

Tennessee Comprehensive Assessment Program / Mathematics

# TCAP/CRA PILOT 2012



## Task 1 : 48 Gumdrops Scoring Guide

### Task 1. 48 Gumdrops Task

Two children are sharing 48 gumdrops.

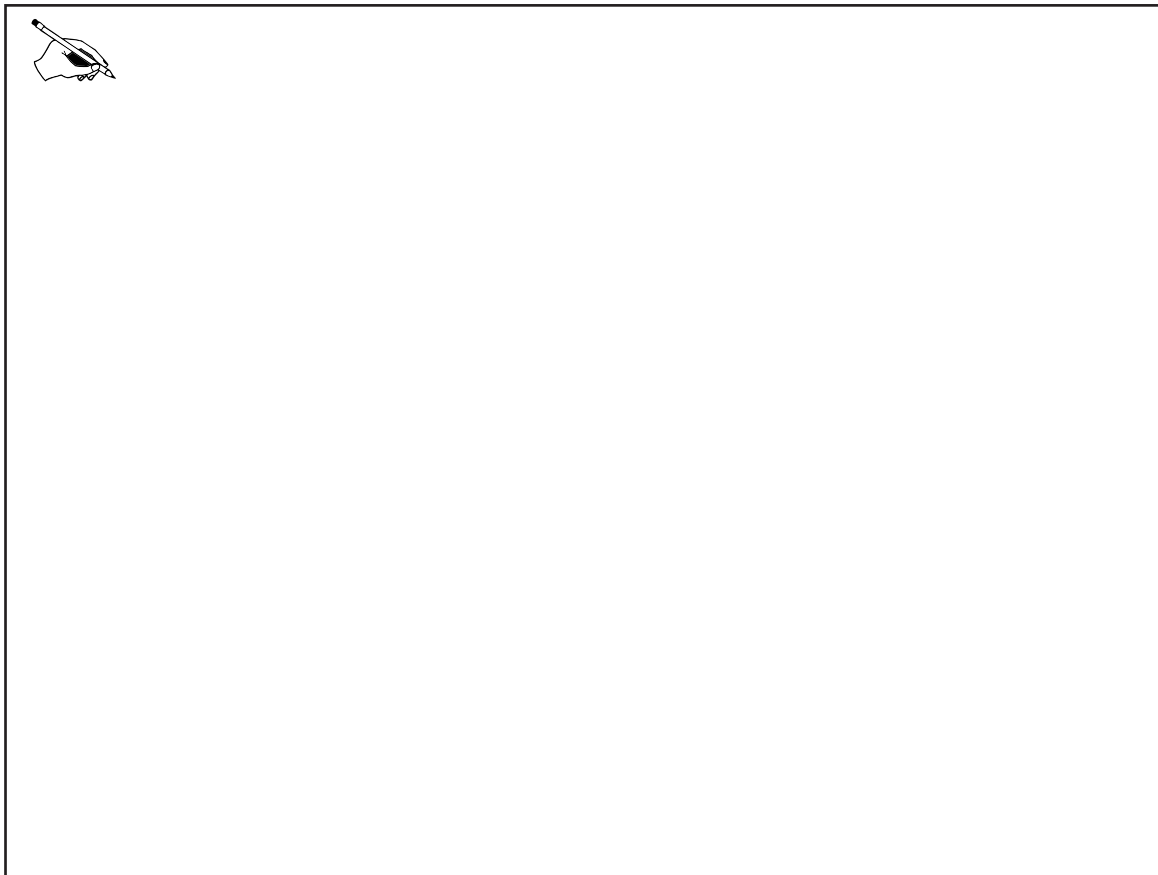
Jessica says, "I want  $\frac{2}{4}$  of the set of 48 gumdrops."

Samuel says, "I want  $\frac{2}{3}$  of the set of 48 gumdrops."

- a. Is it possible for Jessica and Samuel to each have the fraction of the gumdrops they want?

Answer: \_\_\_\_\_

- b. If you respond *yes*, use diagrams and equations to explain how you know they can each receive the share of gumdrops they want. If you respond *no*, use diagrams and equations to explain why the children cannot receive the number of gumdrops they each want.



## 1. 48 Gumdrops Task Scoring Guide

### The CCSS for Mathematical Content (1 point)

5.NF.2 The student uses correct mathematical reasoning to determine that Jessica and Samuel cannot each have the fraction they want, since  $\frac{2}{4} + \frac{2}{3}$  is greater than one whole. \_\_\_\_\_

OR

5.NF.4a The student finds the number of gumdrops when taking  $\frac{2}{4}$  of 48. \_\_\_\_\_

The student finds the number of gumdrops when taking  $\frac{2}{3}$  of 48.

**Total Content Points** \_\_\_\_\_

### The CCSS for Mathematical Practices (5 points)

MP1 The student determines and represents the correct shares of 48 gumdrops or determines that the sum of the fractions of gumdrops that each child will receive is greater than one whole set (the 48 gumdrops). \_\_\_\_\_

(MP1: Make sense of problems and persevere in solving them.)

MP2 The student writes either a multiplication or division equation, and labels the quantities appropriately, or recognizes the given amounts represent more than one whole. \_\_\_\_\_

(MP2: Reason abstractly and quantitatively.)

MP4 The student provides diagrams, equations and/or words that demonstrate reasoning related to the equal partitioning of the whole indicated by the denominator and the number of groups indicated by the numerator. The student may: \_\_\_\_\_

- divide 48 by 4 to get 12 and then add two groups of 12 or multiply 12 by 2.
- divide 48 by 3 to get 16 and then add two groups of 16 or multiply 16 by 2.

(MP4: Model with mathematics.)

MP6 Division or multiplication equations are accurate. Work may show: \_\_\_\_\_

- an accurate number of gumdrops is found for Jessica and Samuel.
- reasoning indicating that  $\frac{2}{3}$  is more than half and  $\frac{2}{4}$  is half; therefore, each child cannot receive his/her share of the gumdrops.

(MP6: Attend to precision.)

MP7 Work indicates that the whole can be partitioned into fractional parts based on the denominator and the numerator describes the share of the equal parts to be received. \_\_\_\_\_

(MP7: Look for and make use of structure.)

**Total Practice Points** \_\_\_\_\_

**Total Awarded Points** \_\_\_\_\_

## The CCSS for Mathematical Content Addressed in This Task

### Use equivalent fractions as a strategy to add and subtract fractions.

- 5.NF.2 Solve word problems involving addition and subtraction of fractions referring to the same whole, including cases of unlike denominators, e.g., by using visual fraction models or equations to represent the problem. Use benchmark fractions and number sense of fractions to estimate mentally and assess the reasonableness of answers. *For example, recognize an incorrect result  $2/5 + 1/2 = 3/7$ , by observing that  $3/7 < 1/2$ .*

### Apply and extend previous understandings of multiplication and division to multiply and divide fractions.

#### Apply and extend previous understandings of multiplication to multiply a fraction or whole number by a fraction.

- 5.NF.4a Interpret the product  $(a/b) \times q$  as a parts of a partition of  $q$  into  $b$  equal parts; equivalently, as the result of a sequence of operations  $a \times q \div b$ . *For example, use a visual fraction model to show  $(2/3) \times 4 = 8/3$ , and create a story context for this equation. Do the same with  $(2/3) \times (4/5) = 8/15$ . (In general,  $(a/b) \times (c/d) = ac/bd$ .)*

## The CCSS for Mathematical Practices\*

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

\*Gray text indicates Mathematical Practices that are not addressed in this task.

Students' responses to a mathematical task provide evidence of what they understand and are able to do in relation to the standards and practices. Across tasks, this cumulative evidence shows students' understanding and abilities within a domain. When students do not respond completely to all parts of a task, they provide insufficient evidence of their mathematical understanding and abilities and therefore do not fully demonstrate the expectations of the standards and practices aligned with that task.

Task 1. 48 Gumdrops Task

Two children are sharing 48 gumdrops.


Jessica says, "I want  $\frac{2}{4}$  of the set of 48 gumdrops."

Samuel says, "I want  $\frac{2}{3}$  of the set of 48 gumdrops."

- a. Is it possible for Jessica and Samuel to each have the fraction of the gumdrops they want?

Answer: no

- b. If you respond yes, use diagrams and equations to explain how you know they can each receive the share of gumdrops they want. If you respond no, use diagrams and equations to explain why the children cannot receive the number of gum drops they each want.

  $\frac{2}{4} = \frac{1}{2}$  If 1 kid wants a half of 48 gumdrops they will get 24 gumdrops. If the other kid wants  $\frac{2}{3}$  they would get 32 gumdrops because  $\frac{2}{3} \times 48 = 32$ . If  $24 + 32 = 56$  there will not be enough gumdrops.

Guide 1

Litho. 50002

Total Content Points: 1 (5.NF.4a)

Total Practice Points: 4 (MP2, MP4, MP6, MP7)

This student has clearly determined the number of gumdrops each child wants (5.NF.4a) through the use of a multiplication equation as well as by simplifying  $\frac{2}{4}$  to  $\frac{1}{2}$  (MP4).

The correct multiplication of the fractions shows an understanding of structure (MP7). The work shown is accurate (MP6) and the student has clearly contextualized the answer by pointing out that there will not be enough gumdrops for both children (MP2). The student has not drawn a diagram, as required by the task, so not all aspects of the task have been successfully addressed, which demonstrates a lack of persevering in the task (no credit for MP1).

Total Awarded Points: 5 out of 6

Task 1. 48 Gumdrops Task

Two children are sharing 48 gumdrops.

$$\frac{6}{12} \times \frac{4}{4} = \frac{24}{48}$$

$$\frac{8}{12} + \frac{6}{12} = \frac{14}{12}$$

$$\frac{8}{12} \times \frac{4}{4} = \frac{32}{48}$$

Jessica says, "I want  $\frac{2}{4}$  of the set of 48 gumdrops."  
 $= \frac{14}{12}$

Samuel says, "I want  $\frac{2}{3}$  of the set of 48 gumdrops."

- a. Is it possible for Jessica and Samuel to each have the fraction of the gumdrops they want?

Answer: no

- b. If you respond yes, use diagrams and equations to explain how you know they can each receive the share of gumdrops they want. If you respond no, use diagrams and equations to explain why the children cannot receive the number of gum drops they each want.

$\frac{8}{12} + \frac{6}{12} = \frac{14}{12}$ 
 $\frac{32}{48} + \frac{24}{48} = \frac{56}{48}$

Samuel

= 32

= 16



Guide 2

Litho. 50145

Total Content Points: 1 (5.NF.4a)

Total Practice Points: 3 (MP1, MP4, MP7)

This response clearly indicates the number of gumdrops desired by each student (5.NF.4a). The multiplication equations in the upper left are correctly performed and accurately model the situation presented (MP4), but there is no clearly-stated connection between the fractions in the work and the fractions of the set of gumdrops given in the task. The student could have been more precise in her explanation (no credit for MP6). Both the multiplication shown and the diagram demonstrate an understanding of the structure of fractions (MP7). The student has addressed all parts of the task (MP1). The lack of a clear and direct comparison of the total number of gumdrops to the number desired shows that the student could have more clearly contextualized her work (no credit for MP2).

Total Awarded Points: 4 out of 6

**Task 1. 48 Gumdrops Task**

Two children are sharing 48 gumdrops.

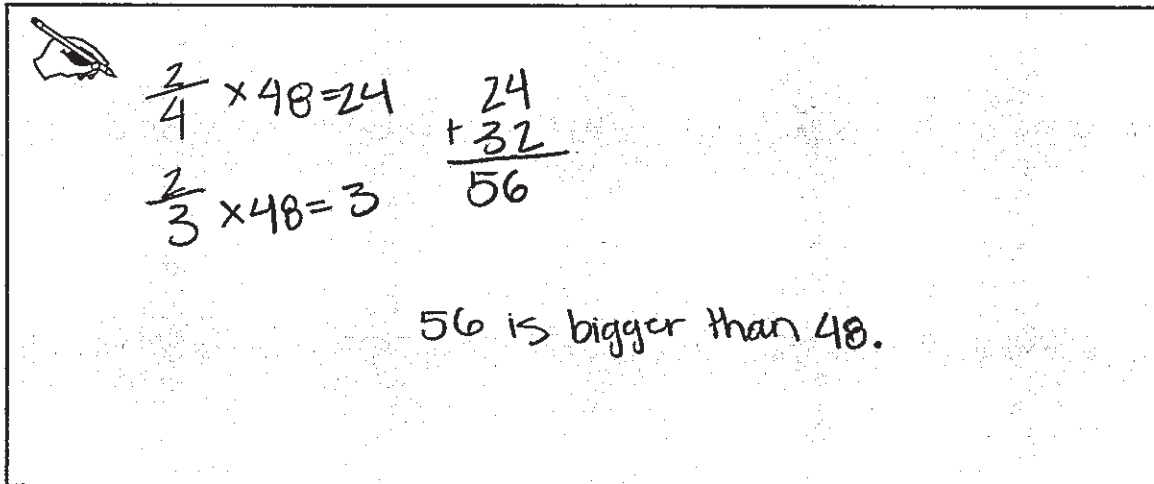
Jessica says, "I want  $\frac{2}{4}$  of the set of 48 gumdrops."

Samuel says, "I want  $\frac{2}{3}$  of the set of 48 gumdrops."

- a. Is it possible for Jessica and Samuel to each have the fraction of the gumdrops they want?

Answer: NO

- b. If you respond *yes*, use diagrams and equations to explain how you know they can each receive the share of gumdrops they want. If you respond *no*, use diagrams and equations to explain why the children cannot receive the number of gum drops they each want.



$\frac{2}{4} \times 48 = 24$   
 $\frac{2}{3} \times 48 = 32$   

$$\begin{array}{r} 24 \\ + 32 \\ \hline 56 \end{array}$$
  
 56 is bigger than 48.

Guide 3

Litho. 50228

Total Content Points: 1 (5.NF.4a)

Total Practice Points: 3 (MP2, MP4, MP7)

This response clearly demonstrates that the student has found the correct number of gumdrops that each child wants (5.NF.4a). Although one of the answers given is incomplete and shows a lack of precision (no credit for MP6), the context shows that the student understands the process of multiplying whole numbers by fractions (MP7). The equations given correctly model the situation (MP4), and the student has contextualized the equations by noting that the total number desired is greater than the whole set of gumdrops (MP2). However, because the task requires that the students provide diagrams, the student cannot be said to have persevered and attended to all parts of the task (no credit for MP1).

Total Awarded Points: 4 out of 6

Task 1. 48 Gumdrops Task

Guide 4

Two children are sharing 48 gumdrops.

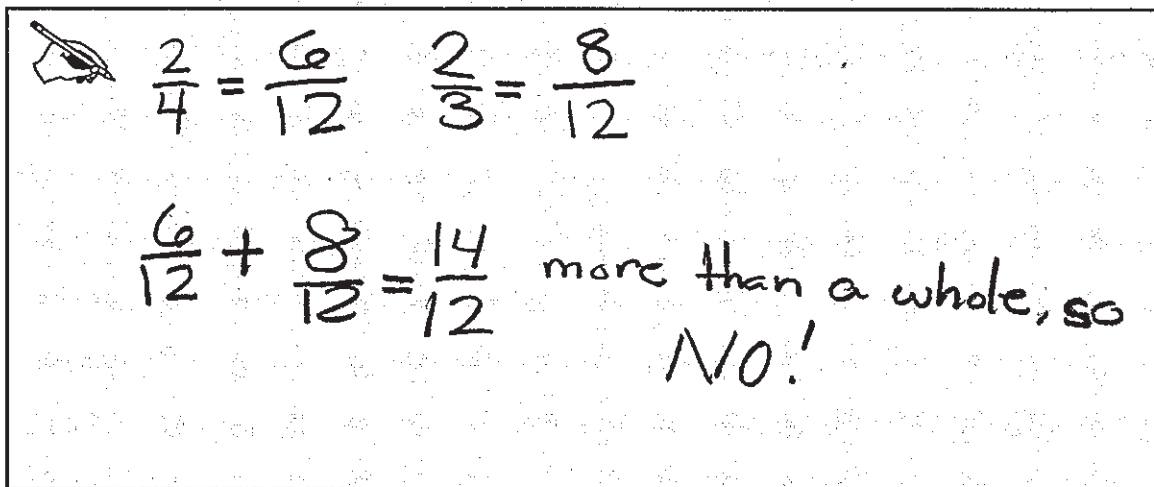
Jessica says, "I want  $\frac{2}{4}$  of the set of 48 gumdrops."

Samuel says, "I want  $\frac{2}{3}$  of the set of 48 gumdrops."

- a. Is it possible for Jessica and Samuel to each have the fraction of the gumdrops they want?

Answer: no

- b. If you respond yes, use diagrams and equations to explain how you know they can each receive the share of gumdrops they want. If you respond *no*, use diagrams and equations to explain why the children cannot receive the number of gum drops they each want.



Handwritten work showing the conversion of fractions and their sum:

$$\frac{2}{4} = \frac{6}{12} \quad \frac{2}{3} = \frac{8}{12}$$
$$\frac{6}{12} + \frac{8}{12} = \frac{14}{12} \text{ more than a whole, so NO!}$$

Guide 4

Litho. 50154

Total Content Points: 1 (5.NF.2)

Total Practice points: 3 (MP2, MP6, MP7)

This student has shown work indicating that the children will not each be able to have the candy they want, since  $\frac{2}{3}$  plus  $\frac{2}{4}$  is greater than one whole (5.NF.2). Lacking multiplication equations or a diagram, the student has not acceptably modeled the overall set of gumdrops (no credit for MP4), nor has she addressed all parts of the task (no credit for MP1). Finding common denominators for the two fractions shows reasoning related to the multiplicative structure of fractions (MP7). The student finds common denominators for the two fractions, and then she compares the total of the fractions desired to one whole, showing both quantitative and abstract thinking (MP2). The student uses the given fractions to show that Jessica and Samuel cannot both have the number of gumdrops that they want, based on appropriate reasoning involving adding  $\frac{2}{4}$  to  $\frac{2}{3}$  and getting a sum greater than one whole (MP6).

Total Awarded Points: 4 out of 6

Task 1. 48 Gumdrops Task

Two children are sharing 48 gumdrops.

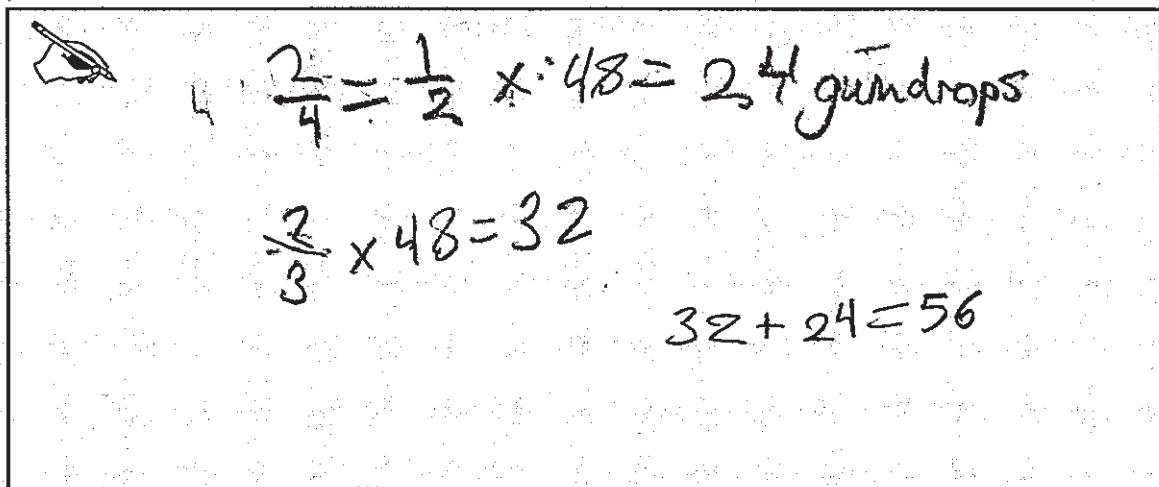
Jessica says, "I want  $\frac{2}{4}$  of the set of 48 gumdrops."

Samuel says, "I want  $\frac{2}{3}$  of the set of 48 gumdrops."

- a. Is it possible for Jessica and Samuel to each have the fraction of the gumdrops they want?

Answer: no

- b. If you respond yes, use diagrams and equations to explain how you know they can each receive the share of gumdrops they want. If you respond no, use diagrams and equations to explain why the children cannot receive the number of gum drops they each want.



Handwritten work showing calculations:

$\frac{2}{4} = \frac{1}{2} \times 48 = 24$  gumdrops

$\frac{2}{3} \times 48 = 32$

$32 + 24 = 56$

Guide 5

Litho. 50118

Total Content Points: 1 (5.NF.4a)

Total Practice Points: 3 (MP2, MP4, MP7)

This student correctly finds the number of gumdrops each student wants (5.NF.4a). The student labels the first value found, and connects the total number of gumdrops the children want to the answer of "no" in part a (MP2). Although the student presents some correct multiplication that shows an understanding of the structure of fractions and models the situation (MP4, MP7), there is an error in notation that shows a lack of concern for precision when he incorrectly states " $\frac{2}{4} = \frac{1}{2} \times 48 = 24$  gumdrops" (no credit for MP6). Also, the student has not addressed all parts of the task, as there is no diagram (no credit for MP1).

Total Awarded Points: 4 out of 6

Task 1. 48 Gumdrops Task

Guide 6

Two children are sharing 48 gumdrops.

Jessica says, "I want  $\frac{2}{4}$  of the set of 48 gumdrops."

Samuel says, "I want  $\frac{2}{3}$  of the set of 48 gumdrops."

- a. Is it possible for Jessica and Samuel to each have the fraction of the gumdrops they want?

Answer: Yes

- b. If you respond yes, use diagrams and equations to explain how you know they can each receive the share of gumdrops they want. If you respond *no*, use diagrams and equations to explain why the children cannot receive the number of gum drops they each want.

Handwritten equations showing calculations for the gumdrops task:

$$\frac{48}{1} \times \frac{2}{4} = 24$$
$$\frac{48}{1} \times \frac{2}{3} = 32$$



Guide 6

Litho. 50050

Total Content Points: 1 (5.NF.4a)

Total Practice Points: 2 (MP4, MP7)

Although this student has correctly set up multiplication equations that correctly model the situation given in the task (MP4), and successfully performed the calculations (MP7) to find the number of gumdrops each child wants (5.NF.4a), the response demonstrates an inability to contextualize the given calculations. As the student has answered "Yes" to part a, the student has not successfully completed all parts of the task (no credit for MP1). The two products are never added, which also indicates that the student is unable to contextualize the given functions (no credit for MP2). The overall meaning of the numbers given is unclear, which shows a lack of precision in the response (no credit for MP6).

Total Awarded Points: 3 out of 6

**Task 1. 48 Gumdrops Task**

Two children are sharing 48 gumdrops.

Jessica says, "I want  $\frac{2}{4}$  of the set of 48 gumdrops."

Samuel says, "I want  $\frac{2}{3}$  of the set of 48 gumdrops."

- a. Is it possible for Jessica and Samuel to each have the fraction of the gumdrops they want?

Answer: No

- b. If you respond yes, use diagrams and equations to explain how you know they can each receive the share of gumdrops they want. If you respond *no*, use diagrams and equations to explain why the children cannot receive the number of gum drops they each want.

$\triangle = \text{gumdrop}$

Diagram showing 48 gumdrops arranged in two rows of 24. A horizontal line is drawn between the rows. Above the line, the fraction  $\frac{2}{3}$  is written. Below the line, the fraction  $\frac{1}{3}$  is written. To the right of the diagram, the calculation  $\frac{2}{3}$  of 48 = 32 is shown, followed by  $32 - 48 = -16$ .

Total Content Points: 0

Total Practice Points: 3 (MP4, MP6, MP7)

Although this student has correctly modeled the total gumdrops and the number that Samuel wants, there is no attempt to model or calculate the number that Jessica wants (no credit for 5.NF.4a). There is no clear attempt to display or model the fraction of gumdrops that Jessica wants (no credit for 5.NF.2). The student is able to use her model of the total number of gumdrops and the number that Samuel wants to demonstrate that there will not be enough for both children (MP4). Although the response does not answer all parts of the task (no credit for MP1), the given diagram clearly shows the gumdrops divided into thirds, with Samuel's 32 taking up two groups (MP7), which is clearly more than half (MP6). The student does not perform any multiplication or division equations and does not attempt to compare the amounts Samuel and Jessica want to one whole (no credit for MP2).

Total Awarded Points: 3 out of 6

Task 1. 48 Gumdrops Task

Two children are sharing 48 gumdrops.

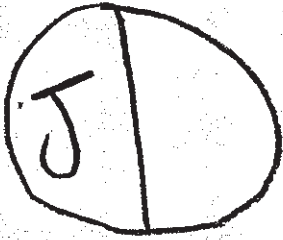
Jessica says, "I want  $\frac{2}{4}$  of the set of 48 gumdrops."

Samuel says, "I want  $\frac{2}{3}$  of the set of 48 gumdrops."

- a. Is it possible for Jessica and Samuel to each have the fraction of the gumdrops they want?

Answer: NO

- b. If you respond yes, use diagrams and equations to explain how you know they can each receive the share of gumdrops they want. If you respond no, use diagrams and equations to explain why the children cannot receive the number of gum drops they each want.

 Well  $\frac{2}{4}$  is half well  $\frac{2}{3}$  is a little more than half. Samuel can't have his  $\frac{2}{3}$  to share.

Guide 8

Litho. 50081

Total Content Points: 1 (5.NF.2)

Total Practice Points: 2 (MP2, MP6)

This student does not find the total number of gumdrops each student wants, but he does demonstrate reasoning that uses some understanding of fractions to explain why Jessica and Samuel cannot both have the number of gumdrops that they want (5.NF.2). While the student does show some understanding that the two children cannot both get the number of gumdrops they want because  $\frac{2}{3}$  is greater than  $\frac{2}{4}$  (MP6), the modeling he uses to demonstrate this is unclear and ineffective (no credit for MP4). There are no equations presented, so the student has not attended to all parts of the task (no credit for MP1). Although he has not performed any equations, the student recognizes that the given amounts are greater than one whole (MP2). The recognition that  $\frac{2}{4}$  is equivalent to  $\frac{1}{2}$  is basic knowledge, and is not enough to clearly demonstrate that the student understands the structure of fractions (no credit for MP7).

Total Awarded Points: 3 out of 6

Task 1. 48 Gumdrops Task

Two children are sharing 48 gumdrops.

Jessica says, "I want  $\frac{2}{4}$  of the set of 48 gumdrops."

Samuel says, "I want  $\frac{2}{3}$  of the set of 48 gumdrops."

- a. Is it possible for Jessica and Samuel to each have the fraction of the gumdrops they want?

Answer: Yes

- b. If you respond yes, use diagrams and equations to explain how you know they can each receive the share of gumdrops they want. If you respond *no*, use diagrams and equations to explain why the children cannot receive the number of gum drops they each want.

Handwritten work showing three calculations:

- 1.  $48 \times \frac{2}{4} = 24$
- 2.  $48 \times \frac{2}{3} = 32$
- 3.  $48 \div 2 = 24$

Guide 9

Litho. 50338

Total Content Points: 1 (5.NF.4a)

Total Practice Points: 1 (MP7)

Although this student has found the correct number of gumdrops each child wants (5.NF.4a) through correct multiplication (MP7), there is a lack of labeling and no clear connection of the results to the total set of gumdrops, or to the answer in part a. These weaknesses demonstrate a lack of contextualization of the multiplication expressions (no credit for MP2) and a lack of precision in communication (no credit for MP6). There are no diagrams or equations given (no credit for MP4), just expressions, so the student has not successfully attended to all parts of the task (no credit for MP1).

Total Awarded Points: 2 out of 6

Two children are sharing 48 gumdrops.

Jessica says, "I want  $\frac{2}{4}$  of the set of 48 gumdrops."

Samuel says, "I want  $\frac{2}{3}$  of the set of 48 gumdrops."

- a. Is it possible for Jessica and Samuel to each have the fraction of the gumdrops they want?

Answer: No

- b. If you respond yes, use diagrams and equations to explain how you know they can each receive the share of gumdrops they want. If you respond *no*, use diagrams and equations to explain why the children cannot receive the number of gum drops they each want.

When Jessica said she wanted  $\frac{2}{4}$  of the gumdrops, she was asking for 24 gumdrops. When Samuel said he wanted  $\frac{2}{3}$  of the gumdrops, she was asking for 32 gumdrops.



Guide 10

Litho. 50258

Total Content Points: 1 (5.NF.4a)

Total Practice Points: 0

This student has found the correct number of gumdrops each student wants (5.NF.4a), but the lack of equations, diagrams, and explanations makes it unclear how he has arrived at those numbers (no credit for MP4). With no work shown or explained, there is no way to adequately judge whether the student understands the multiplicative structure of fractions (no credit for MP7), and no clear attempt to explain the connection between parts a and b of the task, showing a lack of precision (no credit for MP6) and of understanding of the context (no credit for MP2). The student has not attended to all parts of the task (no credit for MP1).

Total Awarded Points: 1 out of 6

Task 1. 48 Gumdrops Task

Two children are sharing 48 gumdrops.


Jessica says, "I want  $\frac{2}{4}$  of the set of 48 gumdrops."

Samuel says, "I want  $\frac{2}{3}$  of the set of 48 gumdrops."

- a. Is it possible for Jessica and Samuel to each have the fraction of the gumdrops they want?

Answer:  ~~$\frac{2}{4} + \frac{2}{3} = 1\frac{1}{6}$~~   $1\frac{1}{6} \div 2 = \frac{7}{12}$

- b. If you respond yes, use diagrams and equations to explain how you know they can each receive the share of gumdrops they want. If you respond *no*, use diagrams and equations to explain why the children cannot receive the number of gum drops they each want.

 I think Jessica gets  $\frac{7}{12}$  of gumdrops, and Samuel gets  $\frac{7}{12}$  too. Because I did  $\frac{2}{4} + \frac{2}{3}$  and got  $1\frac{1}{6}$ . But I divid  $1\frac{1}{6} \div 2 =$  and I got  $\frac{7}{12}$ .

Total Content Points: 0

Total Practice Points: 0

This student has not found the correct number of gumdrops that each child wants (no credit for 5.NF.4a), nor has he clearly identified that the total amount Jessica and Samuel want is greater than the whole set (no credit for 5.NF.2). The student demonstrates an inability to understand the correct approaches to this task (no credit for MP1). Although there is some knowledge of arithmetic demonstrated, the student's work shows no understanding of how to contextualize the numbers he is adding (no credit for MP2), and his explanation lacks precision (no credit for MP6). The student's modeling is incorrect for the task (no credit for MP4), and there is not enough work shown to indicate that he understands the multiplicative nature of fractions (no credit for MP7).

Total Awarded Points: 0 out of 6