Task: Exploring Polynomials Task	Algebra II

Choose three distinct, positive integers. Call these numbers r, s, and t:

r = ______________________ s = _________________________ t = ____________________________

a) Suppose your values of r, s, and t are the zeroes of a third-degree polynomial whose leading coefficient is 1. Write down the factors of your polynomial, then write your polynomial in the form $P(x) = x^3 - A x^2 + B x - C$, where A, B, and C are whole numbers.

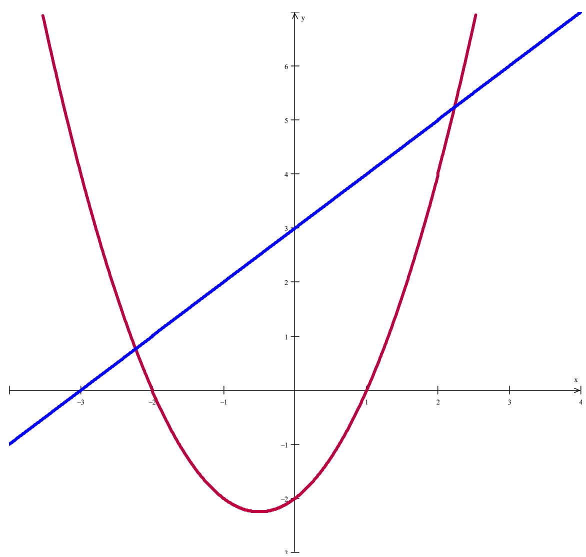
b) How does your choice of r, s, and t relate to the value of C in your polynomial in part (a)? Why?

c) How does your choice of r, s, and t relate to the value of A in your polynomial in part (a)? Why?

d) How does your choice of r, s, and t relate to the value of B in your polynomial in part (a)? Why?

e) In the graph below, the parabola represents the function f(x), the line represents the function g(x), and the marks on the axes are one unit apart. Use the graph to find an expression for H(x), where H(x) is the product of f(x) and g(x).

f) Your expression for H(x) should be written in the same format as your polynomial in part (a). Are the conclusions you reached in parts (b), (c), and (d) valid for your polynomial H(x), using the zeroes of f(x) as the values r and s and the zero of g(x) as the value of t? Explain your reasoning.



Teacher Notes:	
Students explore the connections between the zeroes of a polynomial and the coefficients of the polynomial. In this exploration, students are focusing on polynomials with integer coefficients and leading coefficient 1. Questions following the exploration should focus on what would happen if the polynomial does not have integer coefficients or if the leading coefficient is not 1. Teachers may want to spread the exploration over more than one day by using parts (a)-(d) as an initial exploration, then using parts (e) and (f) as a second exploration in order to make connections with graphing and polynomials.	
Common Core State Standards for Mathematical Content	**Common Core State Standards for Mathematical Practice**
(A-APR) Understand the relationship between zeros and factors of polynomials **(F-BF) Build a function that models a relationship between two quantities** 1. b. Combine standard function types using arithmetic operations. *For example, build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model.*	Mathematical Practices 1. Make sense of problems and persevere in solving them. 2. Reason abstractly and quantitatively. 3. Construct viable arguments and critique the reasoning of others. 4. Model with mathematics. 5. Use appropriate tools strategically. 6. Attend to precision. 7. Look for and make use of structure. 8. Look for and express regularity in repeated reasoning.
Essential Understandings	
• Functions can be combined by adding, subtracting, multiplying, dividing, and composing them. Functions sometimes have inverses. Functions can often be analyzed by viewing them as made from other functions. • Functions can be represented in multiple ways, including algebraic (symbolic), graphical, verbal, and tabular representations. Links among these different representations are important to studying relationships and change.	
Explore Phase	
Possible Solution Paths	**Assessing and Advancing Questions**
a) Students can choose their own values for r, s, and t. Factors should be of the form (x – r), (x – s), and (x – t). Students should then multiply their factors together. They may choose to multiply the factors in any order: (x – r) * (x – s) then multiply by (x – t)	a) Ask students what values they chose. How is a zero related to its corresponding factor? **Assessing questions:** How did you multiply the factors? Would it have made a difference if you multiplied in a different order? Why or why not? **Advancing questions:**

OR (x – s) * (x – t) then multiply by (x – r) OR (x – r) * (x – t) then multiply by (x – s) *Example: Suppose students choose r = 2, s = 3, and t = 5. Then* $(x - 2) * (x - 3) = x^2 - 5x + 6$ *and* $(x^2 - 5x + 6) * (x - 5) = x^3 - 10x^2 + 31x - 30.$	Have you tried concentrating on only two factors? How would you start with these two factors? Once you have multiplied two factors together, how do you multiply this by the third factor? What property are you using?
b) Students can use a "guess and check" approach using the values of r, s, and t OR Students can use their calculations from part (b) and track the value of C as the multiplication occurs OR Students can drop back to the quadratic (say, (x – r)*(x – s)) to determine how the constant is "built". In any case, students should determine that the value of C is the product of r, s, and t. *Example: In the example from part (a), C = 30 = 2 * 3 * 5*	b) **Assessing questions:** How did you find the expression for C? Will this always work? Why or why not? **Advancing questions:** What does C represent? (the constant term) If you only had two factors, how do the zeroes of the factors determine the constant term?
c) Students can use a "guess and check" approach using the values of r, s, and t OR	c) **Assessing questions:** How did you find the expression for A? Will this always work? Why or why not?

Students can use their calculations from part (b) and track the value of A as the multiplication occurs OR Students can drop back to the quadratic (say, (x – r)*(x – s)) to determine how the coefficient of x is “built”. In any case, students should determine that the value of A is the sum of r, s, and t. *Example: In the example in part (a), A = 10 = 2 + 3 + 5.*	**Advancing questions:** What does A represent? (the coefficient of x^2) If you only had two factors, how do the zeroes of the factors determine the coefficient of the x^2 term?
d) Students can use a “guess and check” approach using the values of r, s, and t OR Students can use their calculations from part (b) and track the value of A as the multiplication occurs. (Note: Dropping back to the quadratic (say, (x – r)*(x – s)) to determine how the coefficient is “built” will not work here since the quadratic does not have a term that is “built” in the same way.) In any case, students should determine that the value of B is given by taking r*s + r*t + s*t. (Note: This is most likely the hardest for students to determine. They may need more guidance here than on the other parts.) *Example: In the example from part (a), B = 31 = 2* 3 + 2*5 + 3*5.*	d) **Assessing questions:** How did you find the expression for B? Will this always work? Why or why not? **Advancing questions:** What does B represent? (the coefficient of x) When you found an expression for A, you used your zeroes “one at a time” and added. When you found an expression for C, you multiplied all three zeroes. What would happen if you only looked at two zeroes at a time?
e) Students should find that $f(x) = (x + 2) * (x – 1) = x^2 + x – 2$. They can either use the zeroes to create the factors then multiply these together or they can use the form $f(x) = a x^2 + b x + c$ and use values on the graph to determine the values of a, b, and c by solving a system of equations.	e) **Assessing questions:** How did you determine your polynomials for f(x) and g(x)? How did you find H(x)? **Advancing questions:**

Students should then find that $g(x) = x + 3$, most probably by using either the point-slope or the slope-intercept form. They should then multiply f(x) by g(x) using a process similar to the process they used in part (a). So: $H(x) = (x^2 + x - 2) * (x + 3) = x^3 + 4x^2 + x - 6$.	What is your polynomial associated with f(x)? With g(x)? How would (did) you find these? How is H(x) related to f(x) and g(x)? What do you need to do to find H(x)?
f) Students should check that their solutions to parts (b), (c), and (d) are valid for the polynomial H(x). Here, $A = -4 = -2 + 1 + (-3)$ $B = 1 = (-2 * 1) + (1 * -3) + (-2 * -3)$ $C = 6 = -2 * 1 * (-3)$	f) **Assessing questions:** What patterns did you find in parts (b), (c), and (d)? Do these patterns fit your polynomial H(x)? Why or why not? **Advancing questions:** What are your values of r, s, and t in H(x)? What pattern did you find in part (b)? Does this pattern fit your value of C in H(x)? Why or why not? (Ask similar questions as above for parts (c) and (d).)
Possible Student Misconceptions	
Sign errors can occur. For example, if r is a zero, then $(x - r)$ is the corresponding factor, but students may write $(x + r)$ as the factor.	**Questions:** If r is your zero, what should be your factor? Why is your factor $(x - r)$ instead of $(x + r)$?
Students may also make sign errors in multiplying the factors together.	**Questions:** Have you checked your multiplication? Pay careful attention to your signs as you multiply.
Students may make errors in identifying the values of A, B, and C. In both part (a) and part (f), the polynomial should have the form $x^3 - Ax^2 + Bx - C$, and students may not notice the subtraction in front of A and C. This would cause the calculations using r, s, and t to give the incorrect signs.	**Questions:** What is the value of A (resp. B, C)? Did you notice the "form" your polynomial should be in? Pay attention to the signs in front of A, B, and C—how does this affect your answers and your calculations?
Entry/Extensions	**Assessing and Advancing Questions**
If students can't get started….	Try a simpler case. What would happen if you only used r and s? How would you "build" your factors? How would you multiply these together?
If students finish early….	What would happen if you chose 4 zeroes? What form would your

	polynomial take if you multiplied the four factors? How do your zeroes relate to the coefficients of your fourth-degree polynomial? Would your patterns still work if the leading coefficient of your third-degree polynomial was not 1? Could you adjust your patterns in this case? Will your patterns work no matter what form your zeroes take (positive, negative, fractions, irrational numbers, or complex numbers)?

Discuss/Analyze

Whole Group Questions

Key understanding:

The coefficients of a third degree polynomial with a leading coefficient 1 are related to the zeroes of the polynomial.

Questions:

How are these coefficients related to the zeroes of the polynomial?

Does it matter what form the zeroes take (positive, negative, fractions, irrational numbers, or complex numbers)? Why or why not?

Why do these patterns rely on the leading coefficient being 1? How are your patterns affected if the leading coefficient is not 1?