| Task: Going Green | Algebra 1/Core Math I |
|---|---|

The city of Cautionville has decided to make the city more environmentally friendly. They have introduced several city-wide initiatives that provide incentives for individuals and businesses to make changes that impact the environment. The city council's latest idea is to restructure the way that vehicle registration fees are calculated. As in the past, the fee will be calculated based on a percentage of a car's current value. The new formula will also include a discount based on the fuel efficiency of the vehicle. The city has decided to use the following formula and criteria for calculating the fee:

$c = 0.01v - df$, where $c$ = registration fee (\$), $v$ = current value of vehicle (\$), $d$ = category multiplier (see table below), and $f$ = fuel economy (mpg).

| Fuel Economy Range in mpg | Category multiplier |
|---|---|
| 0-9.9 | 0 |
| 10-14.9 | 0.1 |
| 15-19.9 | 0.2 |
| 20-26.9 | 1 |
| 27-35.9 | 4 |
| 36-44.5 | 6 |
| 45 and above | 8 |

A. Based on the information above, determine the cost of the fee for the criteria below. Show how you determined your answer.

- \$10,000 vehicle with a fuel economy of 22 mpg
- \$36,000 vehicle with a fuel economy of 42 mpg
- \$40,000 vehicle with a fuel economy of 11 mpg

B. The city wants to build a website where people can go to input their information in order to estimate their vehicle registration fee. The programmers would like to develop the site so that three of the four values can be entered and the fourth value would be calculated. They have used the information from the original equation to set up the first calculation below.

| Cost of fee calculation | Value of Vehicle | Category Multiplier | Fuel Economy |
|---|---|---|---|
| $c = 0.01v - df$ | v | d | f |

Fill in each table below with the correct equation that would calculate the missing value if the other three values were entered. Show your work.

| Cost of fee calculation | Value of Vehicle | Category Multiplier | Fuel Economy |
|---|---|---|---|
| c | v | | f |

| Cost of fee calculation | Value of Vehicle | Category Multiplier | Fuel Economy |
|---|---|---|---|
| c | | d | f |

| Cost of fee calculation | Value of Vehicle | Category Multiplier | Fuel Economy |
|---|---|---|---|
| c | v | d | |

C. David just bought a car and claims that he will have no charge for his vehicle registration fee. Is this possible? Use mathematics to justify your answer.

**Teacher Notes**

This problem is written to have students work with solving literal equations in a real-world context. Students should have had some practice with rearranging formulas prior to this task. Part A is designed to provide an entry point for all students and to have them work with the table and concrete numbers prior to attempting to rearrange or manipulate the equation. Some students will need the numeric structures of Part A to make sense of the problem and to manipulate the equation later. Part B moves students into rearranging the formula. It is important to note that this is a teaching task. So, let students attempt Part B with little assistance and provide scaffolding where necessary to individual students or groups. Students will solve the literal equation for the missing value in each of the tables. Part C will provide students a chance to grapple with the meaning of the literal equation conceptually. Although Part C can be answered by providing discrete values for a car that would make the claim true, it is important to push students to justify in the general case. This will help students begin to see the relationship of the variables and quantities to each other within the equation.

This task is written to be done with limited technology and with paper-and-pencil. However, there are several ways that this task could incorporate more resources and technology. The key is to keep the task focused on the essential understandings. Below are a few ways to incorporate more technology.
- Have students go to a website such as [www.kbb.com](www.kbb.com) to research different cars. Using the real-time values provided could make the task more relevant. The teacher would need to determine the specific criteria to be used. For example, you may decide to use suggested retail, excellent condition, and combined fuel economy.
- Have students use a program such as Excel to build the calculation tables so that any three of the four values can be entered and the fourth will be calculated. Students would still need to solve the literal equation in order to input the formula for the missing value cell.

| Common Core State Standards for Mathematical Content | Common Core State Standards for Mathematical Practice |
|---|---|
| A – CED.A.4 Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. *For example, rearrange Ohm's law V=IR to highlight resistance R.*<br><br>A – CED.A.3 Represent constraints by equations and interpret solutions as viable or nonviable options in a modeling context. *For example, represent inequalities describing nutritional and cost constraints on combinations of different foods.*<br><br>A – REI.B.3 Solve linear equations in one variable, including equations with coefficients represented by letters. | 1. Make sense of problems and persevere in solving them.<br>2. Reason abstractly and quantitatively.<br>3. Construct viable arguments and critique the reasoning of others.<br>4. Model with mathematics.<br>5. Use appropriate tools strategically.<br>6. Attend to precision.<br>7. Look for and make use of structure.<br>8. Look for and express regularity in repeated reasoning. |

**Essential Understandings**

- Equations and Formulas can be used to model real-world scenarios.
- Equations and formulas may be rearranged to highlight a quantity of interest by using the same reasoning as in solving equations.

**Explore Phase**

| Possible Solution Paths | Assessing and Advancing Questions |
|---|---|
| Part A.<br><br>Students will calculate the cost of the registration fee by substituting the known values into the formula.<br><br>$v = 10,000 \quad f = 22 \quad d = 1$<br>$\quad c = 0.01(10,000) - 1(22)$<br>$\quad c = 100 - 22$<br>$\quad c = \$78$ | Assessing Questions:<br>- Describe what each number in your formula represents. Where is the value of the vehicle? Where is the multiplier? Where is the fuel economy of the vehicle?<br>- How did you determine what the multiplier would be?<br>- Talk me through your calculations.<br>Advancing Questions: |

| | |
|---|---|
| v = 36,000    f = 42    d = 6<br>　c = 0.01(36,000) – 6(42)<br>　c = 360 – 252<br>　c = $108<br><br>v = 40,000    f = 11    d = 0.1<br>　c = 0.01(40,000) – 0.1(11)<br>　c = 400 – 1.1<br>　c = $398.90 | • What would happen if the value of the vehicle stayed the same, but the fuel efficiency increased?  Decreased? |

**Part B.**

Students will rearrange the formulas to highlight the quantity of interest for each table.  Some examples are below.

| Cost of fee calculation | Value of Vehicle | Category Multiplier | Fuel Economy |
|---|---|---|---|
| c | v | $d = -\dfrac{c - 0.01v}{f}$ | f |

c = 0.01v – df

c – 0.01v = 0.01v – df – 0.01v

c – 0.01v = 0.01v – 0.01v – df

c – 0.01v =  - df

$$\frac{c - 0.01v}{-f} = \frac{-df}{-f}$$

$$d = \frac{c}{-f} - \frac{0.01v}{-f}$$

$$d = -\frac{c}{f} + \frac{0.01v}{f} \qquad\qquad d = -\frac{c - 0.01v}{f}$$

or

| Cost of fee calculation | Value of Vehicle | Category Multiplier | Fuel Economy |
|---|---|---|---|
| c | v = 100(c + df) | d | f |

c = 0.01v – df

c + df = 0.01v – df + df

c + df = 0.01v

$$\frac{c + df}{0.01} = \frac{0.01v}{0.01}$$

$$v = \frac{c + df}{0.01}$$

v = 100(c + df)

Assessing Questions:
• Describe what each part of your equation represents.  What does the v represent? The c? The d? The f?
• Talk me through how you solved the problem.
• How does your solution relate to the scenario?
• Explain why dividing the quantity by 0.01 is the same as multiplying the quantity by 100.

Advancing Questions:
• How could you use your calculations in Part A to help solve this equation?
• How would you solve this equation if the variables had designated values instead of letters?  Show me.

| Cost of fee calculation | Value of Vehicle | Category Multiplier | Fuel Economy |
|:---:|:---:|:---:|:---:|
| c | v | d | $f = -\dfrac{c - 0.01v}{d}$ |

c = 0.01v – df

c – 0.01v = 0.01v – df – 0.01v

c – 0.01v = 0.01v – 0.01v – df

c – 0.01v = - df

$$\frac{c - 0.01v}{-d} = \frac{-df}{-d}$$

$$f = \frac{c - 0.01v}{-d}$$

$$f = \frac{c}{-d} - \frac{0.01v}{-d}$$

$$f = -\frac{c}{d} + \frac{0.01v}{d}$$     or          $$f = -\frac{c - 0.01v}{d}$$

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Part C.  Although students could find specific values for which David's claim is true, push students to use a literal equation to make generalizations about the claim.

Some students may try values for the equation to find ones that will satisfy the conditions.

An example:
 David's claim would be true for a $37,600 car that gets 47 mpg.
 c = 0.01(37600) – 8(47)
 c = 376 – 376
 c = 0

Some students may substitute a value of 0 for c and solve the equation.  Below are two examples.  Any of the variables could be isolated.  The key is that the student can articulate the relationship of the variables to each other and the context.

Some examples:
0.01v – df = 0
0.01v – df + df = 0 + df
0.01v = df

David's claim would be true when one hundredth of the value of the car is equal to the product of the multiplier and the fuel economy.  The car could not be in the 0-9.9 mpg category.

Or

0.01v = df

Assessing Questions:
- How did you find the values that made David's claim true?
- Why did you set the equation equal to zero?
- Talk me through how you solved the equation.
- How does your equation relate to the scenario?
- What restrictions would need to be placed on the equation to fit the scenario?

Advancing Questions:
- How could you use what you know about the equation to help find the values more efficiently?
- How could you solve this equation so that you could make a general statement about David's claim?

$$\frac{0.01v}{0.01} = \frac{df}{0.01}$$

v = 100df

David's claim would be true when the value of the car is one hundred times greater than the product of the multiplier and the fuel economy. The car could not be in the 0-9.9 mpg category.

| Possible Student Misconceptions | |
|---|---|
| 1. In Part A, students assign incorrect values to the variables which will create issues later in the problem. For example, students may assign the 22 mpg as $d$ instead of $f$. This will lead to confusion around how each variable relates to one other and the context later.<br>2. In Part B, students may make computational errors with distributing the negative sign. | Assessing Questions:<br>• Talk me through how you solved the problem.<br>• What values are represented in the table? Which variables do these represent?<br>Advancing Questions:<br>• How would assigning incorrect values to the variables affect the problem? |
| 3. In Part C, students may see their solution as the only solution and not make the generalization. They see guess-and-check as the only means to solve the problem. | Assessing Questions:<br>• Talk me through how you solved the problem.<br>• How do you know that your values prove David's claim?<br>• What does your solution represent?<br>• Are there more values for which David's claim would be true? How do you know?<br>Advancing Questions:<br>• How could you solve this equation so that you could make a general statement about David's claim? |
| **Entry/Extensions** | **Assessing and Advancing Questions** |
| If students can't get started…. | Assessing Questions:<br>• What is the question asking?<br>• What values are represented in the table? Which variables do these represent?<br>• Describe what each number in your equation represents. Where is the value of the vehicle? Where is the multitier? Where is the fuel economy?<br>• Talk me through your calculations.<br>Advancing Questions:<br>• How could you use your calculations in Part A to help solve this equation?<br>• Talk me through how you would find the |

| | vehicle registration fee for any car. |
|---|---|
| If students finish early…. | Assessing Questions:<br>• Show me how your solution relates to the scenario.<br>• Show me how your equations relate to the scenario.<br>• Why did you decide to use this equation to justify David's claim?<br>• How does your equation justify David's claim?<br>Advancing Questions:<br>• Does the city's equation promote a more environmentally conscious decision towards vehicle purchases? Explain.<br>• Is it possible to actually receive a refund or money back from the city?  Justify your answer mathematically. |

**Discuss/Analyze**

**Whole Group Questions**

Select and Sequence refers to when a teacher anticipates possible student strategies ahead of time and then selects and determines the order in which the students' math ideas/strategies will be shared during the whole group discussion.  The purpose of this is to determine which ideas will most likely leverage and advance student thinking about the core math idea(s) of the lesson.

During a whole group discussion, students are sharing their strategies that have been pre-selected and sequenced by the teacher.  Strategies to consider sharing in order to advance student thinking are:

- Methods of Calculation in Part A.
- Different forms of the Equations:  Share various equations and their simplified forms.
- Justifications of David's Claim:  Share various rationales justifying the claim including a pure numerical value justification and the various general justifications using the literal equations.

There are lots of rich discussions that need to occur during the Whole Group Share.  A discussion should occur around the relationships of the variables within the formula.  Another powerful discussion is around generalizable statements about David's claim.  For example, a hundredth of the value of the car would have to be equivalent to the product of the multiplier and the fuel economy.  And, the multiplier could never be zero.  These discussions will provide meaning and context to solving literal equations.  Questions to pose during the discussion:

- How did you calculate the cost of each vehicle registration fee?
- How did you use the calculations in Part A to solve the equations in Part B?
- How do the variables relate to each other in your equation?
- How does your equation relate to the scenario?  Where is each part of your equation in the scenario?
- What are some values for which David's claim would be true?  How did you determine those values?
- Is there more than one value for which David's claim would be true?  Justify.
- How could you manipulate the original equation so that you could make a general statement about David's claim?  What are some of the general statements?

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| Cost of fee calculation | Value of Vehicle | Category Multiplier | Fuel Economy |
|:---:|:---:|:---:|:---:|
| c | | d | f |

| Cost of fee calculation | Value of Vehicle | Category Multiplier | Fuel Economy |
|:---:|:---:|:---:|:---:|
| c | v | d | |

**C.** David just bought a car and claims that he will have no charge for his vehicle registration fee.  Is this possible? Use mathematics to justify your answer.