigo says he wants to ride 12 mph and thinks that the group could keep up with him. Do you agree? Explain your reasoning.

Task 7: The Donut Dilemma — 6th Grade

Every Saturday, you and your friends jog to the donut shop. Your friend Rodrigo just got a new bike and wants to bike along with the running group.

a) You and your friends jog at a rate of 6 minutes per mile, but Rodrigo measures his bike speed in miles per hour. How could you tell Rodrigo what speed he will need to ride to stay with the group?

b) Rodrigo says he wants to ride 12 mph and thinks that the group could keep up with him. Do you agree? Explain your reasoning.

Teacher Notes:

This task is designed to develop students' skills with converting units of measurement using conversion factors expressed as ratios. This unit multiplier method guides students through the converting process in a more concrete way, eliminating the confusion of when to multiply by a conversion factor and when to divide. Students should understand the process of multiplying by unit multipliers to find equivalent fractions before attempting this task, as that understanding is foundational.

Tennessee State Standards for Mathematical Content	Tennessee State Standards for Mathematical Practice
6.RP.A.1 Understand the concept of a ratio and use ratio language to describe a ratio relationship between two quantities. *For example, "The ratio of wings to beaks in the bird house at the zoo was 2:1, because for every 2 wings there was 1 beak." "For every vote candidate A received, candidate C received nearly three votes."* **6.RP.A.2** Understand the concept of a unit rate *a/b* associated with a ratio *a:b* with *b ≠ 0*, and use rate language in the context of a ratio relationship. *For example, "This recipe has a ratio of 3 cups of flour to 4 cups of sugar, so there is 3/4 cup of flour for each cup of sugar." "We paid $75 for 15 hamburgers, which is a rate of $5 per hamburger." (Expectations for unit rates in this grade are limited to non-complex fractions.)* **6.RP.A.3** Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations. **d.** Use ratio reasoning to convert measurement units; manipulate and transform units appropriately when multiplying or dividing quantities.	1. Make sense of problems and persevere in solving them. 2. Reason abstractly and quantitatively. 3. Construct viable arguments and critique the reasoning of others. 4. Model with mathematics. 5. Use appropriate tools strategically. 6. Attend to precision. 7. Look for and make use of structure. 8. Look for and express regularity in repeated reasoning.

Essential Understandings:

- A ratio is a special relationship between two quantities where for every *x* units of one quantity there are *y* units of another quantity.
- Understand that ratios can be represented using ":", fraction notation, and the word "to".
- A given situation may be represented by more than one ratio.

- A rate is a ratio that compares two quantities that are measured in different units.
- A unit rate is a rate that compares a quantity to one unit of another quantity.
- Tape diagrams and double number line diagrams can show ratio relationships and be used to reason about solutions to problems.
- Equivalent ratios can be represented with tables, diagrams, equations, and/or graphs.
- Ratios and unit rates are used to solve for unknown quantities.

Explore Phase

Possible Solution Paths	**Assessing and Advancing Questions**

a) Unit Multiplier:

Since Rodrigo measures his bike speed in miles per hour, we need to change 6 minutes per mile to 10 miles per hour (mph).

To give Rodrigo the speed in miles per hour, students need to convert the speed to these units. They should do this using a unit multiplier ratio, as follows:

$$6\,\text{min}/\,mi = \frac{6\,\text{min}}{1mi} \times \frac{1hr}{60\,\text{min}} = \frac{6hr}{60mi} = \frac{1hr}{10mi}$$

Once students have changed minutes to hours, they still must put the rate in miles per hour. Since this is a ratio, $\frac{1hr}{10mi}$ is 1hr:10mi or 1 hour to 10 miles, which could also be expressed as 10mi:1hr, 10 miles to 1 hour, or $\frac{10mi}{1hr}$. This could be expressed as 10 miles *per* hour.

Assessing Questions:
- How would you tell Rodrigo the speed he needs to ride?
- How did you calculate that speed?
- Why does multiplying the speed by $\frac{1hr}{60\,\text{min}}$ change the numbers used but not change how fast the group is actually moving?
- Is taking 1 hour to ride 10 miles the same as riding 10 miles in 1 hour?

Advancing Questions:
- What values are you given in the problem?
- What are the units on the running speed?
- What units do you need for the biking speed?
- What is different about the running speed units versus the biking speed units?
- How could you change one of those differences?
- How could you change the other?

a) Ratio Diagram:
Students should recognize that the units of the running speed, minutes per mile, are in the reverse order compared to the units of the biking speed, miles per hour. They may either switch the order before changing the units of time or after. If done before, they may create any of the following diagrams to solve this problem:

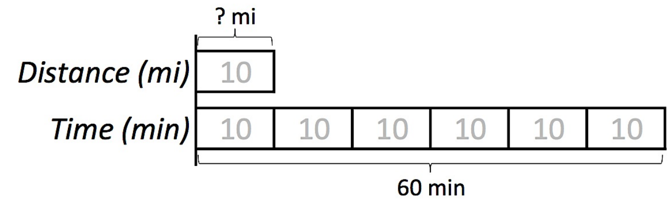
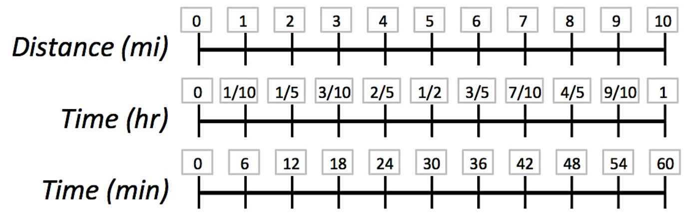
Table

×10

Distance (mi)	1	10
Time (hr)	1/10	1
Time (min)	6	60

Assessing Questions:
- How would you tell Rodrigo the speed he needs to ride?
- How did you find that speed?
- How did you create the diagram you used?
- Can you think of other methods you could have used to solve the problem?
- Can you express the unit conversion factor you used as a ratio?

Advancing Questions:
- What values are you given in the problem?
- What are the units on the running speed?
- What units do you need for the biking speed?
- What is different about the running speed units versus the biking speed units?
- What conversion factor could you use to change the units?

Tape Diagram  *Triple Number Line Diagram*  	• Can you express that unit conversion factor as a ratio? • How can you use that conversion factor ratio to solve the problem? • Could a tape diagram or a group of number lines help to solve this problem?
b) Unit Multiplier: **Since Rodrigo measures his bike speed in miles per hour, we need to change 12 miles per hour (mph) to 5 minutes per mile. This is a very fast running speed, and it is unlikely that the group would be able to keep up on a leisurely jog to get donuts.** **Note that while most students will disagree, as long as students have converted the speed to 5 mins per mile, accept any reasonable justification as to whether they can keep up.** To give Rodrigo's speed in minutes per mile, students need to convert the speed to these units. They should do this using a unit multiplier ratio, as follows: $12mi/hr = \frac{12mi}{1hr} \times \frac{1hr}{60\min} = \frac{12mi}{60\min} = \frac{1mi}{5\min}$ Once students have changed hours to minutes, they still must put the rate in minutes per mile. Since this is a ratio, $\frac{1mi}{5\min}$ is 1mi:5min or 1 miles to 5 minutes, which could also be expressed as 5min:1mi, 5 minutes to 1 mile, or $\frac{5\min}{1mi}$.	**Assessing Questions:** • What speed would the runners have to go in order to keep up with Rodrigo? • How did you calculate that speed? • Why does multiplying the speed by $\frac{1hr}{60\min}$ change the numbers used but not change how fast the group is actually moving? • Is running 5 miles in 1 minute the same as taking 1 minute to run 5 miles? **Advancing Questions:** • What values are you given in the problem? • What are the units on the biking speed? • What units do you need for the running speed? • What is different about the running speed units versus the biking speed units? • How could you change one of those differences? • How could you change the other?
b) Ratio Diagram: Students should recognize that the units of biking speed, miles per hour, are in reverse order compared to the units of running speed, minutes per mile. They may either switch the order before changing the units of time or after. If done before,	**Assessing Questions:** • What speed would the runners have to go in order to keep up with Rodrigo? • How did you find that speed? • How did you create the diagram you used?

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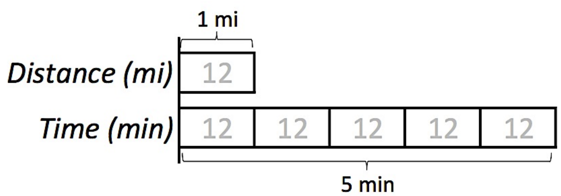
they may create any of the following diagrams to solve this problem:

Table

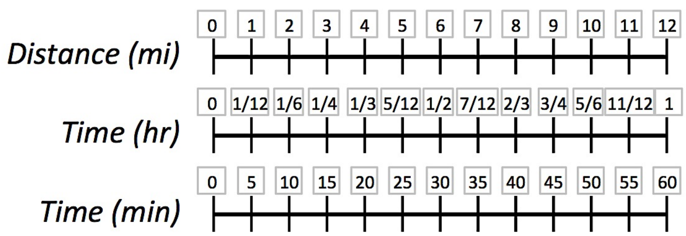
÷12

Distance (mi)	1	12
Time (hr)	1/12	1
Time (min)	5	60

Tape Diagram



Double Number Line Diagram



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- Can you think of other methods you could have used to solve the problem?
- Can you express the unit conversion factor you used as a ratio?

Advancing Questions:

- What values are you given in the problem?
- Could drawing a diagram help?
- What are the units on the biking speed?
- What units do you need for the running speed?
- What is different about the running speed units versus the biking speed units?
- What conversion factor could you use to change the units?
- Can you express that unit conversion factor as a ratio?
- How can you use that conversion factor ratio to solve the problem?

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<th>Possible Student Misconceptions</th>
<th>Assessing and Advancing Questions</th>
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Students may think ratios have to have the same units for each value of the ratio (i.e. cups to cups, feet to feet, etc.).

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- Can you think of a common real-life ratio? (*Guide toward something with units like mph, minutes per mile, words per minute, etc.*)
- What does that ratio literally mean?
- What are the units of the first value? The second?

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Students may not understand that multiplying by a unit multiplier is the same as multiplying by one, and therefore changes the units of the quantity and not the quantity itself.

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- If you need to convert from minutes to hours, what conversion factor do you use?
- What is 1 hour divided by 60 minutes? What is 60 minutes divided by 1 hour?
- Given your answer to the first question, why do you think we would call $\frac{1hr}{60\,\text{min}}$ or $\frac{60\,\text{min}}{1hr}$ *unit* multipliers?
- What is the result when I multiply any quantity by 1?
- Keeping in mind that they are both equal to one, what would you expect the result to be if I

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	multiply a quantity by $\frac{1hr}{60\min}$ or $\frac{60\min}{1hr}$? • How can you explain why the numbers change, even though you essentially multiplied by one?
Entry/Extensions	**Assessing and Advancing Questions**
If students can't get started….	• What values are you given in the problem? • Can you draw a diagram that relates the values in the problem? • What are the units used for running speed? • What are the units used for biking speed? • What is different about the running speed units versus the biking speed units? • What is a unit multiplier/conversion factor? • What conversion factor could you use to change the units? • Can you express that conversion factor as a ratio? • How can you use that conversion factor ratio to solve the problem?
If students finish early….	• If you created a table for either part of the problem, can you graph it and make 2 observations about the graph? • What are 2 additional ways that you could solve both parts of the problem? • Can you and your group write up all of the ways you solved the problem and be prepared to share them with the class?

Discuss/Analyze

Whole Group Questions

Different representations should be selected and sequenced from basic to complex for students to share in this whole group setting. This will allow students to strengthen their understanding as well as give the teacher an opportunity to address misconceptions.

A ratio is a special relationship between two quantities where for every *x* units of one quantity there are *y* units of another quantity.
Understand that ratios can be represented using ":", fraction notation, and the word "to".
A given situation may be represented by more than one ratio.

- What ratios did you use? Did anyone use different ratios?
- What are the units on the running speed?
- What units do you need for the biking speed?
- What is different about the running speed units versus the biking speed units?
- How could you change one of those differences?
- How could you change the other?
- How did you represent your ratios? Did anyone represent them differently?
- Which representation works best for this problem?

- Why does multiplying the speed by $\frac{1hr}{60\min}$ change the numbers used but not change how fast the group is actually moving?
- Is taking 1 hour to ride 10 miles the same as riding 10 miles in 1 hour? Why or why not?

Tape diagrams and double number line diagrams can show ratio relationships and be used to reason about solutions to problems.

Equivalent ratios can be represented with tables, diagrams, equations, and/or graphs.

- What are equivalent ratios? Can you give me some examples? How do we find them?
- Did anyone use a diagram or picture to solve this problem? What type and why?
- What types of pictures and organization can we use to help us visualize equivalent ratios?
- Can you think of other methods you could have used to solve the problem?
- Did you notice that one type of diagram was more useful than another in making observations about the ratio?
- What do we often do with tables that we create?
- Could you graph the values in the tables you created in this problem?
- If you graphed the table values, what did you notice about the graphs of equivalent ratios?

A rate is a ratio that compares two quantities that are measured in different units.

A unit rate is a rate that compares a quantity to one unit of another quantity.

Ratios and unit rates are used to solve for unknown quantities.

- What are rates?
- What rates are given in this problem?
- What is a unit rate?
- Are there unit rates in this problem?
- Does it matter that the units on the quantities in the rate are different (i.e. miles and hours or minutes and miles)? Why or why not?

Name ________________________________

Task 8: Be the Mathe-Magician!

You and your classmates have been asked to solve each of the problems below but can't agree on a solution path. Knowing that you're a math whiz, your group has asked you to decide if they are both correct and provide a diagram or equation to illustrate.

a) *If a dress that originally cost $24 is 25% off, how much is the discount?*
Rosemary says we can turn 25% into the ratio $\frac{1}{4}$ and multiply it by the original cost of the dress, but Jack says 25% should be changed to 0.25 and multiplied.

b) *Convert 3 yards to centimeters.*
Maria says that, since 1 yard = 36 inches and 1 inch is about 2.5 centimeters, the problem can be solved by multiplying 3 times 36 times 2.5, but Rosemary says that the problem should be solved using ratios.

c) *A cleaner needs to be diluted 1 part cleaner to 10 parts water. How much water should be used for 8 cups of cleaner?*
Yungwei says that the equation $y = 10x$ illustrates this situation, but Maria says the equation should be $y = \frac{1}{10}x$.

Task 8: Be the Mathe-Magician — 6th Grade

You and your classmates have been asked to find multiple ways to solve each of the problems below but can't agree on a solution path. Knowing that you're a math whiz, your group has asked you to decide if they are both correct and provide a diagram or equation to illustrate.

a) *If a dress that originally cost $24 is 25% off, how much is the discount?*

Rosemary says we can turn 25% into the ratio $\frac{1}{4}$ and multiply it by the original cost of the dress, but Jack says 25% should be changed to 0.25 and multiplied.

b) *Convert 3 yards to centimeters.*

Maria says that, since 1 yard = 36 inches and 1 inch is about 2.5 centimeters, the problem can be solved by multiplying 3 times 36 times 2.5, but Rosemary says that the problem should be solved using ratios.

c) *A cleaner needs to be diluted 1 part cleaner to 10 parts water. How much water should be used for 8 cups of cleaner?*

Yungwei says that the equation $y = 10x$ illustrates this situation, but Maria says the equation should be $y = \frac{1}{10}x$.

Teacher Notes:

This final task solidifies students' understanding of ratios, including notation, language, rates, unit rates, tables, diagrams, graphs, and equations. Additionally, it solidifies students' skills with representing percentages and conversion factors as ratios. Students should be comfortable with all of the representations of ratios before doing this assignment.

Tennessee State Standards for Mathematical Content	Tennessee State Standards for Mathematical Practice
6.RP.A.1 Understand the concept of a ratio and use ratio language to describe a ratio relationship between two quantities. *For example, "The ratio of wings to beaks in the bird house at the zoo was 2:1, because for every 2 wings there was 1 beak." "For every vote candidate A received, candidate C received nearly three votes."* **6.RP.A.2** Understand the concept of a unit rate *a/b* associated with a ratio *a:b* with *b ≠ 0*, and use rate language in the context of a ratio relationship. *For example, "This recipe has a ratio of 3 cups of flour to 4 cups of sugar, so there is 3/4 cup of flour for each cup of sugar." "We paid $75 for 15 hamburgers, which is a rate of $5 per hamburger." (Expectations for unit rates in this grade are limited to non-complex fractions.)* **6.RP.A.3** Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, tape diagrams,	1. Make sense of problems and persevere in solving them. 2. Reason abstractly and quantitatively. 3. Construct viable arguments and critique the reasoning of others. 4. Model with mathematics. 5. Use appropriate tools strategically. 6. Attend to precision. 7. Look for and make use of structure. 8. Look for and express regularity in repeated reasoning.

double number line diagrams, or equations. **a.** Make tables of equivalent ratios relating quantities with whole-number measurements, find missing values in the tables, and plot the pairs of values on the coordinate plane. Use tables to compare ratios. **b.** Solve unit rate problems including those involving unit pricing and constant speed. *For example, if it took 7 hours to mow 4 lawns, then at that rate, how many lawns could be mowed in 35 hours? At what rate were lawns being mowed?* **c.** Find a percent of a quantity as a rate per 100 (e.g., 30% of a quantity means 30/100 times the quantity); solve problems involving finding the whole, given a part and the percent. **d.** Use ratio reasoning to convert measurement units; manipulate and transform units appropriately when multiplying or dividing quantities.	

Essential Understandings:

- A ratio is a special relationship between two quantities where for every *x* units of one quantity there are *y* units of another quantity.
- Understand that ratios can be represented using ":", fraction notation, and the word "to".
- A given situation may be represented by more than one ratio.
- A rate is a ratio that compares two quantities that are measured in different units.
- A unit rate is a rate that compares a quantity to one unit of another quantity.
- Tape diagrams and double number line diagrams can show ratio relationships and be used to reason about solutions to problems.
- Equivalent ratios can be represented with tables, diagrams, equations, and/or graphs.
- Ratios and unit rates are used to solve for unknown quantities.
- A percent is a special kind of ratio in which a part is compared to a whole with 100 parts.

Explore Phase

Possible Solution Paths	**Assessing and Advancing Questions**
a) Both Rosemary and Jack are correct. Students must provide an explanation and table/diagram similar to the following: *Rosemary* We can represent 25% with the ratio $\frac{25}{100}$, since % means "out of 100." This ratio is equivalent to $\frac{1}{4}$, so 25% of \$24 is $\frac{1}{4}\times\$24=\$6$. *Jack* When 25% is represented as $\frac{25}{100}$, we can change	**Assessing Questions:** • How do you know that both students are correct? • How do the two different methods result in the same answer? • Which form of the percent is the easiest to use in this problem? • What type of table or diagram did you use the represent the problem? • Why did you choose this table/diagram? • Do you think a different type could have worked better? **Advancing Questions:** • What are some other representations for percentages?

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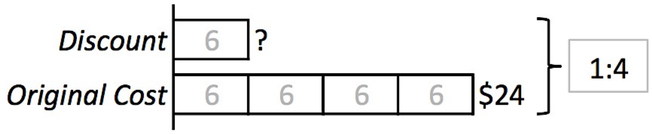
that to the decimal 0.25. Then 25% of $24 is $0.25 \times \$24 = \$6$.

We can also solve this problem using a diagram, like the one below.


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• How are percentages of values found?
• Can you relate these representations of percentages and how percentages are found to what Rosemary and Jack said about the problem? Now can you tell if they are both correct?
• How can you represent ratios using tables and diagrams?
• Can you choose one of these representations and create a table or diagram for this problem?
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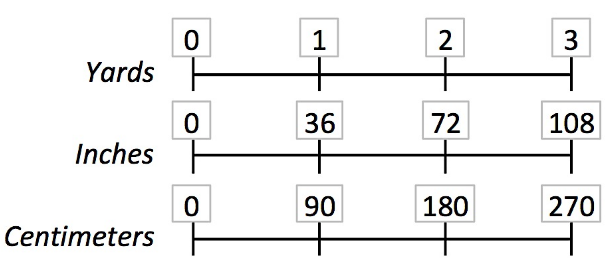
b) Maria and Rosemary are both correct. Maria did the calculations without representing the conversion factors as ratios. However, if the conversion factors are represented as ratios, the calculations that result are the same as what Maria did.

Students must provide an explanation and table/diagram similar to the following:

$1yd = 36in$ can be expressed as $\frac{1yd}{36in}$ or $\frac{36in}{1yd}$

$1in = 2.5cm$ can be expressed as $\frac{1in}{2.5cm}$ or $\frac{2.5cm}{1in}$

Then,
$\frac{3yds}{1} \times \frac{36in}{1yd} \times \frac{2.5cm}{1in} = (3 \times 36 \times 2.5)cm = 270cm$.

A double number line can be used to represent the relationship between units, as seen below.


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Assessing Questions:
• How do you know that both students are correct?
• How do the two different methods result in the same answer?
• Why would it be useful to use ratios to convert units, rather than simply doing the calculations?
• Can you explain the process of using ratios to convert units?
• Why does multiplying by the conversion ratio change the numbers but not the distance represented?
• What type of table or diagram did you use the represent the problem?
• Why did you choose this table/diagram?
• Do you think a different type could have worked better?
Advancing Questions:
• What unit are you starting with? To which unit are you converting?
• What conversion facts do we need to solve this problem?
• Can you represent those with ratios?
• Is there only one ratio for each conversion fact? If not, what is another one?
• What calculations can be done to solve the problem?
• How are unit conversions done with ratios?
• Now can you tell if Maria and Rosemary are both correct?
• How can you represent ratios using tables and diagrams?
• Can you choose one of these representations and create a table or diagram for this problem?
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c) Both Yungwei and Maria are correct, but neither defined the variables, which is what led to the different equations. If we let *x* be the number of
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Assessing Questions:
• How are Yungwei's and Maria's answers different?
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parts of cleaner and *y* be the number of parts of water, then Yungwei's equation would be correct. Reversing the order makes Maria's equation correct. For 8 cups of cleaner, 80 cups of water must be used.

Students must provide an explanation and table/diagram similar to the following:

If *x* is the number of parts of cleaner, and *y* is the number of parts of water, then a possible table of equivalent ratios is the following:

+1

x	1	2	3	…	8
y	10	20	30	…	80

+10

This table can be represented by the equation $y = 10x$. However, if the variables are reversed (i.e. *x* is the number of parts of water, and *y* is the number of parts of cleaner), then a possible table is the one below, and the values can be represented by $y = \frac{1}{10}x$.

+10

x	10	20	30	…	80
y	1	2	3	…	8

+1

- What clarification should they have provided to make the equations they gave more correct?
- What type of equation are both of the equations?
- What type of table or diagram did you use the represent the problem?
- Why did you choose this table/diagram?
- Do you think a different type could have worked better?

Advancing Questions:

- What can we do with the equations that Yungwei and Maria provided?
- Can you make a graph or table from the given equations?
- How are the graphs/tables that we can create from the equations related?
- What do *x* and *y* represent in each equation?

Possible Student Misconceptions	Assessing and Advancing Questions
Students may think ratios have to have the same units for each value of the ratio (i.e. cups to cups, feet to feet, etc.).	• Can you think of a real-life ratio we use every day that has units on it? (*Guide toward something with units like mph, minutes per mile, words per minute, etc.*) • What does that ratio literally mean? • What are the units of the first value? The second?
Students may not understand that multiplying by a unit multiplier is the same as multiplying by one, and therefore changes the units of the quantity and not the quantity itself.	• If you are converting from yards to inches, what conversion factor should you use? • How many times does 36 inches divide into 1 yard? One yard into 36 inches?

	• Why do you think we call $\frac{1yd}{36in}$ or $\frac{36in}{1yd}$ *unit* multipliers? • What is the result when I multiply any quantity by 1? • What would you expect the result to be if I multiply a quantity by $\frac{1yd}{36in}$ or $\frac{36in}{1yd}$? • How can you explain why the numbers change, even though you technically multiplied by one?
Entry/Extensions	**Assessing and Advancing Questions**
If students can't get started….	• Do we always represent ratios the same way? • What are some ways that we can represent the same ratio? • Compare the methods that each of the students used. Can you show that they are using different representations of the same ratio? • What does a percent mean? • How can you convert a percent to a decimal? • What is a unit multiplier/conversion factor? • What conversion factor could you use to change the units? • Can you express that conversion factor as a ratio? • How can you use that conversion factor ratio to solve the problem?
If students finish early….	• Can you create a different type of table or diagram for the problems than the ones you already made? • Are the different types of tables and diagrams equally useful in all ratio problems? • Can you and your group create graphs and equations for each of the ratios in the problems above?

Discuss/Analyze

Whole Group Questions

Different representations should be selected and sequenced from basic to complex for students to share in this whole group setting. This will allow students to strengthen their understanding as well as give the teacher an opportunity to address misconceptions.

A ratio is a special relationship between two quantities where for every *x* units of one quantity there are *y* units of another quantity.
Understand that ratios can be represented using ":", fraction notation, and the word "to".

- What forms of ratio notation appear in this task?
- Do you think it makes a difference which notation you use?
- What notation should you use when you are doing percentage problems? Unit conversions?

- What notations are best when you are writing out a ratio relationship?

A given situation may be represented by more than one ratio.

- How did you represent the conversion factors in part b) with ratios? Did anyone represent them differently?
- Is there more than one correct way to turn conversion factors into ratios?

Equivalent ratios can be represented with tables, diagrams, equations, and/or graphs.

- What does the graph of Yungwei’s equation look like? What does the graph of Maria’s equation look like? Are they similar? Can anyone explain the differences?

Tape diagrams and double number line diagrams can show ratio relationships and be used to reason about solutions to problems.

- What are equivalent ratios? Can you give some examples? How are they found?
- What types of pictures and organization can we use to help us visualize equivalent ratios?
- What type of diagrams did you create for the ratios in this task? Did anyone create different diagrams?
- Did you notice that one type of diagram was more useful than another in making observations about the ratio?
- What do we often do with tables that we create?
- Did you graph the values in the tables you created in this problem?
- If you graphed the table values, what did you notice about the graphs of equivalent ratios?

A rate is a ratio that compares two quantities that are measured in different units.
A unit rate is a rate that compares a quantity to one unit of another quantity.
Ratios and unit rates are used to solve for unknown quantities.

- What is a rate?
- Are there rates in this task? Which ratios are rates?
- What is a unit rate?
- Are there unit rates in this task? Which rates are unit rates?
- Does it matter that the units on the quantities in the rate are different (i.e. yards and inches)? Why or why not?

A percent is a special kind of ratio in which a part is compared to a whole with 100 parts.

- What is the percentage that represents a whole?
- Is a percent a part to part ratio or a part to whole ratio?