Task: Third Degree Polynomial	**Algebra II**

The graph of a third degree polynomial f(x) has exactly two x-intercepts. The x-intercepts are -3 and 2.

a) You are given the table below. This table gives information about the polynomial f (x).

x	x < -3	x = -3	-3 < x < 2	x = 2	x > 2	3
f(x)	positive	0	negative	0	negative	-6

Use this information to come up with an equation for the polynomial function f(x). Use a graph to justify your answer.

b) John says the y-intercept for this function is -6. Find a way to show John if he is correct or incorrect.

Teacher Notes:

This is a good example of a function having a repeated factor.

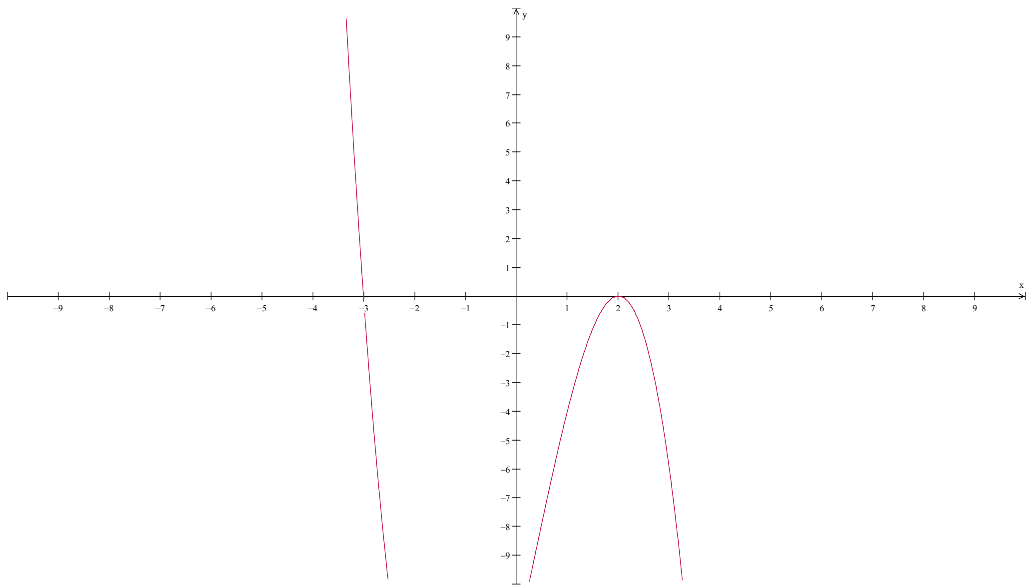
The chart given in this problem is an example of something that is seen in further mathematics courses.

Please note that the students will have to find the leading coefficient of -1.

Common Core State Standards for Mathematical Content	Common Core State Standards for Mathematical Practice
A-APR.3: Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial.	**1. Make sense of problems and persevere in solving them.** 2. Reason abstractly and quantitatively. **3. Construct viable arguments and critique the reasoning of others.** 4. Model with mathematics. 5. Use appropriate tools strategically. 6. Attend to precision. **7. Look for and make use of structure.** **8. Look for and express regularity in repeated reasoning.**

Essential Understandings

- Functions can be represented in multiple ways, including algebraic (symbolic), graphical, verbal, and tabular representations. Links among these different representations are important to studying relationships and change.

<table>
<tr><th colspan="2">Explore Phase</th></tr>
<tr><th>Possible Solution Paths</th><th>Assessing and Advancing Questions</th></tr>
<tr><td>
a)
Students could use the table to sketch a graph. They should also know that x-intercepts of the graph are also called zeros of the function. They use this information to come up with factors (x+3)(x-2).
Using information from the table they could try points on either side of -3 and 2 to see which is a repeated factor.
Since the graph has the same sign on either side of 2, (x-2) must be the repeated factor.

$f(x) = a(x+3)(x-2)^2$ using $f(3) = -6$

$-6 = a(6)(1)$

$-6 = 6a$

$a = -1$

$f(x) = -1(x+3)(x-2)^2$

$f(x) = -x^3+x^2+8x-12$</td><td>Assessing – How do you know that the sketch you made is correct?

Advancing – How can we use the graph to help ensure our equation is correct?</td></tr>
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b) John is incorrect. See below: $f(x) = a(x+3)(x-2)^2$ using $f(3) = -6$ $-6 = a(6)(1)$ $-6 = 6a$ $a = -1$ $f(x) = -1(x+3)(x-2)^2$ $f(x) = -x^3+x^2+8x-12$ y intercept = -12	Assessing – How can you tell if John is correct or incorrect? Advancing – Do you notice a relationship between the x-intercepts and the y-intercept?
Possible Student Misconceptions	
a) Some students may miss the repeated factor and come up with: $(x + 3)(x-2)$ $x^2 + x -6$	Assessing – What was the degree of your answer? What was the degree of the polynomial given in the instructions? Advancing – How can we use the table to help us understand what the equation might be?
b) Some students may miss that the leading coefficient is -1 and may come up with: $(x+3)(x-2)^2$ $x^3-x^2-8x+12$	Assessing – Are you sure your equation matches the information in the table? Advancing – Is there anything you notice in the table around x = 2 that could help you in future problems?
Entry/Extensions	**Assessing and Advancing Questions**
If students can't get started….	Assessing – What is the question asking us to do? Do you understand the term 3rd degree polynomial? Advancing – How can we use the x-intercepts to help us get started?

If students finish early….	Assessing – Why was the information in the table helpful? Advancing – Make a new table for this problem for all integers between -3 and 3. Follow the format in the table above.

Discuss/Analyze

Whole Group Questions

- How can the graph of a 3^{rd} degree polynomial have exactly 2 x-intercepts?
- Was the table helpful in this problem?
- What told you that the leading coefficient must be negative?
- Was sketching a graph helpful?
- How many ways was this function represented by the time you were done?
- Is there anything in the table or graph that told you anything about the factors?
- Is there another way to justify you answer other than a graph?
- What was the relationship in this problem between the x-intercepts and the y- intercept?
- How do you think John came up with his y-intercept?
- What told you that the y-intercept must be negative?