| **Task: Pool Patio Problem** | **Algebra I** |
|---|---|

A hotel is remodeling their grounds and plans to improve the area around a 20 foot by 40 foot rectangular pool. The owner wants to use a new product to resurface a rectangular concrete patio that completely borders and frames the pool. To save money, the owner has some of the resurfacing product left over from another project and plans to only use the leftover materials around the pool. The owner wants to use all of the leftover product which would resurface 1,909 square feet of patio.

A. If the owner wants the resurfaced rectangular patio to have a uniform width all the way around the pool, what would be the width of the resurfaced patio? Draw and label a picture to represent the scenario and show your work.

B. The owner has been able to purchase enough additional products to resurface twice as much area of the patio as before. His initial thought is that this will allow him to double the width that was found in Part A.
- Is the owner correct? Why or why not?
- If the owner is incorrect, what would be the width of the resurfaced patio if the original square footage was doubled (round to the nearest tenth)? Show your work.
- By rounding to the nearest tenth, did the owner use all of the resurfacing material? Explain.

**Teacher Notes**

This problem is written to have students work with quadratic equations in a real-world context. Part A is designed to have a resurfaced rectangular patio that completely borders the pool. The intent of the problem is to have the rectangular patio have a uniform width that completely frames the pool. It will be important to both see the model that students draw and have them make sense of the model before moving into any numeric calculations.

Part B uses the answer from Part A. In Part B, the answer may be incorrect based on when the student rounds the answer within the Quadratic Formula. If the student rounds at the final answer, the new width will be 19.3. If the student rounds to the nearest tenth after taking the square root and then again at the final answer, the new width will be 19.4. A width of 19.4 would be incorrect since it would use more resurfacing product than what the owner has available. The correct answer of 19.3 will not use all of the material.

| **Common Core State Standards for Mathematical Content** | **Common Core State Standards for Mathematical Practice** |
|---|---|
| A – CED.A.1 Create equations ~~and inequalities~~ in one variable and use them to solve problems. *Include equations arising from linear and quadratic functions, and simple rational and exponential functions.*<br><br>A – CED.A.3 Represent constraints by equations ~~or inequalities, and by systems of equations and/or inequalities~~, and interpret solutions as viable or nonviable options in a modeling context. *For example, represent inequalities describing nutritional and cost constraints on combinations of different* | 1. Make sense of problems and persevere in solving them.<br>2. Reason abstractly and quantitatively.<br>3. Construct viable arguments and critique the reasoning of others.<br>4. Model with mathematics.<br>5. Use appropriate tools strategically.<br>6. Attend to precision.<br>7. Look for and make use of structure. |

*foods.*

A – REI.B.4 Solve quadratic equations in one variable.

8. Look for and express regularity in repeated reasoning.

**Essential Understandings**

- Quadratic equations can be used to model real-world scenarios.
- Solutions to a quadratic equation must be considered viable based on the domain and context of the function.
- There are multiple methods to solve quadratic equations. The efficiency and accuracy of each method will vary based on the tools available and the solutions represented by the equation.

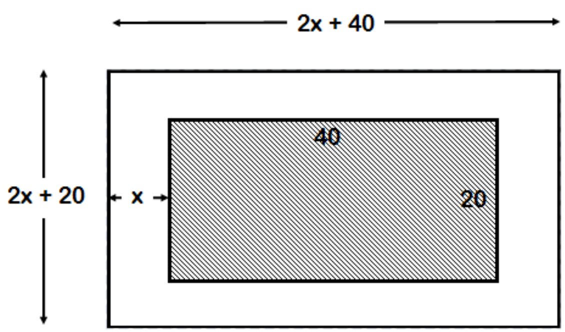
**Explore Phase**

**Possible Solution Paths**

**Part A.**

<u>Drawings:</u>

1. Students may take the total area and subtract the area of the pool.




$(2x + 20)(2x + 40) – (20 \bullet 40) = 4x^2 + 120x + 800 – 800 = 4x^2 + 120x$

$4x^2 + 120x = 1909$

**Assessing and Advancing Questions**

Assessing Questions:

- Describe your drawing. Where is the pool? Where is the patio? What does the x represent?
- Why did you decide to write this equation? Explain how each part of the equation relates to your drawing.

Advancing Questions:

- How could you work the problem without using the area of the pool?
- What is a different equation that could represent this scenario?
- How could you work the problem differently?
- Would your method work for any size pool? Why or why not?

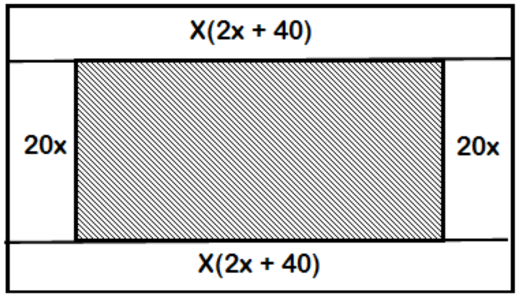
2. Students may partition the actual frame into individual rectangles. Below are several ways that a student may be thinking about the problem.

Assessing questions:

- Why did you decide to partition your drawing this way?
- What is the area of each partition?

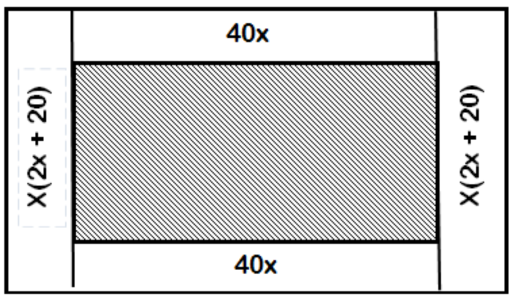
Advancing Questions:

- How could you partition the patio differently? Does this change the equation?
- How could you work the problem another way?




$2[x(2x + 40)] + 2(20x) = 2(2x^2 + 40x) + 40x = 4x^2 + 80x + 40x$
$= 4x^2 + 120x$

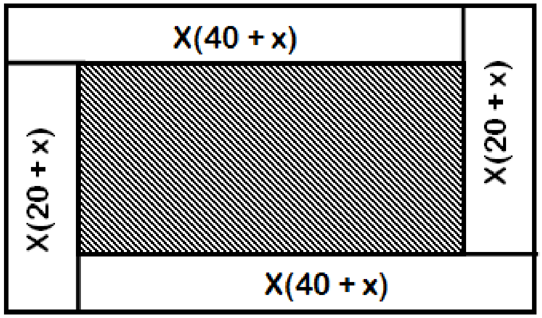
$4x^2 + 120x = 1909$




$2[x(2x + 20)] + 2(40x) = 2(2x^2 + 20x) + 80x = 4x^2 + 40x + 80x$
$= 4x^2 + 120x$

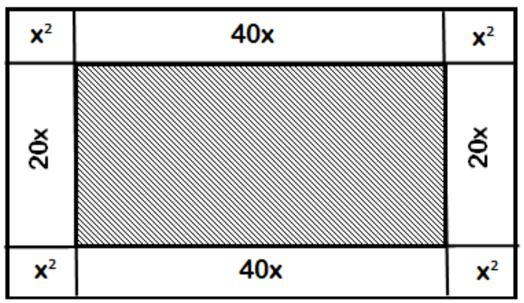
$4x^2 + 120x = 1909$

- Would your method work for any size pool? Why or why not?




$2[x(40 + x)] + 2[x(20 + x)] = 2(40x + x^2) + 2(20x + x^2)$
$= 80x + 2x^2 + 40x + 2x^2 = 4x^2 + 120x$

$4x^2 + 120x = 1909$




$4x^2 + 2(20x) + 2(40x) = 4x^2 + 40x + 80x = 4x^2 + 120x$

$4x^2 + 120x = 1909$

<u>Solving for the Unknown Quantity:</u>

1. Students may decide to use the Quadratic Formula to solve the problem.

$4x^2 + 120x = 1909$

$4x^2 + 120x – 1909 = 0$

Assessing Questions:

- Why did you decide to use the quadratic formula?
- What would be the values for *a*, *b*, and *c*?
- Why are there two potential solutions to this problem?
- Given the scenario, which value for *x* would make sense? Why?

Advancing Questions:

- How could you work the problem a different way?

$$x = \frac{-120 \pm \sqrt{(120)^2 - 4(4)(-1909)}}{2(4)}$$

$$x = \frac{-120 \pm \sqrt{14,400 + 30,544}}{8}$$

$$x = \frac{-120 \pm 212}{8}$$

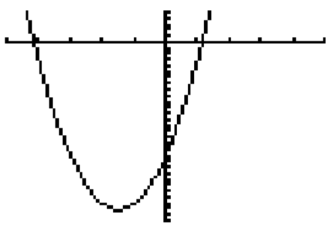
$$x = \frac{92}{8} \text{ or } x = \frac{-332}{8}$$

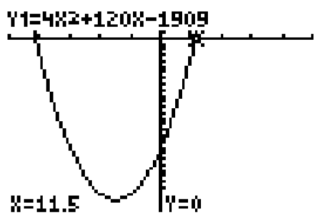
x = 11.5 or x = -41.5
x = 11.5 feet

- How would you know which method to use?

2. Students may use a calculator to solve the problem by graphing or using a table to find x- intercepts.

$4x^2 + 120x – 1909 = 0$






| X | $Y_1$ |
|---|---|
| 10 | -309 |
| 10.5 | -208 |
| 11 | -105 |
| 11.5 | 0 |
| 12 | 107 |
| 12.5 | 216 |
| 13 | 327 |

X=11.5

Assessing Questions:

- Why did you decide to use graphing to solve this problem? Or, why did you decide to use a table to solve this problem?
- Why did you decide to use this equation?
- How did you use your calculator to solve this problem? Show me the process.
- What does the x-value represent in the graph/table? What does the x-value represent in the scenario?
- Why are there two potential solutions to this problem?
- Given the scenario, which value of x would make sense? Why?

Advancing Questions:

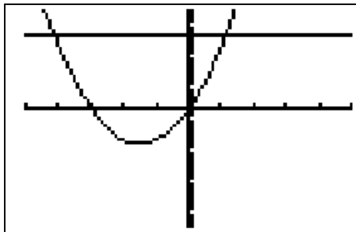
- What if you decided to use $4x^2 + 120x = 1909$?
  a. How would this change the graph/table?
  b. How are the graphs related?
- How could you work the problem another way?
- How would you know which method to use?

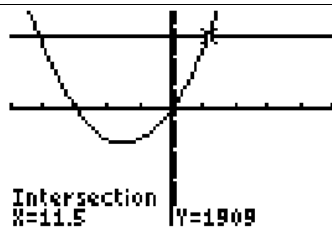
3. Students may use a calculator to solve the problem by graphing or using a table to find the x-values when the equation equals 1909.

$4x^2 + 120x = 1909$

Assessing Questions:

- Why did you decide to use graphing to solve this problem? Or, why did you decide to use a table to solve this problem?
- Why did you decide to use this equation?
- How did you use your calculator to solve this problem? Show me the process.






| X | Y1 | Y2 |
|---|---|---|
| 10 | 1600 | 1909 |
| 10.5 | 1701 | 1909 |
| 11 | 1804 | 1909 |
| 11.5 | 1909 | 1909 |
| 12 | 2016 | 1909 |
| 12.5 | 2125 | 1909 |
| 13 | 2236 | 1909 |

X=11.5

- What does the x-value represent in the graph/table? What does the x-value represent in the scenario?
- Why are there two potential solutions to this problem?
- Given the scenario, which value of x would make sense? Why?

Advancing Questions:

- What if you decided to use $4x^2 + 120x - 1909 = 0$?
  a. How would this change the graph/table?
  b. How are the graphs related?
- How could you work the problem another way?
- How would you know which method to use?

**Part B.**

1. Students may double the width in part A and substitute that value into their equation to show that the solution of 4876 does not equal the owner's claim of 3818.

$4(23^2) + 120(23) = 2116 + 2760 = 4876$
$4876 \neq 3818$

Then, students may solve $4x^2 + 120x = 3818$ using methods shown in Part A to conclude that $19.3 \neq 23$.

*Note that if using the Quadratic Formula, the result may be incorrect based on when the student rounds. If the student rounds at the final answer, the new width will be 19.3. If the student rounds to the nearest tenth after taking the square root and then again at the final answer, the width will be 19.4 which uses more product than available.*

Students then may substitute 19.3 into the expression $4X^2 + 120x$ to conclude that the owner is only resurfacing 3805.96 square feet of patio.

Assessing Questions:

- How did you determine the area of the new patio?
- How would the new picture be labeled given the context?
- How did you determine the width that you used?
- Why did you decide to use substitution to solve the question?
- By rounding to the nearest tenth, how much of the patio would be resurfaced?

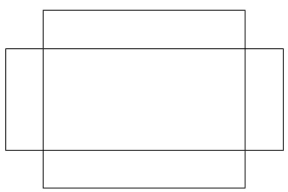
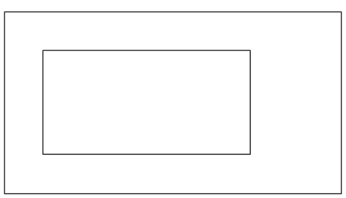
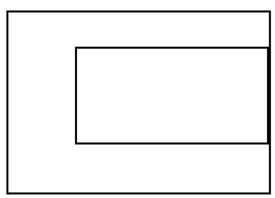
Advancing Questions:

- Why did the owner's claim not work?
- How does the original equation and your new equation compare?

2. Students may just move straight to solving $4x^2 + 120x = 3818$ using methods shown in Part A and conclude that $19.3 \neq 23$.

Assessing Questions:

- How did you determine the area of the new patio?
- How would the new picture be labeled given the context?

| | |
|---|---|
| *Note that if using the Quadratic Formula, the result may be incorrect based on when the student rounds. If the student rounds at the final answer, the new width will be 19.3. If the student rounds to the nearest tenth after taking the square root and then again at the final answer, the width will be 19.4 which uses more product than available.*<br><br>Students then may substitute 19.3 into the expression $4x^2 + 120x$ to conclude that the owner is only resurfacing 3805.96 square feet of patio. | • How did you determine the width that you used?<br>• Why did you decide to use substitution to solve the question?<br>• By rounding to the nearest tenth, how much of the patio would be resurfaced?<br><br>Advancing Questions:<br>• Why did the owner's claim not work?<br>• How does the original equation and your new equation compare? |
| **Possible Student Misconceptions** | |
| 1. Students may draw a visual representation that does not represent the scenario.<br><br>Examples:<br>The patio does not frame the pool.<br><br>The patio does not have a uniform width<br>  | Assessing Questions:<br>• What does it mean to completely frame something?<br>• Label your picture. Explain how each part of your drawing relates to the problem.<br><br>Advancing Questions:<br>• What type of function does your drawing represent?<br>• What would be the area of the four missing corners? What would the equation be if the corners were included?<br><br>Assessing Questions:<br>• What does it mean to have a uniform width?<br>• Label your picture. Explain how each part of your drawing relates to the problem.<br><br>Advancing Questions:<br>• Would the original drawing that you made have a different equation than your new drawing?<br>• What type of function does your drawing represent? |
| 2. Students may only include one side of the patio within the equation. Thus, the equation would result in a leading | Assessing questions:<br>• Describe each part of your drawing. What does the variable |

| | |
|---|---|
| coefficient of 2 instead of 4. | represent in your drawing?<br>• How does your picture relate to the equation that you have created?<br>• What would be the area of each of your partitions?<br><br>Advancing Questions:<br>• How does the area of each of your partitions relate to your equation?<br>• How do you know that you have used up all of the left over material? |
| 3. Students do not use all of the left over product in Part A. | Assessing Questions:<br>• How did you determine the amount of product that you used?<br>• How much of the left over product does the owner want to use?<br><br>Advancing Question:<br>• How do you know that you have used all of the left over material? |
| **Entry/Extensions** | **Assessing and Advancing Questions** |
| If students can't get started…. | Assessing Questions:<br>• What is the question asking?<br>• Draw and label a picture that represents the scenario. Where is the pool in your picture? Where is the patio?<br>• How did you determine what labels to use?<br><br>Advancing Questions:<br>• What would be the area of each partition of your drawing? How did you determine that area?<br>• How could you use each individual area to determine the area of the frame? |
| If students finish early…. | Assessing Questions:<br>• Show me how your equation relates to your drawing.<br>• Why did you decide to partition your drawing this way?<br>• Why did you decide to use this method to solve the problem? |

- Why are there two potential solutions to this problem?
- Given the scenario, which value of x would make sense?  Why?

Advancing Questions:

- Group or pair students who are finished – How are your methods similar and different? How are all of your equations related?
- What if the owner wanted to plant small gardens in the four square corners of the patio.  How would this change the problem scenario?  What type of function would this create?
- Is there a case where the owner's claim will hold true? Mathematically justify your answer.

**Discuss/Analyze**

**Whole Group Questions**

Select and Sequence refers to when a teacher anticipates possible student strategies ahead of time and then selects and determines the order in which the students' math ideas/strategies will be shared during the whole group discussion.  The purpose of this is to determine which ideas will most likely leverage and advance student thinking about the core math idea(s) of the lesson.

During a whole group discussion, students are sharing their strategies that have been pre-selected and sequenced by the teacher.  Strategies to consider sharing in order to advance student thinking are:

Drawing/Model:  Partitioned rectangles (multiple ways to partition) and Taking a part from the whole.
Solution: Quadratic Formula, Graphs, and Tables.

You may want to consider charting on the board or chart paper the different expanded equations throughout the Whole Group Share time.  The third question below asks students to compare the different equations.  Students should see that each equation simplifies to the same equation ($4x^2 + 120x = 1909$).

Questions to pose during the discussion:

- Why did you decide to partition your drawing this way?
- How does your equation relate to your drawing?
- How does each of the equations relate to each other?
- Why did you decide to use your method to solve the problem? What advantages/disadvantages did your method have?
- Your equation had two answers.  How did you determine which answer to use?
- Did the owner use all of the leftover material in Part B?  Justify.