

Tennessee Comprehensive Assessment Program / Mathematics

# TCAP/CRA PILOT 2012



## Task 3 : 24 Cookies Scoring Guide

### Task 3. 24 Cookies Task

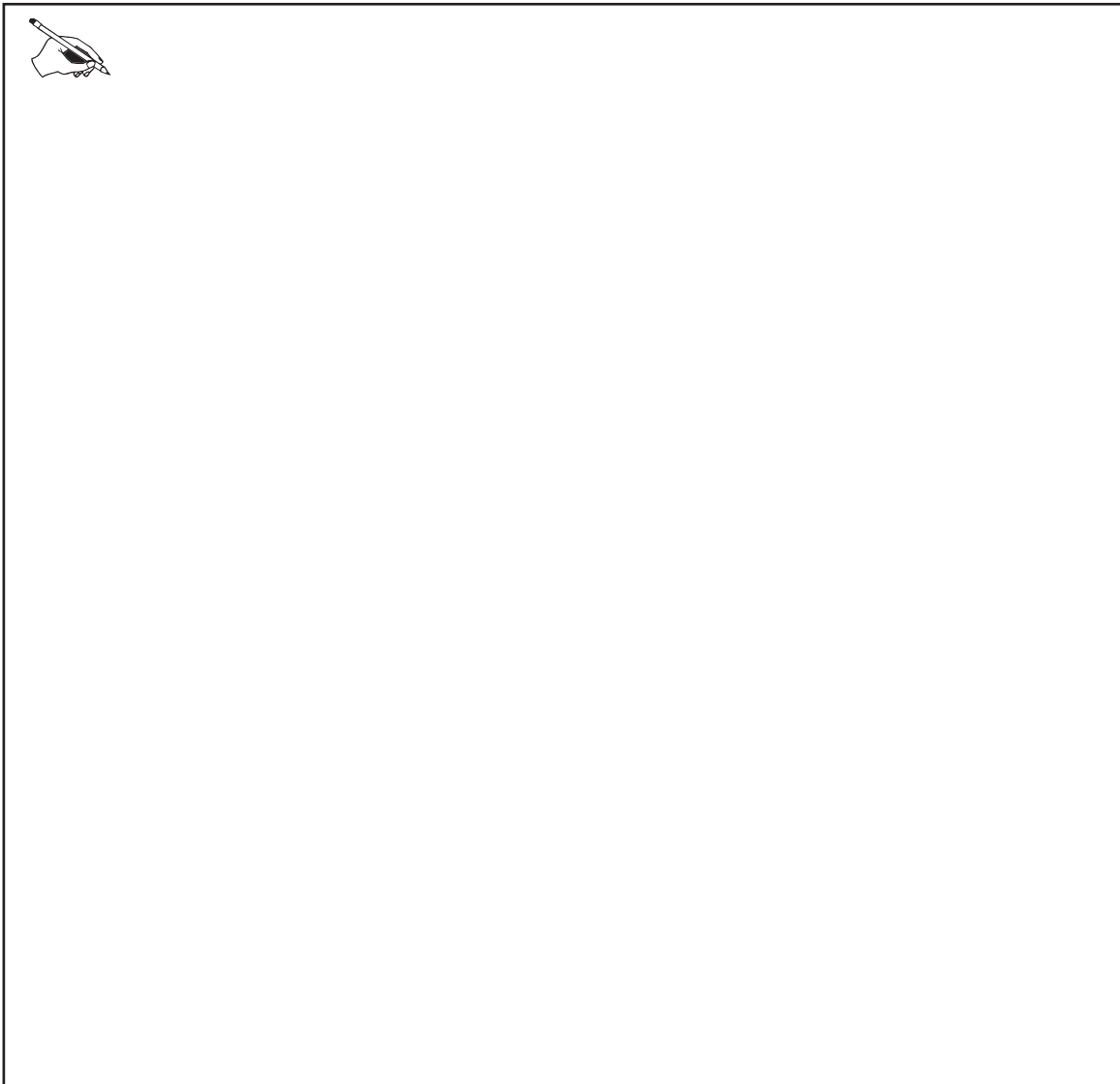
Three children are sharing 24 cookies.

Maria says, "I want  $\frac{1}{3}$  of the set of 24 cookies."

Ted says, "I want  $\frac{1}{2}$  of the set of 24 cookies."

Monica says, "I want  $\frac{1}{4}$  of the set of 24 cookies."

Use diagrams and equations to explain if it is possible for Maria, Ted and Monica to each have the fraction of the cookies they want. Write about the relationship between the 24 cookies that are available and the fraction of cookies each student wants.



### 3. 24 Cookies Task Scoring Guide

#### The CCSS for Mathematical Content (1 point)

4.NF.4c Student recognizes that the denominator of a fraction determines the number of equal groups that 24 will be divided into. In this problem, 24 is divided into 3 groups, 2 groups, or 4 groups, respectively. \_\_\_\_\_

OR

4.NF.3a Student recognizes that all three children cannot get the fraction of cookies that they want because the sum of the fractions desired is greater than one whole. \_\_\_\_\_

**Total Content Points** \_\_\_\_\_

#### The CCSS for Mathematical Practices (5 points)

MP1 Student explains the share that each student will receive, recognizes that the quantity of cookies is not sufficient, and explains why. Attends to all aspects of the task. \_\_\_\_\_

(MP1: Make sense of problems and persevere in solving them.)

MP2 Student writes equations and labels the amounts appropriately. \_\_\_\_\_

(MP2: Reason abstractly and quantitatively.)

MP4 The model shows an understanding of equal groups. \_\_\_\_\_

- Student diagrams show 24 is partitioned into 3 (or 2 or 4, respectively).

- The fraction notation indicates that  $24 \div 3$  (or 2 or 4, respectively).

(MP4: Model with mathematics.)

MP6 Accurate and precise work is shown. \_\_\_\_\_

- Identifies 24 as the whole number of cookies.

- Identifies 6, 8 and 12 as the shares of cookies.

- Identifies the cookies needed by the students as 26 and the total available as 24, and states that there are not enough cookies.

- Reasoning indicates that the sum of fractions that the children want equal more than one whole set (the 24 cookies).

(MP6: Attend to precision.)

MP7 Student's work indicates that the student recognizes that multiplication of a whole number by a fraction means that the whole number is partitioned into equal groups based on the size of the denominator. \_\_\_\_\_

(MP7: Look for and make use of structure.)

**Total Practice Points** \_\_\_\_\_

**Total Awarded Points** \_\_\_\_\_

## The CCSS for Mathematical Content Addressed in This Task

**Build fractions from unit fractions by applying and extending previous understandings of operations on whole numbers.**

**Apply and extend previous understandings of multiplication to multiply a fraction by a whole number.**

4.NF.4c Solve word problems involving multiplication of a fraction by a whole number, e.g., by using visual fraction models and equations to represent the problem. *For example, if each person at a party will eat  $\frac{3}{8}$  of a pound of roast beef, and there will be 5 people at the party, how many pounds of roast beef will be needed? Between what two whole numbers does your answer lie?*

**Understand a fraction  $\frac{a}{b}$  with  $a > 1$  as a sum of fractions  $\frac{1}{b}$ .**

4.NF.3a Understand addition and subtraction of fractions as joining and separating parts referring to the same whole.

## The CCSS for Mathematical Practices\*

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

\*Gray text indicates Mathematical Practices not addressed in this task.

Students' responses to a mathematical task provide evidence of what they understand and are able to do in relation to the standards and practices. Across tasks, this cumulative evidence shows students' understanding and abilities within a domain. When students do not respond completely to all parts of a task, they provide insufficient evidence of their mathematical understanding and abilities and therefore do not fully demonstrate the expectations of the standards and practices aligned with that task.

**Task 3. 24 Cookies Task**

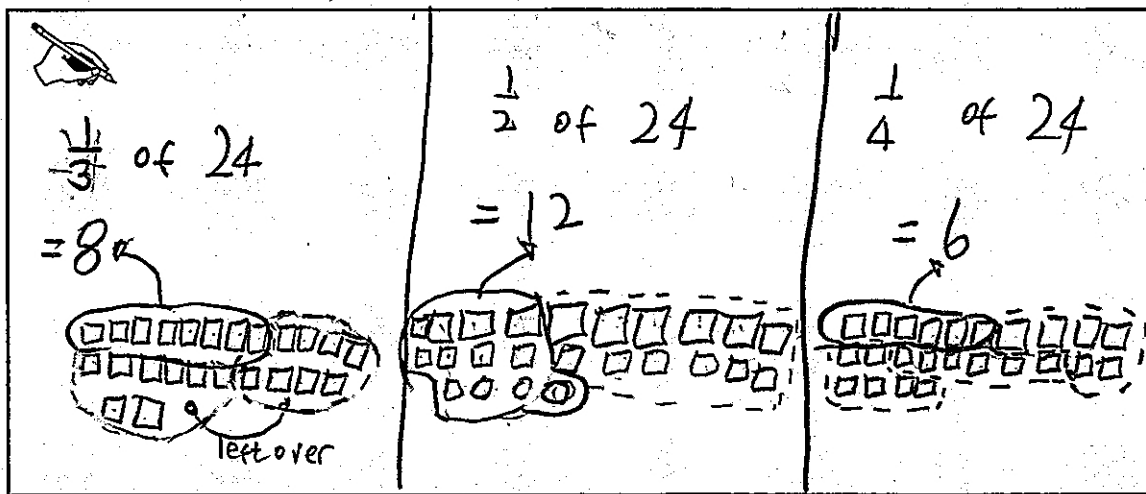
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Use diagrams and equations to explain if it is possible for Maria, Ted and Monica to each have the fraction of the cookies they want. Write about the relationship between the 24 cookies that are available and the fraction of cookies each student wants.



$$8 + 12 + 6 = 26$$

$$26 > 24$$

it is impossible for them to have the fraction of the cookies they want.

Guide 1

Litho 40357

Total Content Points: 1 (4.NF.4c)

Total Practice Points: 5 (MP1, MP2, MP4, MP6, MP7)

The student uses the denominator of the fractions for each student's request to determine the number of equal groups into which 24 cookies will be divided (4.NF.4c). The student explains the share of cookies each student will receive, and recognizes that 26 cookies are necessary and only 24 are available (MP1). The response contains an explanation that makes clear the number of cookies that child wants and that the set of cookies the children want is greater than the whole set of cookies available (MP2). The student models 24 cookies partitioned into equal groups and identifies which group each child wants (MP4). The calculations are clear and precise (MP6) and use multiplication to partition 24 cookies into equal groups based on the size of the denominator (MP7).

Total Awarded Points: 6 of 6

Task 3. 24 Cookies Task

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Ted says, "I want  $\frac{1}{2}$  of the set of 24 cookies."

Monica says, "I want  $\frac{1}{4}$  of the set of 24 cookies."

Use diagrams and equations to explain if it is possible for Maria, Ted and Monica to each have the fraction of the cookies they want. Write about the relationship between the 24 cookies that are available and the fraction of cookies each student wants.

$\frac{1}{3}$ of 24 $\frac{1}{3} \times 24 = 8$ Maria-8	$\frac{1}{2}$ of 24 $\frac{1}{2} \times 24 = 12$ Ted-12	$\frac{1}{4}$ of 24 $\frac{1}{4} \times 24 = 6$ Monica-6
---	---	--

$$\begin{array}{r} 12 \\ + 12 \\ + 6 \\ \hline 26 \end{array}$$

$$26 \neq 24$$

you can't do  
 $26 - 24$

02 over the amount

So that wouldn't work.



Guide 2

Litho 40078

Total Content Points: 1 (4.NF.4c)

Total Practice Points: 4 (MP2, MP4, MP6, MP7)

The student uses the denominator of the fractions for each student's request to determine the number of equal groups into which 24 cookies will be divided (4.NF.4c). The student provides appropriate equations and labels how many cookies each child wants (MP2). The student models 24 cookies partitioned into equal groups using multiplication of the unit fractions (MP4). The calculations are clear and precise (MP6) and use multiplication to partition 24 cookies into equal groups based on the size of the denominator (MP7). The lack of a diagram demonstrates that the student did not persevere in completing all aspects of the task (no credit for MP1).

Total Awarded Points: 5 of 6

**Task 3. 24 Cookies Task**


Three children are sharing 24 cookies.

Maria says, "I want  $\frac{1}{3}$  of the set of 24 cookies."

Ted says, "I want  $\frac{1}{2}$  of the set of 24 cookies."

Monica says, "I want  $\frac{1}{4}$  of the set of 24 cookies."

Use diagrams and equations to explain if it is possible for Maria, Ted and Monica to each have the fraction of the cookies they want. Write about the relationship between the 24 cookies that are available and the fraction of cookies each student wants.

 No, because one of them won't get the amount they want. There are 2 cookies short for  $\frac{1}{3}$ ,  $\frac{1}{2}$ , and  $12\frac{1}{4}$  of the cookies.

$24 \div 3 = 8$	$+ 8$	amount of cookies: 24
$24 \div 2 = 12$	$26$	
$24 \div 4 = 6$		



Guide 3

Litho 40099

Total Content Points: 1 (4.NF.4c)

Total Practice Points: 4 (MP2, MP4, MP6, MP7)

The student uses the denominator of the fractions for each student's request to determine the number of equal groups into which 24 cookies will be divided (4.NF.4c). Using division equations, the student models 24 cookies partitioned into three, two, and four equal groups, respectively (MP4). The student relates the total number of cookies the children want to the number available ("There are 2 cookies short"), and the calculations are correct, clear and precise (MP2, MP6). The student uses multiplication to partition 24 cookies into equal groups based on the size of the denominator (MP7). The lack of a diagram demonstrates that the student did not persevere in completing all aspects of the task (no credit for MP1).

Total Awarded Points: 5 of 6

Task 3. 24 Cookies Task

Three children are sharing 24 cookies.

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Ted says, "I want  $\frac{1}{2}$  of the set of 24 cookies."

Monica says, "I want  $\frac{1}{4}$  of the set of 24 cookies."

Use diagrams and equations to explain if it is possible for Maria, Ted and Monica to each have the fraction of the cookies they want. Write about the relationship between the 24 cookies that are available and the fraction of cookies each student wants.

The student's work is contained within a rectangular box. On the left side, there is a small drawing of a pencil tip. Below it, the word "no" is written, followed by "because". Below that, the fractions  $\frac{1}{3} + \frac{1}{2} + \frac{1}{4}$  are written, with a horizontal line underneath and the result  $\frac{26}{24}$  or  $1\frac{2}{24}$  written below the line. To the right of this, three equations are written vertically:  $\frac{1}{3} = \frac{8}{24}$ ,  $\frac{1}{2} = \frac{12}{24}$ , and  $\frac{1}{4} = \frac{6}{24}$ . On the right side of the box, there is a hand-drawn diagram of a tall, narrow stack of cookies, represented by horizontal lines. The number "24" is written at the top of the stack. On the right side of the stack, three brackets indicate the portions: the top portion is labeled  $\frac{1}{3}$ , the middle portion is labeled  $\frac{1}{2}$ , and the bottom portion is labeled  $\frac{1}{4}$ .

Guide 4

Litho 40227

Total Content Points: 1 (4.NF.4c)

Total Practice Points: 3 (MP2, MP4, MP7)

The student uses the denominator of the fractions for each student's request to determine the number of equal groups into which 24 cookies will be divided and recognizes that 24 cookies will not be sufficient (4.NF.4c). The student explains the share of cookies each student will receive, but lacks precision in explaining the relationship between 24 and 26 (no credit for MP6). The response models 24 cookies partitioned into three, two, and four equal groups, respectively, which demonstrates an understanding of fractions as dividing the whole into equal groups (MP4). Although the student fails to label any total as a number of cookies, or show who will receive each group, she does add the fractions to show the total that the children want is greater than one full set (MP2). The student uses multiplication to partition 24 into equal groups based on the size of the denominator (MP7). The lack of an effective diagram demonstrates that the student did not persevere in completing all aspects of the task (no credit for MP1).

Total Awarded Points: 4 of 6

Task 3. 24 Cookies Task

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Ted says, "I want  $\frac{1}{2}$  of the set of 24 cookies."

Monica says, "I want  $\frac{1}{4}$  of the set of 24 cookies."

Use diagrams and equations to explain if it is possible for Maria, Ted and Monica to each have the fraction of the cookies they want. Write about the relationship between the 24 cookies that are available and the fraction of cookies each student wants.

Handwritten work showing calculations for each child's share:

- For Maria ( $\frac{1}{3}$ ):  $\frac{6}{24}$  (written as  $\frac{6}{24}$  with a horizontal line above the 6 and a vertical line to the right of the 24, indicating 6 groups of 4).
- For Ted ( $\frac{1}{2}$ ):  $\frac{12}{24}$  (written as  $\frac{12}{24}$  with a horizontal line above the 12 and a vertical line to the right of the 24, indicating 12 groups of 2).
- For Monica ( $\frac{1}{4}$ ):  $\frac{6}{24}$  (written as  $\frac{6}{24}$  with a horizontal line above the 6 and a vertical line to the right of the 24, indicating 6 groups of 4).

Handwritten note: "No because it is 2 cookies over."

Guide 5

Litho 40016

Total Content Points: 1 (4.NF.4c)

Total Practice Points: 3 (MP2, MP4, MP7)

The student uses the denominator of the fractions that each student wants to determine the number of equal groups into which 24 cookies will be divided (4.NF.4c) and models 24 cookies partitioned into three, two, and four equal groups, respectively (MP4). The student uses equations to find a common denominator and show that the sum of the desired fractions is greater than one whole (MP2). The student uses multiplication to partition 24 into equal groups based on the size of the denominator (MP7), but the explanation provided does not clearly identify 26 as the number of cookies that are required and that only 24 are available (no credit for MP6). The lack of a diagram demonstrates that the student did not persevere in completing all aspects of the task (no credit for MP1).

Total Awarded Points: 4 of 6

Task 3. 24 Cookies Task

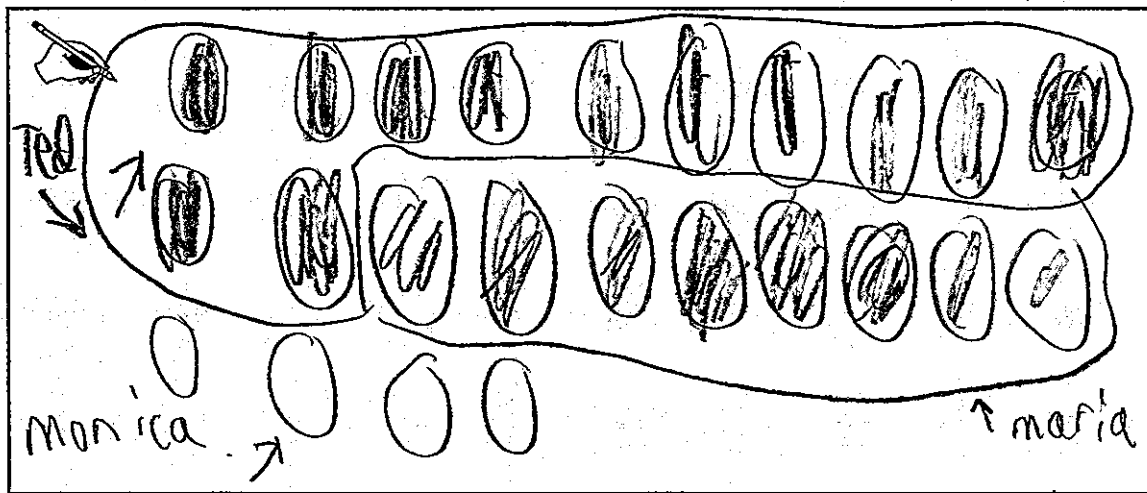
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Use diagrams and equations to explain if it is possible for Maria, Ted and Monica to each have the fraction of the cookies they want. Write about the relationship between the 24 cookies that are available and the fraction of cookies each student wants.



They can't because monica needs six but she only got 4. Ted wants  $\frac{1}{2}$  of the cookies, that's 12. Monica wants  $\frac{1}{4}$ , that's 6. Maria wants  $\frac{1}{3}$ , that's 8.





Guide 6

Litho 40248

Total Content Points: 1 (4.NF.4c)

Total Practice Points: 2 (MP4, MP7)

The student uses the denominator of the fractions for each student's request to determine the number of equal groups into which 24 cookies will be divided (4.NF.4c). The student has drawn a diagram that divides the available cookies into groups, showing that 24 cookies is not sufficient (MP4), but fails to include equations (no credit for MP1, no credit for MP2). The student's explanation shows that he has used multiplication to partition 24 into equal groups based on the size of the denominator (MP7), but does not clearly state that 26 cookies are needed and only 24 are available (no credit for MP6).

Total Awarded Points: 3 of 6

**Task 3. 24 Cookies Task**


Three children are sharing 24 cookies.

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Monica says, "I want  $\frac{1}{4}$  of the set of 24 cookies."

Use diagrams and equations to explain if it is possible for Maria, Ted and Monica to each have the fraction of the cookies they want. Write about the relationship between the 24 cookies that are available and the fraction of cookies each student wants.

	$\frac{1}{3}$ or	$\frac{4}{12}$	<i>about</i>	<i>about</i>	
	$\frac{1}{2}$ or	$\frac{6}{12}$			
	$\frac{1}{4}$ or	$\frac{3}{12}$			
		$\frac{13}{12}$			← totals

It isn't possible because  
 $\frac{1}{2}$  (or  $\frac{6}{12}$ ) +  $\frac{1}{3}$  (or  $\frac{4}{12}$ ) +  $\frac{1}{4}$  (or  $\frac{3}{12}$ )  
 $= \frac{13}{12}$  (or 25 cookies)  
 One more cookie is needed  
 so everybody can have  
 what they want.

Guide 7

Litho 40369

Total Content Points: 1 (4.NF.3a)

Total Practice Points: 2 (MP4, MP7)

The student finds common denominators for the given fractions, adds them to show that their sum is greater than one whole, and indicates that she understands that this means that the children want more cookies than are available (4.NF.3a). While the student uses multiplication to partition a whole number into equal groups based on the size of the denominator (MP7), there is an error when she attempts to calculate the number of cookies needed (no credit for MP6). The student's inability to correctly relate the number of cookies desired by the children to the set of 24 cookies demonstrates a lack of making sense of the problem, and an inability to correctly contextualize the data (no credit for MP1, no credit for MP2). The student uses a chart to model an understanding of equal groups (MP4).

Total Awarded Points: 3 of 6

**Task 3. 24 Cookies Task**

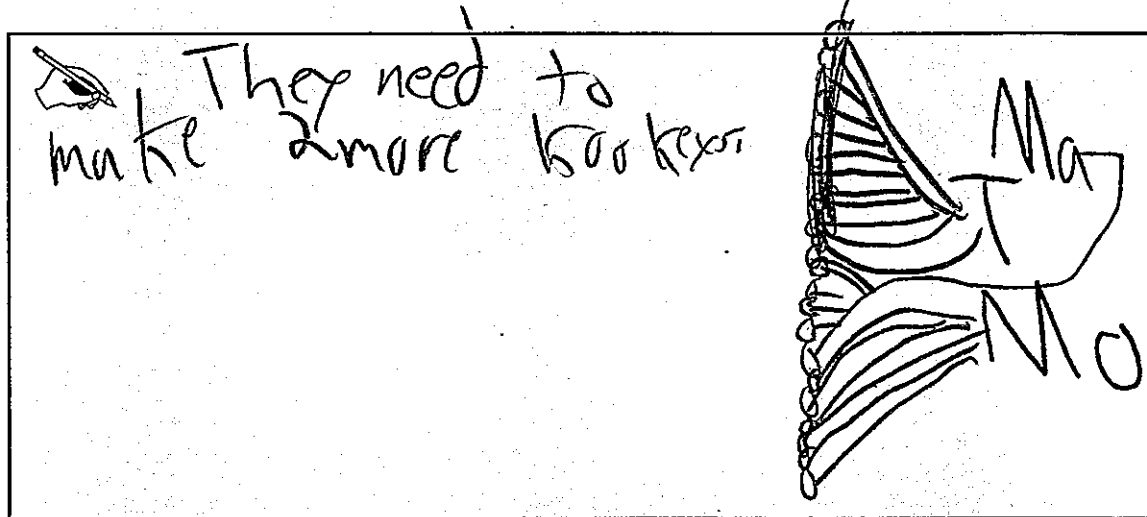
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Use diagrams and equations to explain if it is possible for Maria, Ted and Monica to each have the fraction of the cookies they want. Write about the relationship between the 24 cookies that are available and the fraction of cookies each student wants.



Guide 8

Litho 40093

Total Content Points: 1 (4.NF.4c)

Total Practice Points: 1 (MP4)

This student uses the denominator of the fractions for each student's request to determine the number of equal groups into which 24 cookies will be divided (4.NF.4c). The student models 24 cookies partitioned into equal groups, indicating that the three children will not all receive the number of cookies that they want (MP4). The student does not provide the equations necessary to determine each student's share of cookies (no credit for MP2), which demonstrates a failure to attend to all aspects of the task (no credit for MP1) and a lack of precision and clarity (no credit for MP6). There is not enough work shown to indicate that the student understands the meaning of multiplication of a whole number by a fraction (no credit for MP7).

Total Awarded Points: 2 of 6

### Task 3. 24 Cookies Task


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Monica says, "I want  $\frac{1}{4}$  of the set of 24 cookies."

Use diagrams and equations to explain if it is possible for Maria, Ted and Monica to each have the fraction of the cookies they want. Write about the relationship between the 24 cookies that are available and the fraction of cookies each student wants.


$$\frac{1}{3} = \frac{8}{24}$$
$$\frac{1}{2} = \frac{12}{24}$$
$$\frac{1}{4} = \frac{6}{24}$$

There is not enough cookies for Maria, Ted, and Monica.

Guide 9

Litho 40364

Total Content Points: 1 (4.NF.4c)

Total Practice Points: 1 (MP7)

The student uses the denominator of the fractions for each student's request to determine the number of equal groups into which 24 cookies will be divided (4.NF.4c) through multiplication of a whole number by a fraction (MP7). The student does not adequately label who will receive each group of cookies (no credit for MP2). The lack of a diagram or addition of the given fractions to form a total means that the attempted model does not demonstrate an understanding of dividing the original set of cookies into equal groups (no credit for MP4), and the lack of a diagram and equations explaining the relationship between 24 and 26 demonstrates a lack of precision (no credit for MP6) and a failure to attend to all aspects of the task (no credit for MP1).

Total Awarded Points: 2 of 6

### Task 3. 24 Cookies Task


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Monica says, "I want  $\frac{1}{4}$  of the set of 24 cookies."

Use diagrams and equations to explain if it is possible for Maria, Ted and Monica to each have the fraction of the cookies they want. Write about the relationship between the 24 cookies that are available and the fraction of cookies each student wants.

 No because there is not enough.  $\frac{1}{3} + \frac{1}{2} + \frac{1}{4} = 1\frac{1}{4}$  it is not enough.



Guide 10

Litho 40109

Total Content Points: 1 (4.NF.3a)

Total Practice Points: 1 (MP4)

The student uses correct mathematical reasoning to show that the sum of the fractions representing the cookies the children want is greater than one whole (4.NF.3a). Although the student has provided a correct addition equation that demonstrates his reasoning (MP4), he does not contextualize the fractions shown, or compare the sum to one whole set of 24 cookies (no credit for MP2). The work shown is correct, but lacking context, the student's explanation is not precise (no credit for MP6). The student does not show a diagram, demonstrating an inability to make sense of the problem and persevere in solving it (no credit for MP1). Although the equation given demonstrates that the total fraction of cookies desired is greater than the set of available cookies, the student fails to show an understanding of the meaning of multiplication of a whole number by a fraction (no credit for MP7).

Total Awarded Points: 2 of 6

Task 3. 24 Cookies Task

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Monica says, "I want  $\frac{1}{4}$  of the set of 24 cookies."

Use diagrams and equations to explain if it is possible for Maria, Ted and Monica to each have the fraction of the cookies they want. Write about the relationship between the 24 cookies that are available and the fraction of cookies each student wants.

No, it's not possible for each of them to have the fraction of the cookies that they want. There is enough for two of them to

have the amount of cookies they want, but not enough for all three.

Guide 11

Litho 40296

Total Content Points: 1 (4.NF.4c)

Total Practice Points: 0

The student uses the denominator of the fractions for each student's request to determine the number of equal groups into which 24 cookies will be divided (4.NF.4c). Although the student attempts to use a diagram to respond to the task, it lacks equations, and does not model the number of cookies Maria wants (no credit for MP4). The student does not use equations to find the number of cookies each child wants or to demonstrate that the sum of the given fractions is greater than one whole (no credit for MP1, no credit for MP2). The work shown is insufficient to indicate that the student recognizes the meaning of multiplication of a whole number by a fraction (no credit for MP7). The response does not show the number of cookies that Maria wants, or the total number or fraction desired by the three children together (no credit for MP6).

Total Awarded Points: 1 of 6

Task 3. 24 Cookies Task


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Ted says, "I want  $\frac{1}{2}$  of the set of 24 cookies."

Monica says, "I want  $\frac{1}{4}$  of the set of 24 cookies."

Use diagrams and equations to explain if it is possible for Maria, Ted and Monica to each have the fraction of the cookies they want. Write about the relationship between the 24 cookies that are available and the fraction of cookies each student wants.

 No they can't have the fraction they wanted because they would have to have more than 24 cookies

Total Content Points: 0

Total Practice Points: 0

The student does not use the denominator of the fractions for each student's request to determine the number of equal groups into which 24 cookies will be divided (no credit for 4.NF.4c), and no equations or diagrams addressing the task are provided (no credit for MP1, no credit for MP2, no credit for MP4). The student answers correctly that there will not be enough cookies to fulfill each student's request, but fails to identify each student's share or the total number of cookies needed (no credit for MP6), or to use multiplication of a whole number by a fraction (no credit for MP7).

Total Awarded Points: 0 of 6