Tennessee Comprehensive Assessment Program

TCAP/CRA 2013

A2

Anchor Set

One Rocket, Three Equations Task

SECURE MATERIAL - Reader Name: ______________________________

Tennessee Comprehensive Assessment Program

One Rocket, Three Equations Task

For his science project, Grady designs and launches a model rocket from the rooftop of an empty building. He places a sensor on the rocket to provide him with data about the height (from ground level, in feet) of the rocket over time (from the time of the launch, in seconds).

From the data returned by the sensor, Grady was able to write and graph a function modeling the height of the rocket over time.

$$H(t) = -16t^2 + 96t + 112$$

a. Rewrite the function in vertex form.

b. Describe what the vertex form of the function reveals in the context of the rocket situation.

c. Rewrite the function by factoring completely.

d. Describe what the factored form of the function reveals in the context of the rocket situation.

REVIEW YOUR WORK IF YOU HAVE TIME.

Scoring Guide

The CCSS for Mathematical Content (4 points)

A-SSE.B.3(w) Writes the function $H(t) = -16(t-3)^2 + 256$ or an equivalent function in vertex form. ______
(1 Point)

A-SSE.B.3(x) Indicates that this form reveals the maximum height. ______
(1 Point)

A-SSE.B.3(y) Writes the function $H(t) = -16(t-7)(t+1)$ or an equivalent function in completely factored form. ______
(1 Point)

A-SSE.B.3(z) Indicates that this form reveals the time when height of the rocket is 0 or when the rocket is on the ground. ______
(1 Point)

The CCSS for Mathematical Practice (2 points)

MP6 Manipulates symbols accurately and uses mathematical language precisely. ______
(1 Point)
(MP6: Attend to precision.)

MP7 Recognizes and makes use of the structure of completing the square or $h = -\frac{b}{2a}$ to rewrite the equation in vertex form. ______
(1 Point)
(MP7: Look for and make use of structure.)

TOTAL POINTS: 6

2

The CCSS for Mathematical Content Addressed In This Task

Seeing Structure in Expressions	A-SSE
Write expressions in equivalent forms to solve problems.	
A-SSE.B.3	Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.

The CCSS for Mathematical Practice*

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

* Gray type indicates Mathematical Practices not addressed in this assessment.

One Rocket, Three Equations Task

For his science project, Grady designs and launches a model rocket from the rooftop of an empty building. He places a sensor on the rocket to provide him with data about the height (from ground level, in feet) of the rocket over time (from the time of the launch, in seconds).

From the data returned by the sensor, Grady was able to write and graph a function modeling the height of the rocket over time.

$$H(t) = -16t^2 + 96t + 112$$

a. Rewrite the function in vertex form.

$y = -16t^2 + 96t + 112$

$= -16(t^2 - 6t) + 112$

$= -16(t^2 - 6t + 9) + 256$

$= -16(t - 3)^2 + 256$

A-1b

b. Describe what the vertex form of the function reveals in the context of the rocket situation.

The vertex form, $y = -16(t-3)^2+256$, gives the time when the rocket reaches its highest altitude. From the function, I can observe that the rocket reach its highest point at 256 feet 3 seconds after the launch.

c. Rewrite the function by factoring completely.

$$fy = -16t^2+96t+112$$
$$= -16(t^2-6t-7)$$
$$= -16(t^2+t-7t-7)$$
$$= -16[(t^2+t)-(7t+7)]$$
$$= -16[t(t+1)-7(t+1)]$$
$$= -16(t-7)(t+1)$$

Litho#: 0078

d. Describe what the factored form of the function reveals in the context of the rocket situation.

The factored form, $y=-16(t-7)(t+1)$, gives the real zeroes of the function. In another word, the time when the rocket reach the ground. In this case, the zeroes are 7 and -1. Since the second can not be -1, 7 second is the preferred solution in this situation.

Anchor 1 Litho 0078

Total Content Points: 4 (A-SSE.B.3(w), A-SSE.B.3(x), A-SSE.B.3(y), A-SSE.B.3(z))

Total Practice Points: 2 (MP6, MP7)

The student correctly writes the function in vertex form ($-16(t-3)^2 + 256$) (A-SSE.B.3(w)). The student correctly explains that vertex form reveals the maximum height ("when the rocket reaches its highest altitude") (A-SSE.B.3(x)). The student correctly factors the function ($-16(t-7)(t+1)$) (A-SSE.B.3(y)) and correctly explains that this forms reveals "the time when the rocket reach the ground" (A-SSE.B.3(z)). The student manipulates symbols accurately and uses precise mathematical language (MP6). The student recognizes and makes use of the structure of completing the square to find the vertex form (MP7).

Total Awarded Points: 6 out of 6

A-2a

One Rocket, Three Equations Task

For his science project, Grady designs and launches a model rocket from the rooftop of an empty building. He places a sensor on the rocket to provide him with data about the height (from ground level, in feet) of the rocket over time (from the time of the launch, in seconds).

From the data returned by the sensor, Grady was able to write and graph a function modeling the height of the rocket over time.

$$H(t) = -16t^2 + 96t + 112$$

a. Rewrite the function in vertex form.

$t(y) = -16x^2 + 96x + 112$

$y - 112 = -16x^2 + 96x$

$y - 112 = -16x(x - 6)$

Litho#: 0069

b. Describe what the vertex form of the function reveals in the context of the rocket situation.

It shows the point where the rocket stops asending and starts descending. It gives the highest point the rocket went before it started to fall.

c. Rewrite the function by factoring completely.

$y = -16x^2 + 96x + 112$

$y = -16(x^2 - 6x - 7)$

$y = -16(x+1)(x-7)$

Litho#: 0069

A-2c

d. Describe what the factored form of the function reveals in the context of the rocket situation.

It shows the point where it left the ground, and it shows the point where it hit the ground

Litho#: 0069

Anchor 2 Litho 0069

Total Content Points: 3 (A-SSE.B.3(x), A-SSE.B.3(y), A-SSE.B.3(z))

Total Practice Points: 1 (MP6)

The student does not correctly write the function in vertex form in Part A (no credit for A-SSE.B.3(w)). However, the student correctly explains that vertex form reveals the maximum height ("gives the highest point the rocket went") (A-SSE.B.3(x)). In Part C, the function is correctly factored $(-16(x + 1)(x - 7)$ (A-SSE.B.3(y)). The student correctly explains that the factored form "shows where the rocket hit the ground" (A-SSE.B.3(z)). The student manipulates symbols accurately and uses precise mathematical language (MP6). The student does not show recognition that the structure of completing the square or of $h = -\frac{b}{2a}$ can be used to find the vertex form (no credit for MP7).

Total Awarded Points: 4 out of 6

One Rocket, Three Equations Task

For his science project, Grady designs and launches a model rocket from the rooftop of an empty building. He places a sensor on the rocket to provide him with data about the height (from ground level, in feet) of the rocket over time (from the time of the launch, in seconds).

From the data returned by the sensor, Grady was able to write and graph a function modeling the height of the rocket over time.

$$H(t) = -16t^2 + 96t + 112$$

a. Rewrite the function in vertex form.

$y = -16(x-3)^2 + 256$

A-3b

b. Describe what the vertex form of the function reveals in the context of the rocket situation.

the point were the rocket stops and goes back down

c. Rewrite the function by factoring completely.

$y = -16(x-3)^2 + 256$

$y = (x^2 - 9) + 240$

Litho#: 0091

d. Describe what the factored form of the function reveals in the context of the rocket situation.

that it went 256 feet in the air.

Anchor 3 Litho 0091

Total Content Points: 2 (A-SSE.B.3(w), A-SSE.B.3(x))

Total Practice Points: 0

The function in vertex form in Part A is correctly written (A-SSE.B.3(w)). The student correctly explains in Part B that vertex form reveals the maximum height ("the point were the rocket stops and goes back down") (A-SSE.B.3(x)). The student does not correctly factor the function in Part C (no credit for A-SSE.B.3(y)). In Part D, the student does not correctly explain what the factored form reveals (no credit for A-SSE.B.3(z)). The student does not manipulate the equation correctly in Part C, subtracting the factor of –16 and expanding $(x - 3)^2$ incorrectly (no credit for MP6). The student does not show any work in Part A, providing no evidence that indicates a recognition that the structure of either completing the square or of $h = -\frac{b}{2a}$ can be used to find the vertex form (no credit for MP7).

Total Awarded Points: 2 out of 6

A-4a

One Rocket, Three Equations Task

For his science project, Grady designs and launches a model rocket from the rooftop of an empty building. He places a sensor on the rocket to provide him with data about the height (from ground level, in feet) of the rocket over time (from the time of the launch, in seconds).

From the data returned by the sensor, Grady was able to write and graph a function modeling the height of the rocket over time.

$$H(t) = -16t^2 + 96t + 112$$

a. Rewrite the function in vertex form.

$-16(x-3)^2 + 256$

$\frac{-(96)}{2(-16)} = 3$

aos = 3
vertex = (3, 256)

Litho#: 0088

A-4b

b. Describe what the vertex form of the function reveals in the context of the rocket situation.

$-16(x-3)^2+256$

it is showing that ~~every~~ every 3 minutes it passes theough 256 meters

c. Rewrite the function by factoring completely.

$\frac{-16t^2}{-16}+\frac{96t}{-16}+\frac{112}{-16}$

t^2-6t-7

$(t-6)(t-1)$

Litho#: 0088

d. Describe what the factored form of the function reveals in the context of the rocket situation.

It shows that the time and height may be different each time.

Anchor 4 Litho 0088

Total Content Points: 1 (A-SSE.B.3(w))

Total Practice Points: 1 (MP7)

The student correctly writes the function in vertex form in Part A (A-SSE.B.3(w)). In Part B, the student does not explain that vertex form reveals the maximum height (no credit for A-SSE.B.3(x)). The function is not correctly written in factored form in Part C (no credit for A-SSE.B.3(y)). In Part D, the student does not correctly explain what the factored form reveals (no credit for A-SSE.B.3(z)). The student does not include the left side of the function or the equal sign in Part C ($H(t) =$) and does not manipulate the equation correctly, dropping the negative 16 while factoring, which demonstrates a lack of precision (no credit for MP6). The student recognizes and makes use of the structure of $h = -\frac{b}{2a}$ to find the vertex form of the equation (MP7).

Total Awarded Points: 2 out of 6

One Rocket, Three Equations Task

For his science project, Grady designs and launches a model rocket from the rooftop of an empty building. He places a sensor on the rocket to provide him with data about the height (from ground level, in feet) of the rocket over time (from the time of the launch, in seconds).

From the data returned by the sensor, Grady was able to write and graph a function modeling the height of the rocket over time.

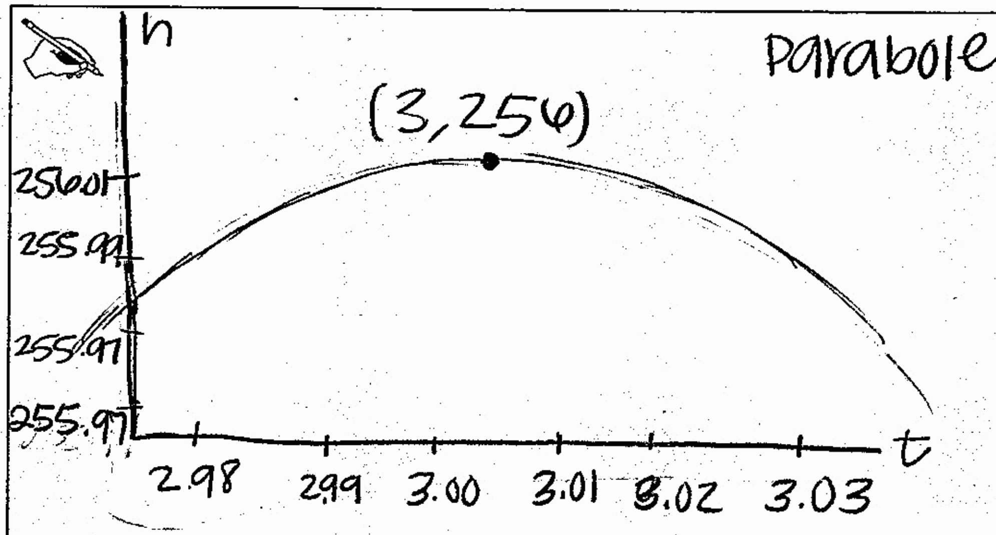
$$H(t) = -16t^2 + 96t + 112$$

a. Rewrite the function in vertex form.

Form parabola

(3,256)

b. Describe what the vertex form of the function reveals in the context of the rocket situation.



c. Rewrite the function by factoring completely.

$ht = -16t^2 + 96 + 112$

$h(t) = -16(t-7)(t+1)$

d. Describe what the factored form of the function reveals in the context of the rocket situation.

Anchor 5 Litho 0062

Total Content Points: 2 (A-SSE.B.3(x), A-SSE.B.3(y))

Total Practice Points: 0

The student does not write the function in vertex form in Part A (no credit for A-SSE.B.3(w)). However, the student shows a visual representation indicating that vertex form reveals the maximum height in Part B (A-SSE.B.3(x)). The function is written in correctly factored form in Part C ($-16(t-7)(t+1)$) (A-SSE.B.3(y)). The student does not attempt to explain what this form reveals (no credit for A-SSE.B.3(z)). The student incorrectly notates the function in Part C and forgets the variable t ($ht = -16t^2 + 96 + 112$) (no credit for MP6). The student does not show recognition that the structure of completing the square or of $h = -\frac{b}{2a}$ can be used to find the vertex form (no credit for MP7).

Total Awarded Points: 2 out of 6

A-6a

One Rocket, Three Equations Task

For his science project, Grady designs and launches a model rocket from the rooftop of an empty building. He places a sensor on the rocket to provide him with data about the height (from ground level, in feet) of the rocket over time (from the time of the launch, in seconds).

From the data returned by the sensor, Grady was able to write and graph a function modeling the height of the rocket over time.

$$H(t) = -16t^2 + 96t + 112$$

a. Rewrite the function in vertex form.

$y = (h + 1)^2 + (q + 7)^2$

Litho#: 0040

A-6b

b. Describe what the vertex form of the function reveals in the context of the rocket situation.

The vertex form reveals the points in which the rocket launched and when it fell.

c. Rewrite the function by factoring completely.

$H(t) = -16t^2 + 96t + 112$

$-4t^2 + 24t + 28$

$-t^2 + 6t + 7$

$-4(t+1)(t-7)$

$t = -1, 7 = 0$

Litho#: 0040

A-6c

d. Describe what the factored form of the function reveals in the context of the rocket situation.

The factored form reveals the time when the rocket launched and when it landed.

Litho#: 0040

Anchor 6 Litho 0040

Total Content Points: 1 (A-SSE.B.3(z))

Total Practice Points: 0

The function is not correctly written in vertex form in Part A (no credit for A-SSE.B.3(w)). The student does not explain that vertex form reveals the maximum height in Part B (no credit for A-SSE.B.3(x)). The function is not correctly written in factored form in Part C (no credit for A-SSE.B.3(y)). However, the explanation given correctly indicates that the factored form shows “when it [the rocket] landed” (A-SSE.B.3(z)). The student does not manipulate the equation correctly in Part C, losing a factor of 4 when factoring the function, which demonstrates a lack of precision (no credit for MP6). The student does not show recognition that the structure of completing the square or of $h = -\frac{b}{2a}$ can be used to rewrite the equation in vertex form (no credit for MP7).

Total Awarded Points: 1 out of 6

One Rocket, Three Equations Task

For his science project, Grady designs and launches a model rocket from the rooftop of an empty building. He places a sensor on the rocket to provide him with data about the height (from ground level, in feet) of the rocket over time (from the time of the launch, in seconds).

From the data returned by the sensor, Grady was able to write and graph a function modeling the height of the rocket over time.

$$H(t) = -16t^2 + 96t + 112$$

a. Rewrite the function in vertex form.

$$\frac{-96}{2(-16)} + 112 = V$$

b. Describe what the vertex form of the function reveals in the context of the rocket situation.

The vertex reveals where the rocket was the highest.

c. Rewrite the function by factoring completely.

$h(t) = -16t^2 + 96 + 112$

$(x + 2)\ (x + 56)$

Litho#: 0023

A-7c

d. Describe what the factored form of the function reveals in the context of the rocket situation.

where on a graph it would
cross the X axis

Litho#: 0023

Anchor 7 Litho 0023

Total Content Points: 1 (A-SSE.B.3(x))

Total Practice Points: 0

The function is not correctly written in vertex form in Part A (no credit for A-SSE.B.3(w)). However, the student does correctly explain what vertex form reveals in Part B ("where the rocket was the highest") (A-SSE.B.3(x)). In Part C, the student does not write the function in correctly factored form (no credit for A-SSE.B.3(y)). While the student correctly explains in Part D that factored form shows where the graph of a function would cross the *x*-axis, there is not an explanation of what the form reveals in the context of the rocket situation (no credit for A-SSE.B.3(z)). In Part C the student factors incorrectly and changes the variable from *t* to *x*, demonstrating a lack of precision (no credit for MP6). The student does not show recognition that the structure of completing the square or of $h = -\frac{b}{2a}$ can be used to rewrite the equation in vertex form (no credit for MP7).

Total Awarded Points: 1 out of 6

A-8a

One Rocket, Three Equations Task

For his science project, Grady designs and launches a model rocket from the rooftop of an empty building. He places a sensor on the rocket to provide him with data about the height (from ground level, in feet) of the rocket over time (from the time of the launch, in seconds).

From the data returned by the sensor, Grady was able to write and graph a function modeling the height of the rocket over time.

$$H(t) = -16t^2 + 96t + 112$$

a. Rewrite the function in vertex form.

$-16 \cdot 112 = -1792$

$(-16t^2 + 112t)(-16t + 112)$

$-16t(t-7) - 16(t-7)$

$(-16t - 16)(t-7)$

Litho#: 0032

A-8b

b. Describe what the vertex form of the function reveals in the context of the rocket situation.

It shows how much speed the rocket gained

c. Rewrite the function by factoring completely.

$(-16t-16)(t-7)$

Litho#: 0032

d. Describe what the factored form of the function reveals in the context of the rocket situation.

Shows how much curve in speed-time ratio that the rocket had

Anchor 8 Litho 0032

Total Content Points: 1 (A-SSE.B.3(y))

Total Practice Points: 0

The function is not correctly written in vertex form in Part A (no credit for A-SSE.B.3(w)). In Part B, the student does not explain that vertex form reveals the maximum height (no credit for A-SSE.B.3(x)). While not completely factored, the function in Part C is correctly factored enough to allow the *x*-intercepts to be determined (A-SSE.B.3(y)). However, in Part D the student does not correctly explain what the factored form reveals (no credit for A-SSE.B.3(z)). The student uses imprecise mathematical language in the explanation in Part D ("how much curve in speed-time ratio that the rocket had") and does not include the left side of the function or the equal sign in Part C ($H(t) =$) (no credit for MP6). The student does not show recognition that the structure of completing the square or of $h = -\frac{b}{2a}$ can be used to rewrite the equation in vertex form (no credit for MP7).

Total Awarded Points: 1 out of 6

One Rocket, Three Equations Task

For his science project, Grady designs and launches a model rocket from the rooftop of an empty building. He places a sensor on the rocket to provide him with data about the height (from ground level, in feet) of the rocket over time (from the time of the launch, in seconds).

From the data returned by the sensor, Grady was able to write and graph a function modeling the height of the rocket over time.

$$H(t) = -16t^2 + 96t + 112$$

a. Rewrite the function in vertex form.

$-16 = a$

$96 = h$

$112 = k$

$\dfrac{-96 = \sqrt{16384}}{-32}$

$\dfrac{-96 \pm 128}{-32}$

$x = -1 \quad x = 7$

A-9b

b. Describe what the vertex form of the function reveals in the context of the rocket situation.

It show were the rocket hit its point.

c. Rewrite the function by factoring completely.

$$\frac{-96 \pm \sqrt{96^2 - 4(-16)(112)}}{2(-16)}$$

$$\frac{-96 \pm 128}{-32}$$

Litho#: 0037

d. Describe what the factored form of the function reveals in the context of the rocket situation.

factored fer shows points that the line pass through

Anchor 9 Litho 0037

Total Content Points: 0

Total Practice Points: 0

In Part A, the function is not correctly written in vertex form (no credit for A-SSE.B.3(w)). The student does not explain that vertex form reveals the maximum height in Part B (no credit for A-SSE.B.3(x)). In Part C, the function is not written in correctly factored form (no credit for A-SSE.B.3(y)). The student does not correctly explain what the factored form reveals in Part D (no credit for A-SSE.B.3(z)). The student uses the quadratic formula in Part C, but does not include the $-4ac$ $(-4(-16)(112))$ under the radical, and uses imprecise mathematical language in explanations in Parts B and D (no credit for MP6). The student does not show recognition that the structure of completing the square or of $h = -\frac{b}{2a}$ can be used to rewrite the equation in vertex form (no credit for MP7).

Total Awarded Points: 0 out of 6

A-10a

One Rocket, Three Equations Task

For his science project, Grady designs and launches a model rocket from the rooftop of an empty building. He places a sensor on the rocket to provide him with data about the height (from ground level, in feet) of the rocket over time (from the time of the launch, in seconds).

From the data returned by the sensor, Grady was able to write and graph a function modeling the height of the rocket over time.

$$H(t) = -16t^2 + 96t + 112$$

a. Rewrite the function in vertex form.

$H(t) = -16t^2 + 96t + 112$

Litho#: 0030

b. Describe what the vertex form of the function reveals in the context of the rocket situation.

It reveals the height of the rocket.

c. Rewrite the function by factoring completely.

$-16t^2 + 96t + 112$

$-2 \quad 8 \qquad 8 \quad 12 \qquad 2 \quad 56$

$2 \quad 4 \qquad 2 \quad 4 \quad 2 \quad 6 \qquad 4 \quad 14$

$2 \quad 2 \qquad 2 \quad 2 \qquad 2 \quad 3 \qquad 2 \quad 2 \quad 2 \quad 7$

A-10c

d. Describe what the factored form of the function reveals in the context of the rocket situation.

It reveals the height of the rocket over time.

Anchor 10 Litho 0030

Total Content Points: 0

Total Practice Points: 0

The function is not correctly written in vertex form in Part A (no credit for A-SSE.B.3(w)). In Part B, the student does not correctly explain that vertex form reveals the maximum height (no credit for A-SSE.B.3(x)). The student does not write the function in correctly factored form in Part C, instead factoring the coefficients and the constant into prime numbers (no credit for A-SSE.B.3(y)). In Part D, the student does not correctly explain what the factored form reveals (no credit for A-SSE.B.3(z)). The student copies the given function in Part A and overall shows insufficient work to demonstrate precision (no credit for MP6). The student does not show recognition that the structure of completing the square or of $h = -\frac{b}{2a}$ can be used to rewrite the equation in vertex form (no credit for MP7).

Total Awarded Points: 0 out of 6